

AC 2009-2128: CIRCUIT ANALYSIS DON'TS: INSTILLING ERROR-AVOIDANCE SKILLS WHILE TEACHING PROPER TECHNIQUES

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Circuit Analysis Don'ts: Instilling Error-Avoidance Skills While Teaching Proper Techniques

Abstract

Circuit analysis is a discipline that is fraught with pitfalls for the beginning student. Common errors include misidentification of element configurations, unsuccessful efforts to create a simplified equivalent circuit, incorrect application of the source conversion principle, and omission or incorrect formulation of terms when writing a mesh or nodal equation to characterize circuit operation. This paper describes six circuit analysis errors that students frequently make, it suggests some proactive strategies that can be used to help students avoid these errors, and it describes the assessment techniques that have been used to evaluate the effectiveness of these strategies.

Introduction

Implicit in this discussion is the belief that students are better prepared to solve problems (e.g., to analyze circuits) when warnings about common errors are interwoven into instruction about proper techniques. The essence of this approach is to tell students to "do it this way, but be careful not to do it that way." Warnings about common errors serve to bring proper techniques into clearer focus, which better defines the correct approach.

Difficulty Identifying Series and Parallel Elements in a Series-Parallel Circuit

One of the most fundamental circuit concepts that students struggle to master is the identification of series and/or parallel circuit elements in series-parallel networks. Students often treat components that are graphically parallel as if they are electrically parallel, and they frequently consider any two elements that share a single common junction to be in series. Some students also have the mindset that components that are not in series must be in parallel, and vice versa.

Proactive Strategy

Present precise, but concise definitions of series and parallel elements in terms of both connections and circuit parameters:

Series elements that are adjacent to one another exclusively share a common junction; all series elements share a common current.

Parallel elements span two common junctions; parallel elements share a common voltage.

Emphasize that the identification of series and/or parallel element configurations is a critical skill in circuit analysis. Reinforce study in this area by giving students extra practice with series-parallel resistor combination problems.

Assessment

One course the author has taught on multiple occasions is the second of a three course sequence, which builds on the introduction to series-parallel circuit analysis provided by its predecessor. After observing many of the entering students struggle with concepts they were supposed to have mastered in the previous course, the author instituted a review quiz at the start of the second week of class to assess student competency in fundamental skills and provide students with an incentive to review foundational material.

When the review quiz was first administered in Fall 2005, the performance of students on a four-element series-parallel resistor combination problem was appalling: 70% of the students misidentified at least one resistor configuration as being parallel when it should have been identified as series, or vice versa. The proactive strategy previously described was hastily developed in response to this finding; this strategy has been utilized during the first week of class ever since. Figure 1 records the incidence of misidentified series or parallel elements on similar quiz problems given over the course of four successive semesters.

	Fall 2005	REVISIONS	Spring 2006	Fall 2006	Spring 2007
Students who misidentified element configuration(s)	23		4	11	13
Number of students taking the quiz	33	32	32	32	
Percentage of students making this error	70	13	34	41	

Figure 1: Misidentified Series and/or Parallel Elements in a Series-Parallel Circuit

Note that the incidence of misidentified elements was significantly lower after the proactive strategy was implemented. Room for improvement still exists, but the time available for remediation is limited by the timing of the quiz, and little can be done to help students who miss part or all of the first week of class. Only a small number of students have been observed to experience lingering difficulties with configuration identification since the proactive strategy was adopted.

Terminals Lost while Reducing a Circuit

Figure 2 illustrates the approach that some students take to simplify a circuit using component reduction techniques.

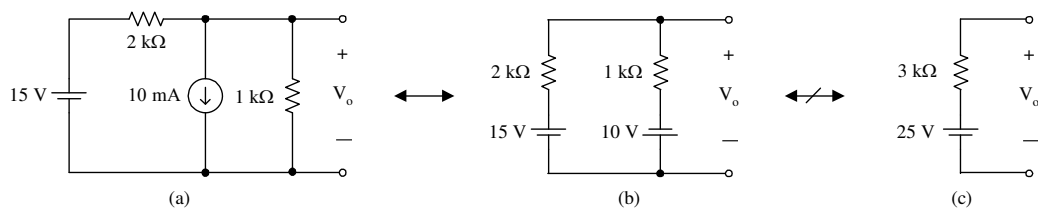


Figure 2: Loss of Terminals during Circuit Reduction

First, the students observe that the parallel combination of 10 mA and 1 k Ω in Figure 2a can be treated as a practical current source. Noting that $(10 \text{ mA})(1 \text{ k}\Omega) = 10 \text{ V}$, the students perform a source conversion^{1,8} to create Figure 2b. The students then proceed to combine the resistors and voltage

sources in series to produce Figure 2c. Although the latter step does provide a legitimate approach for computing the current flowing in Figure 2b, it is not a valid means of reducing the original circuit, since combining the respective elements in series causes the terminals to become lost. Despite this fact, it is common for students to present the circuit shown in Figure 2c (complete with bogus terminals) as the culmination of their efforts to reduce Figure 2a.

