

## **Classroom Models for Illustrating Dynamics Principles Part I. - Particle Kinematics and Kinetics**

**Michael A. Magill, Ph.D., P.E.  
Purdue University**

### **ABSTRACT**

This paper is part I in a two part series that describes a collection of ten classroom models used to illustrate basic Dynamics principles. The models discussed in part I of the series cover the topics of Particle Kinematics and Kinetics while part II covers Rigid Body Kinematics and Kinetics. These models are excellent tools for communicating basic Engineering Mechanics concepts while also stimulating interest and enthusiasm. These devices were developed for undergraduate engineering technology students but they are equally valuable for engineering students. Most of these models are inexpensive or can be constructed easily.

### **INTRODUCTION**

Dynamics is one of the more difficult courses that engineering and engineering technology students encounter during their undergraduate study. As a result, mechanics instructors are trying continually to find or develop techniques that enhance student learning. One of the greatest challenges is creating student interest and enthusiasm. It is well known that students learn more and work harder when they are interested in a topic. A good technique for breaking the monotony of classroom lectures and creating student interest is to introduce exciting classroom models. These models teach basic mechanics principles but more importantly they get students involved, stimulate interest and give a change of pace. The time required to properly present a model is roughly the same as presenting an example problem.

### **THE CLASSROOM MODELS**

The models discussed in this paper (Part I) cover the topics of Particle Kinematics and Kinetics. All the information necessary for developing these models and presenting them in the classroom is provided within the paper. The details for each model are provided on separate pages to facilitate duplicating and using them as classroom handouts. The description of each model includes an interesting problem statement, descriptive diagrams, and the analytical solution.

The five classroom models\* presented in this paper are:

- Equation of Motion - Rotary Table with Weights
- Conservation of Energy - Weight Suspended from a Frame
- Conservation of Energy/Coefficient of Restitution - Bouncing Ball
- Conservation of Momentum/Impact - Collision of a Large Object with a Small Object
- Conservation of Momentum/Impact/System of Particles - Suspended Steel Balls

## PROCEDURE/APPROACH

The approach used for presenting these models will control their ability to stimulate interest and communicate mechanics principles. Just showing the model and throwing the analytical solution on the overhead will not produce the desired results. The following is an outlined approach that the author has found to be successful.

- Start by introducing the model and posing one or two interesting questions.
- Ask for a show of hands on each of the possible solutions. Establish a competitive spirit in the classroom. Have a couple students offer a solution using their “gut feeling.”
- The next step is to lead the students through the analytical solution. This phase should be performed quickly without covering a lot of detail. Too much detail at this point will distract the students and they may lose interest. This phase moves more efficiently and is more effective if copies of the analytical solution are distributed and also shown on the overhead. This phase of reviewing the analytical solution can include the following:
  - Determine if the problem requires a particle or rigid body solution. (i.e., Is the body rotating? Is the body’s size of consequence? Make a clear distinction between rotation and curvilinear motion.)
  - Determine if the problem requires a kinematic or kinetic solution or both. (i.e., Does this problem involve a force analysis or just motion geometry?)
  - If the solution requires a kinetic solution determine which procedure is best.
  - Establish the proper diagrams.
  - Set up the basic governing equation(s).
- Obtain one or two volunteers to assist with demonstrating the model. Make the demonstration fun and interesting.
- Return to the analytical solution to clear up details and ask additional questions. This is also a good time to discuss the validity of any assumptions.

There are many creative techniques for using models. Try having students present the models. Better yet, have each student develop a model and then present it and the analytical solution to the class. The best student models can be saved for future semesters.

## CONCLUSION

The key to making these models successful is making them fun and interesting. Models are excellent tools for communicating basic Engineering Mechanics concepts while also stimulating interest and enthusiasm.

**MICHAEL MAGILL** is an Associate Professor in Mechanical Engineering Technology at Purdue University. Prior to joining Purdue University he was on the faculty at Oklahoma State University for eleven years. Professor Magill’s areas of interest are solid mechanics, applied structural analysis and computer-aided analysis. He has a B.S. and M.S. in Mechanical Engineering and a Ph.D. in Civil Engineering, all from Oklahoma State University.

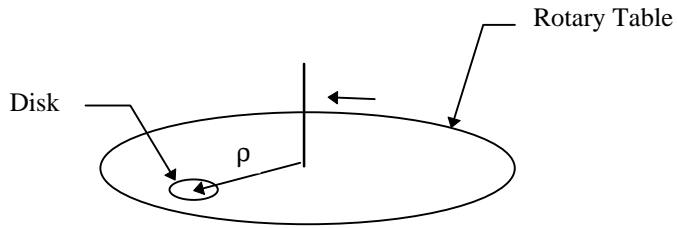
\*The ideas for these models came from various sources so the author does not claim ownership or copyright.