Figure 2a is replicated in Figure 3a. The correct approach to reducing this circuit is to treat the series combination of 15 V and 2 kΩ as a practical voltage source, and to use the fact that $15V/2k\Omega = 7.5 \text{ mA}$ to accomplish the source conversion illustrated in Figure 3b. The parallel sources and parallel resistors can then be combined as shown in Figure 3c.

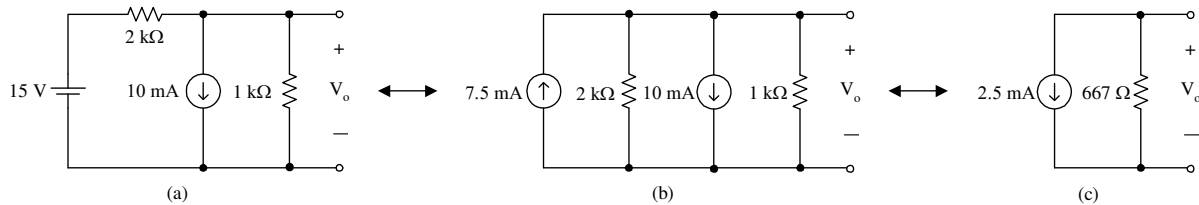


Figure 3: Retention of Terminals during Circuit Reduction

Proactive Strategy

Repeatedly emphasize that terminals are an important part of a circuit, and that, in general, terminals must be retained during circuit reduction efforts. Work an example such as the one in this section to demonstrate the ease with which terminals can be "lost" through careless circuit reduction. Provide homework problems with terminals that enable students to practice careful circuit reduction techniques.

Assessment

Each semester, a quiz problem is used to assess student competency in performing source conversions while retaining terminals. The problems usually contain both a practical voltage source and a practical current source, but the element positions, orientations, and values vary. Seven semesters of these quiz problems have been analyzed to determine the number of students who lost one or both terminals in their effort to solve the problem. The results from this investigation appear in Figure 4.

	F05	Sp06	F06	REVISIONS	Sp07	F07	Sp08	F08
Students who lost terminals during a source conversion	10	8	11			5	5	7
Number of students taking the quiz	31	31	32		33	35	40	30
Percentage of students making this error	32	26	34		15	14	18	17
	Mean = 31				Mean = 16			

Figure 4: Terminals Lost while Performing a Source Conversion

Note that difficulties with lost terminals plagued about 31% of the students before the strategy was implemented, but only about 16% of the students lost terminals after the strategy was put in place.

Incorrect Application of Source Conversions

A more subtle mistake occurs in a circuit such as Figure 5a if students are asked to compute the current (or power) associated with the 390 Ω resistor. Recognizing that this circuit is but a source conversion away from a series circuit, the students convert the practical current source into the practical voltage source in Figure 5b by noting that $(12 \text{ mA})(390 \Omega) = 4.68 \text{ V}$.

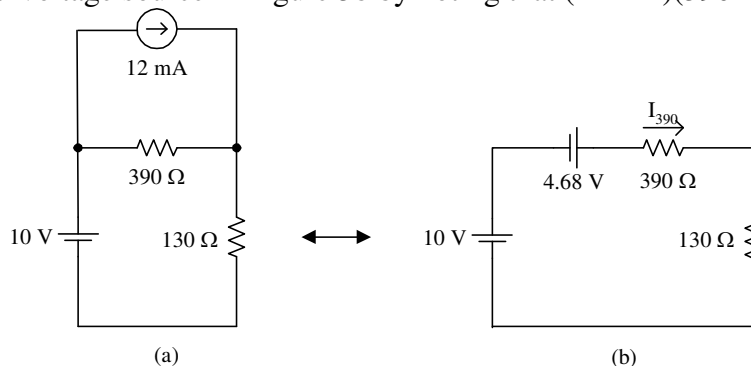


Figure 5: Incorrect Application of a Source Conversion

The students then proceed to correctly determine the current in Figure 5b using

$$I_{390} = \frac{10 \text{ V} + 4.68 \text{ V}}{390 \Omega + 130 \Omega} = 28.2 \text{ mA.}$$

What the students fail to realize about their solution is that the current flowing in the 390 Ω in Figure 5b is not the same as the current flowing in the 390 Ω in Figure 5a. The equivalence provided by a source conversion exists only at its terminals, and the 390 Ω resistor in Figure 5b is an internal element of the converted source.

One means to obtain the correct solution to this problem is to apply the superposition theorem^{2,9}. Although the solution can proceed directly from Figure 5a (replicated in Figure 6a), Figures 6b and 6c have been provided in the interest of clarity. Application of the current division principle^{3,10} to Figure 6b and series circuit analysis to Figure 6c yields

$$I_{390} = I_{390A} + I_{390B} = -\left(\frac{130 \Omega}{390 \Omega + 130 \Omega}\right)(12 \text{ mA}) + \frac{10 \text{ V}}{390 \Omega + 130 \Omega} = 16.2 \text{ mA,}$$

a result that differs significantly from the previous (erroneous) answer of 28.2 mA.

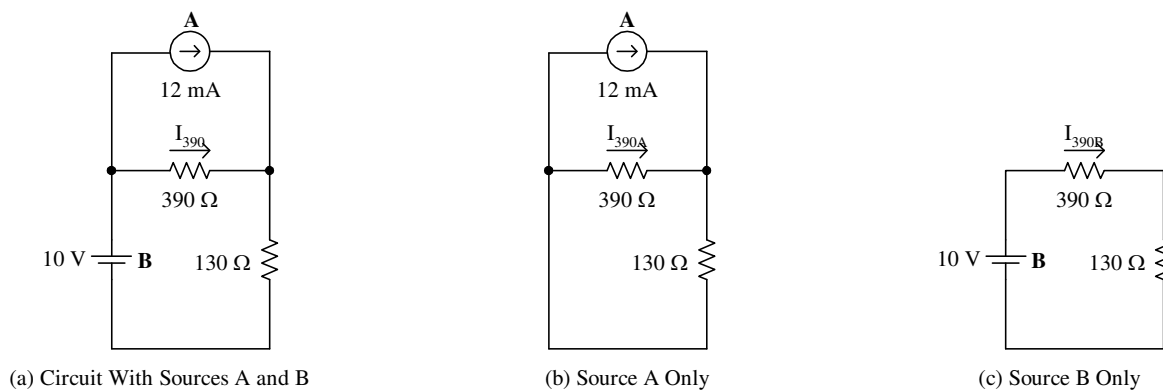


Figure 6: Application of Superposition to Find the Current in the 390 Ω

Proactive Strategy

Demonstrate the difference in the internal behavior of practical voltage and current sources when the source conversion technique is introduced. Work an example such as solving for the current in the $390\ \Omega$ resistors of Figure 5a and Figure 5b to show that the answers are different, and then apply superposition or mesh analysis to Figure 5a to demonstrate which solution is correct. Repeatedly emphasize that the equivalency between converted sources only exists at their terminals. Assign homework problems that tempt students to misapply source conversions.

Reflection

A quiz problem to evaluate whether a student is likely to misapply source conversion was added in Fall 2008, but no such problem was in use prior to implementation of the current strategy. The motivation for addressing this problem came not from student difficulties on a specific type of problem, but from an observed pattern of misapplied source conversions when students applied the technique to various problems throughout the course. Thus, there is no basis of comparison for making a quantitative assessment as to the effectiveness of the strategy in use. The author's qualitative assessment, based on student work that was graded during Fall 2008, is that the use of the strategy thus reported has successfully reduced the number of misapplied source conversions.

Terms Omitted while Writing Mesh or Nodal Equations

Two practices that appear to greatly increase the likelihood that students will omit terms when performing mesh or nodal analysis are using a "shortcut approach" to write the equations, and writing the terms "out of order." In the author's experience, students who use a shortcut approach are considerably more likely to omit terms than students who use the traditional technique, and the incidence of "term omission errors" associated with a shortcut method seems to increase as the size of the circuit increases. Equations written piece-meal with terms "out of order" exhibit fewer omissions, but they still constitute an avoidable source of errors.