**PARTICLE: EQUATION OF MOTION - ROTARY TABLE WITH WEIGHTS**

Supplies: Rotating table (i.e. kitchen “Lazy-Susan”) and miscellaneous weights of equivalent contacting surfaces

Given: Several disks of different weight are placed on the rotary table at a distance  $\rho$  from the center. The table is rotated slowly increasing the velocity so that angular acceleration is negligible.

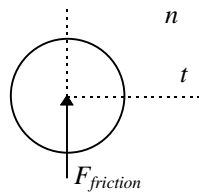
- $\mu_s = 0.2$
- $W_1 = \frac{1}{4} \text{ lb}$
- $W_2 = \frac{1}{2} \text{ lb}$
- $W_3 = \frac{3}{4} \text{ lb}$
- $W_4 = 1 \text{ lb}$



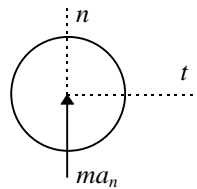
Find: Which disk will slide off first?

Solution:

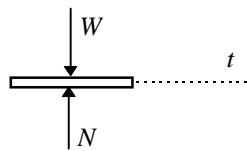
FBD of disk (top view)



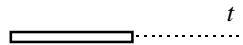
KD (top view)



FBD (side view)



KD (side view)



$$\sum F_n = F_{friction} = ma_n \qquad \leftarrow F_{friction} = N\mu_s \text{ and } a_n = \frac{v^2}{\rho}$$

$$N\mu_s = m \left( \frac{v^2}{\rho} \right) \qquad \leftarrow N = W = mg$$

$$mg\mu_s = m \left( \frac{v^2}{\rho} \right)$$

$$v^2 = \mu_s g \rho$$

Sliding is a function of  $v$ ,  $\mu_s$ , and  $\rho$  only!

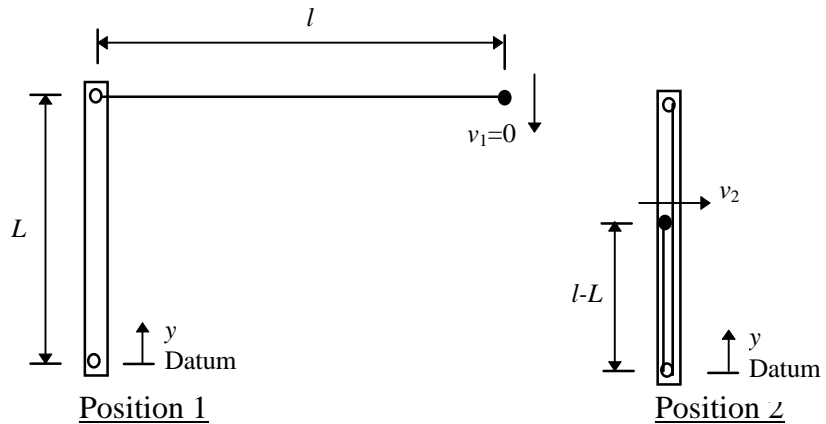
$\therefore$  All the disks will slide off at the same velocity!!

**PARTICLE: CONSERVATION OF ENERGY -  
WEIGHT SUSPENDED FROM A FRAME**

Supplies: Wooden stick with two pegs, string, and a golf ball.

Given:

$m$  = Mass of golf ball  
 $L$  = Distance between pegs  
 $l$  = Length of string



- Find:
- String length so that  $v_2$  will be zero.
  - String length so that  $v_2$  will be greater than zero.
  - String length so that weight doesn't reach position 2.