Application of the traditional mesh analysis technique⁴ to Mesh 2 of Figure 7 yields:

$$\text{Mesh 2: } R_2(I_2 - I_1) + R_4 I_2 + R_6(I_2 - I_5) + R_5(I_2 - I_4) + V_a = 0$$

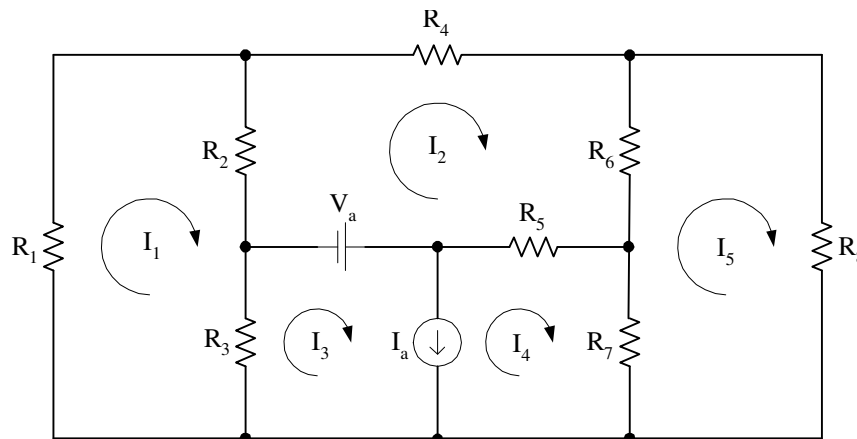


Figure 7: Mesh Analysis Demonstration Circuit

Application of the shortcut method in the same location yields:

$$\underline{\text{Mesh 2:}} \quad -R_2 I_1 + (R_2 + R_4 + R_5 + R_6) I_2 - R_5 I_4 - R_6 I_5 = -V_a$$

Considering that the traditional approach to mesh analysis produces one term per component, and that terms appear in the same order as their respective circuit elements in the mesh, the traditional approach is easier to check and is therefore less error-prone than the shortcut method.

Application of the traditional nodal analysis technique⁵ to Node 3 of Figure 8 yields:

$$\underline{\text{Node 3:}} \quad \frac{V_3 - V_2}{R_3} - I_a + \frac{V_3 - V_4}{R_6} + \frac{V_3}{R_4} = 0$$

Application of the shortcut approach in the same location yields:

$$\underline{\text{Node 3:}} \quad -\frac{V_2}{R_3} + V_3 \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_6} \right) - \frac{V_4}{R_6} = I_a$$

As with mesh analysis, the traditional one-term-per-component nodal analysis technique is easier to check, and is therefore less error-prone than the shortcut method.

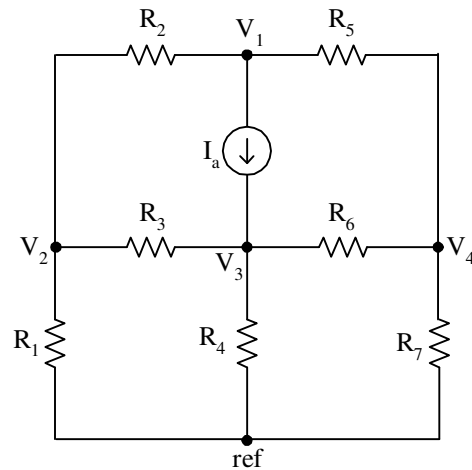


Figure 8: Nodal Analysis Demonstration Circuit

Proactive Strategy

Urge students to avoid using shortcut methods for mesh and nodal analysis. Convince students that the slight time savings offered by these methods is overshadowed by the error-avoidance attributes of the traditional methods. Encourage students to write mesh analysis terms in the order the elements are encountered during a unidirectional traversal of the mesh; encourage students to write nodal analysis terms in the order that branches are encountered during a unidirectional traversal about the node.

Mesh Equations Written Across Current Sources

Students frequently fail to recognize the need to define a supermesh⁶ in circuits having current sources in their internal branches. A representative equation, written for Mesh 2 of Figure 9a is

$$\text{Mesh 2: } (1.1\text{k}\Omega)I_2 + (2.7\text{k}\Omega)(I_2 - I_1) + 1.2\text{ A} = 0$$

This equation is, of course, invalid, since a basic mesh equation cannot be written across a current source. The proper approach for applying mesh analysis to this circuit is to define a supermesh as indicated in Figure 9b, and to write:

$$\text{Supermesh 2/3: } (1.1\text{k}\Omega)I_2 + (2.7\text{k}\Omega)(I_2 - I_1) + (3.3\text{k}\Omega)(I_3 - I_1) - 10\text{ V} = 0$$

and:

$$\text{Supermesh 2/3 Supplemental: } I_2 - I_3 = 1.2\text{ A}$$

A related common error occurs when students recognize the need to write a supermesh equation in Supermesh 2/3, but also write separate (invalid) equations for Meshes 2 and 3 in the erroneous manner exemplified above.

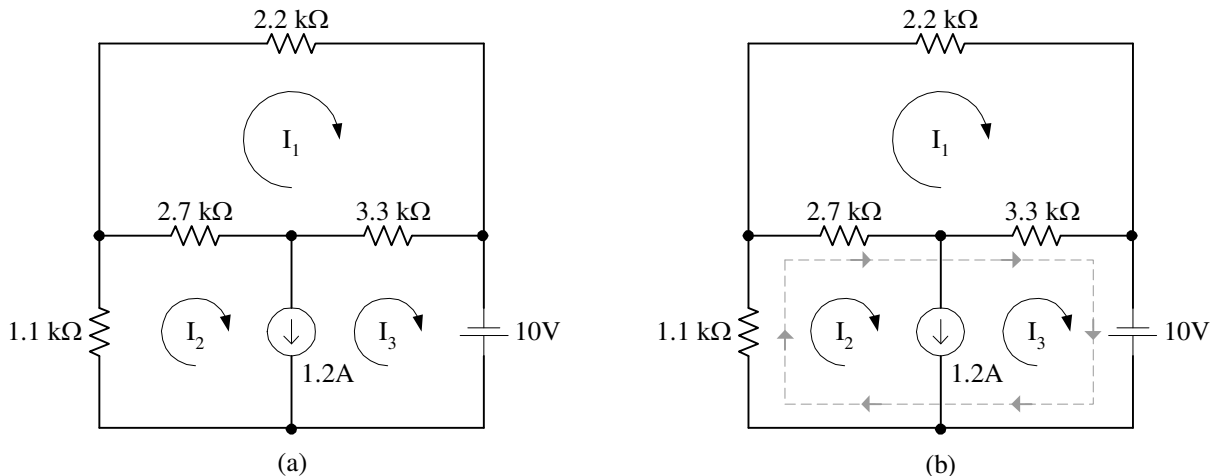


Figure 9: Incorrect and Correct Approaches for Dealing with Current Sources in Mesh Analysis

Proactive Strategy

Remind students that mesh analysis is really an enhanced version of Kirchhoff's Voltage Law. Note that since each equation represents the sum of all the voltages in its respective mesh, each of its terms must either be a voltage or an "IR" term. Distribute the flowchart in Figure 10 to integrate the three sub-methods of mesh analysis into a coherent whole. Encourage students to develop a "sense" that writing a mesh equation across a current source is invalid. Assign homework problems that include current sources in inner branches to force usage of the supermesh approach.

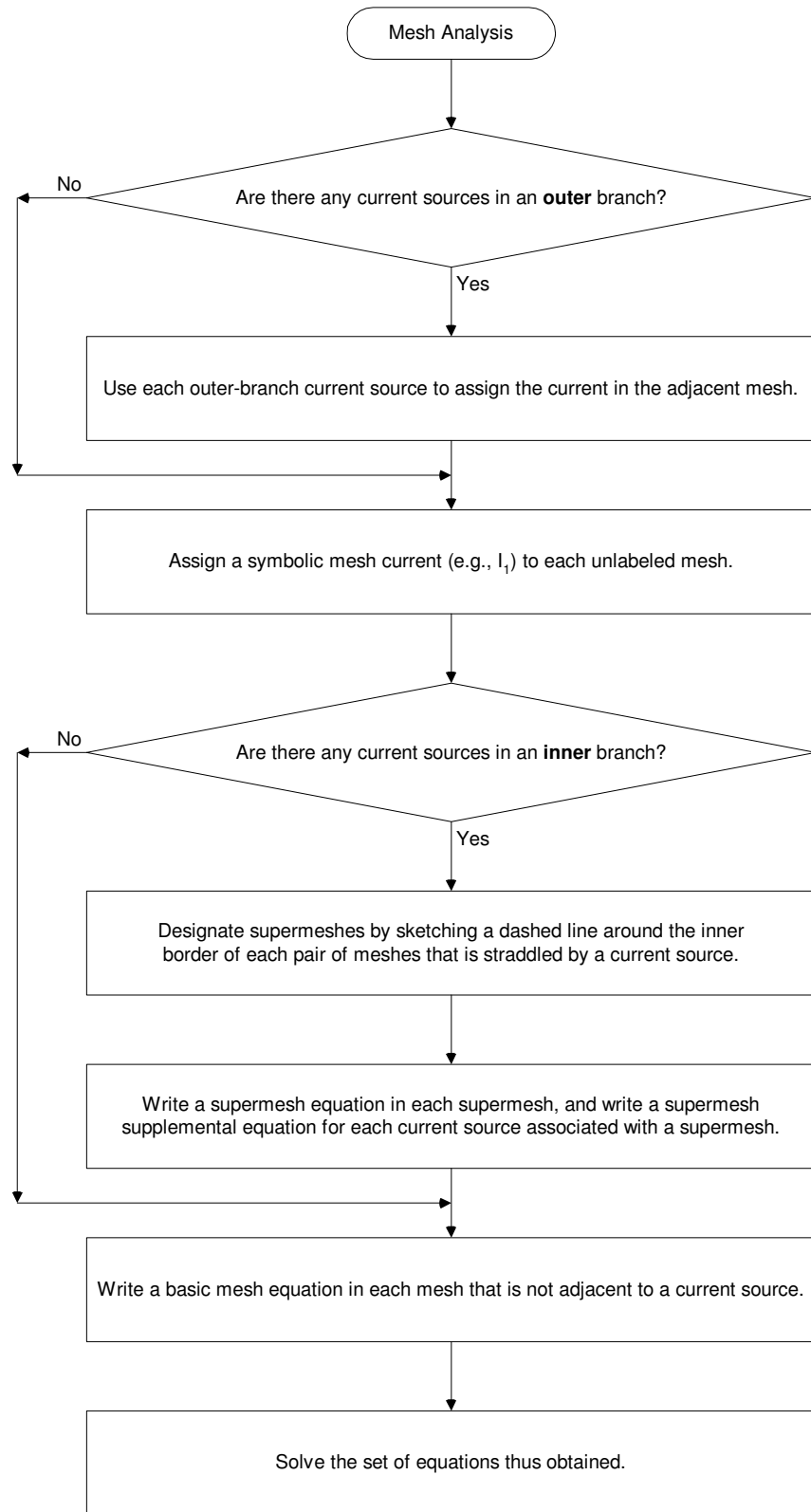


Figure 10: Mesh Analysis Flowchart

Assessment

The flowchart in Figure 10 was developed during the spring semester of 2008. Mesh analysis problems on quizzes and final exams have been analyzed to determine the number of students who attempted to write a mesh equation across a current source. Quizzes and final exams were also examined to determine the number of students who wrote an equation in a mesh containing an outer-branch current source; the students should have recognized that the current in such a mesh is determined by the magnitude and direction of the outer-branch current source, and that a valid equation cannot be written in this mesh. Figure 11 and Figure 12 present the findings for the fall semesters before and after the mesh analysis flowchart was introduced.

		Fall 2007	REVISIONS	Fall 2008
Quiz	Students writing a mesh equation across a current source	11		5
	Number of students taking the quiz	32		30
	Percentage of students making this error	34		17
Final	Students writing a mesh equation across a current source	6		2
	Number of students taking the exam	29		24
	Percentage of students making this error	21		8

Figure 11: Mesh Equations Written Across a Current Source

Note that Figure 11 provides a before-and-after comparison of the percentage of students who wrote mesh equations across current sources; a decreased incidence of this error can be seen when comparing the quiz and final exam in 2008 to their counterparts in 2007. It is also worth noting that the improvements achieved on the final exam exceeded those observed on the quiz.

		Fall 2007 (Quiz)	REVISIONS	Fall 2008 (Quiz)	Fall 2008 (Final)
Students writing an equation in a mesh having a "known" current		13			13
Number of students taking the quiz		32		27	24
Percentage of students making this error		41		48	8

Figure 12: Equations in Meshes Having Current Set by an Outer Branch Source

Figure 12 shows that the incidence of students writing equations in meshes having an outer-branch current source actually increased slightly from 2007 to 2008. This finding is believed to be due to the fact that the applicable problem was on Quiz 2 in 2007 and on Quiz 1 in 2008; students in 2008 had less time to process the information on the flowchart and become acclimated to the course before taking the quiz. Also note that the students made considerably fewer of the targeted errors on the 2008 final exam compared to the incidence of these errors on either quiz.

Nodal Equations Written Through Voltage Sources

An analogous error occurs when students attempt to write a nodal equation through a voltage source, as is exemplified by the following equation written for Figure 13a:

$$\text{Node 1: } \frac{V_1}{1.1\text{k}\Omega} + \frac{V_1 - (-10\text{V})}{2.2\text{k}\Omega} - 5.0\text{V} = 0$$

This equation is invalid, since a basic nodal equation cannot be written through a voltage source. The proper method for applying nodal analysis to this circuit is to define a supernode⁷ as indicated in Figure 13b, and to write:

$$\text{Supernode 1/2: } \frac{V_1}{1.1\text{k}\Omega} + \frac{V_1 - (-10\text{V})}{2.2\text{k}\Omega} + \frac{V_2 - (-10\text{V})}{3.3\text{k}\Omega} + 1.2\text{A} = 0$$

and:

$$\text{Supernode 1/2 Supplemental: } V_2 - V_1 = 5.0\text{V}$$

A related common error occurs when students recognize the need to write a supernode equation at Supernode 2/3, but also write separate (invalid) equations at Nodes 2 and 3 in the erroneous manner exemplified above.

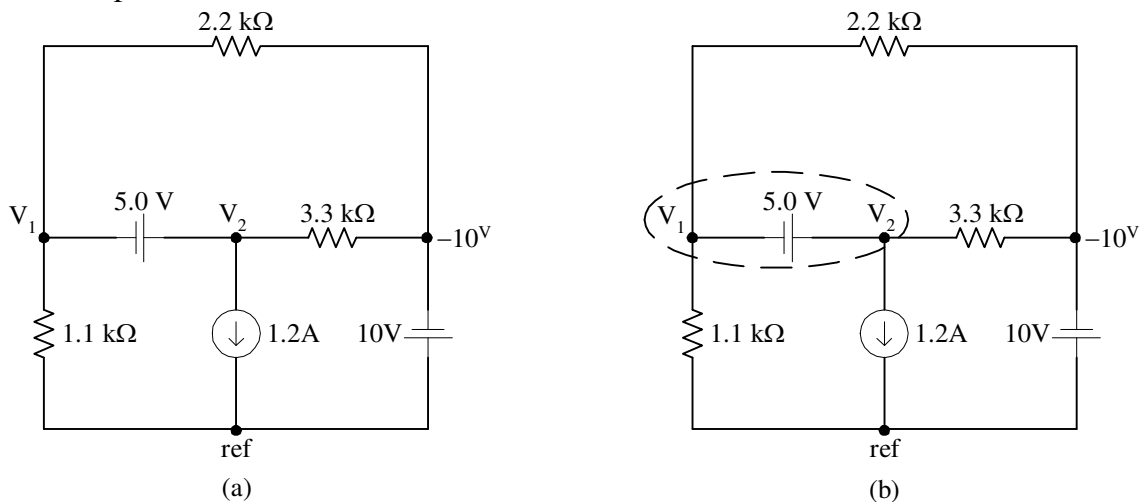


Figure 13: Incorrect and Correct Approaches for Dealing with Current Sources in Nodal Analysis

Proactive Strategy

Remind students that nodal analysis is really an enhanced version of Kirchhoff's Current Law. Note that since each equation represents the sum of the currents leaving its respective node, each of its terms must either be a current or a "V over R" term. Distribute the flowchart in Figure 14 to integrate the three sub-methods of nodal analysis into a coherent whole. Encourage students to develop a "sense" that writing a nodal equation through a voltage source is invalid. Assign homework problems that include multiple unconnected voltage sources to force usage of the supernode approach.

Assessment

Like its mesh analysis counterpart, the flowchart in Figure 14 was developed during the spring semester of 2008. Nodal analysis problems on quizzes given before and after its introduction have been analyzed to determine the number of students who attempted to write a nodal equation through a voltage source. These quizzes were also examined to determine the number of

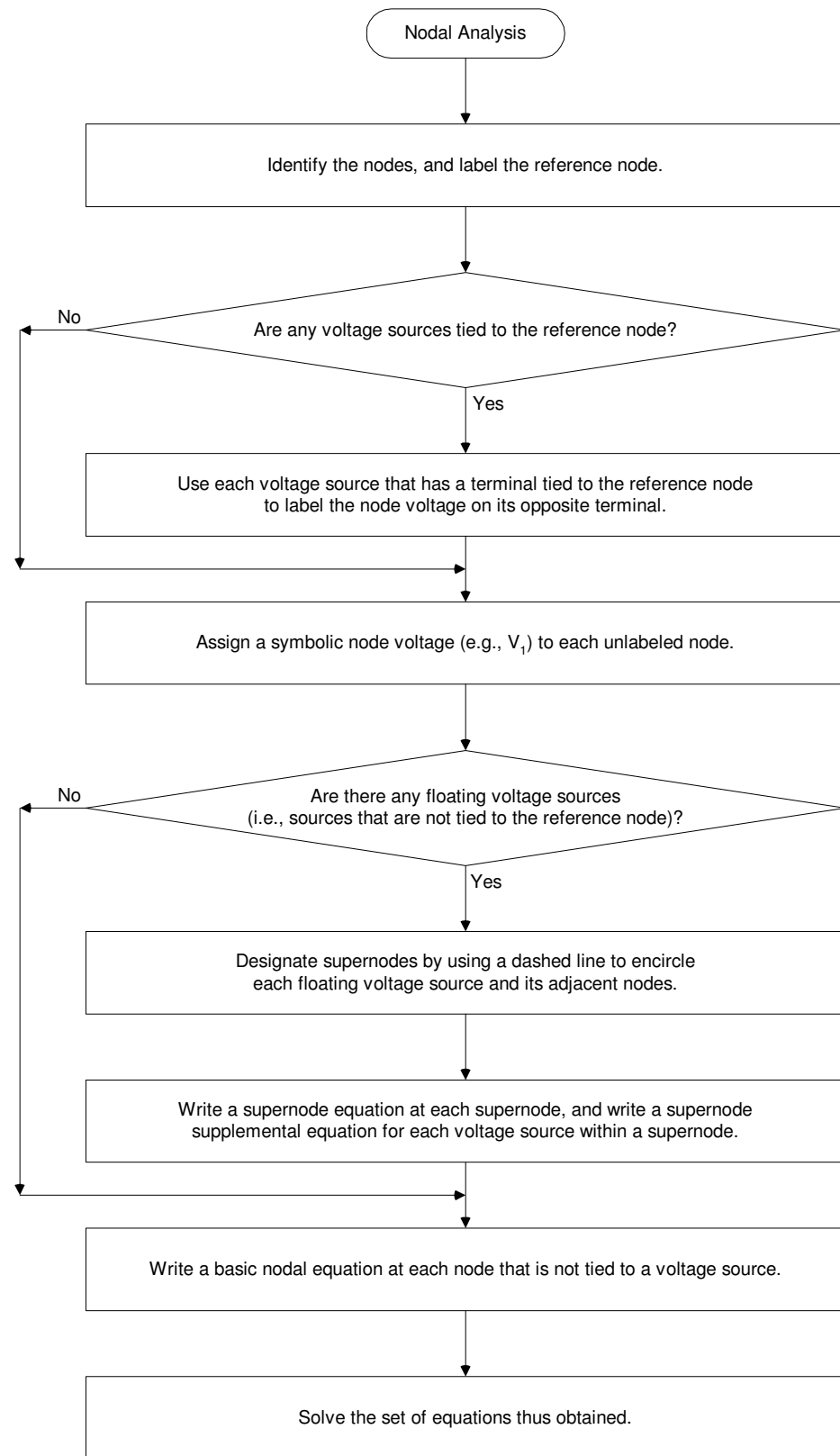


Figure 14: Nodal Analysis Flowchart

students who wrote an equation at a node containing a reference-node-connected voltage source. Given a source in this configuration, the students should have recognized that the source sets the voltage of the node opposite the reference node, and no equation can be written at this "opposite" node. Figure 15 and Figure 16 present the findings for the fall semesters before and after the nodal analysis flowchart was introduced.

	Fall 2007 (Quiz Problem)	REVISIONS	Fall 2008 (Quiz Problem)
Students writing a nodal equation through a voltage source	12		5
Number of students taking the quiz/exam	32		27
Percentage of students making this error	38		19

Figure 15: Nodal Equations Written Through a Voltage Source

Note from Figure 15 that in the semester immediately following the introduction of the flowchart, about half as many students attempted to write an equation through a voltage source as did during the semester before the flowchart was introduced.

	Fall 2007 (Quiz)	REVISIONS	Fall 2008 (Quiz)
Students writing an equation in node having a "known" voltage	13		3
Number of students taking the quiz	30		27
Percentage of students making this error	43		11

Figure 16: Equations at Nodes Having Voltage Set by a Reference-Node-Connected Source

As seen in Figure 16, a comparison of quizzes from the Fall 2007 and Fall 2008 semesters reveals a fourfold reduction in the number of students who attempted to write an equation at the opposite end of a reference-node-connected voltage source.

Conclusion

Most of the proactive strategies that appear in the preceding sections were developed in response to student errors that have been repeatedly observed while teaching circuit analysis or electronics. Although a rigorous evaluation of these strategies has not been conducted, the evidence accumulated thus far strongly suggests that the strategies are effective. Quizzes and final exams given before and after the implementation of the strategies have been examined to determine the incidence of targeted errors. In almost every case, the incidence of targeted errors fell by a factor of two or more after the adoption of a particular strategy.

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