Solution:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m v_1^2 + m g L = \frac{1}{2} m v_2^2 + m g (l - L)$$

$$0 + m g L = \frac{1}{2} m v_2^2 + m g (l - L)$$

$$g L = \frac{1}{2} v_2^2 + g (l - L)$$

$$2 g L = v_2^2 + 2 g (l - L)$$

$$v_2^2 = 2 g L - 2 g (l - L)$$

$$v_2^2 = 2 g (2 L - l)$$

$$v_2 = [2 g (2 L - l)]^{1/2} \leftarrow$$

a) Set  $v_2 = 0$

$$0 = [2 g (2 L - l)]^{1/2}$$

$$0 = (2 L - l)$$

$$l = 2 L \leftarrow$$

b) When  $v_2 > 0$

$$v_2 = [2 g (2 L - l)]^{1/2} > 0$$

$$(2 L - l) > 0$$

$$l < 2 L \leftarrow$$

$\therefore$  Shorten String!!

c) If  $v_2$  doesn't exist, then position 2 is unreachable

$$v_2 = [2 g (2 L - l)]^{1/2}$$

If  $(2 L - l) < 0$ ,  $v_2$  doesn't exist.

$$l > 2 L \leftarrow$$

$\therefore$  Lengthen String!!

**PARTICLE: CONSERVATION OF ENERGY / COEFFICIENT OF RESTITUTION -  
BOUNCING BALL**

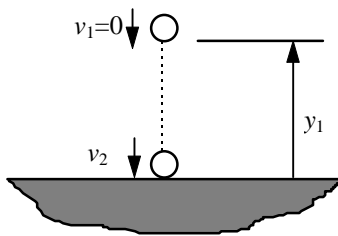
Supplies: Any type of ball (i.e. golf, tennis, basketball, etc.).

Given: Bouncing ball

Find: Coefficient of Restitution ( $e$ ) of the ball and surface

Solution: Drop ball from a known height and measure the return height.

Find velocity before ball hits surface



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m v_1^2 + m g y_1 = \frac{1}{2} m v_2^2 + m g y_2$$

$$0 + m g y_1 = \frac{1}{2} m v_2^2 + 0$$

$$g y_1 = \frac{1}{2} v_2^2$$

$$v_2 = -\sqrt{2 g y_1}$$

← Negative root because moving in direction opposite positive  $y$ .

$$e = \frac{(v_B)_3 - (v_A)_3}{(v_A)_2 - (v_B)_2}$$

←  $v_A$  = Velocity of ball and  $v_B$  = Velocity of ground

$$e = \frac{0 - (v_A)_3}{(v_A)_2 - 0}$$

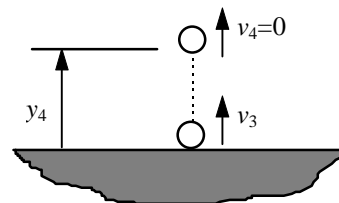
$$e = -\frac{(v_A)_3}{(v_A)_2}$$

$$e = -\frac{\sqrt{2 g y_4}}{(-\sqrt{2 g y_1})}$$

$$e = \sqrt{\frac{y_4}{y_1}}$$

← ∴ Coefficient of Restitution function of height only!

Find velocity after ball hits surface



$$T_3 + V_3 = T_4 + V_4$$

$$\frac{1}{2} m v_3^2 + m g y_3 = \frac{1}{2} m v_4^2 + m g y_4$$

$$\frac{1}{2} m v_3^2 + 0 = 0 + m g y_4$$

$$\frac{1}{2} v_3^2 = g y_4$$

$$v_3 = \sqrt{2 g y_4}$$

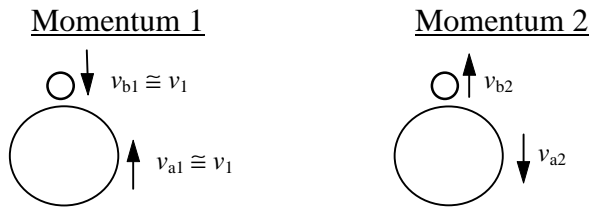
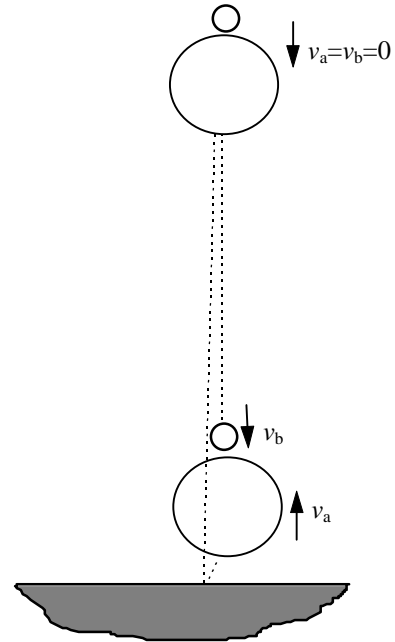
**PARTICLE: CONSERVATION OF MOMENTUM / IMPACT -  
COLLISION OF A LARGE OBJECT WITH A SMALL OBJECT**

Supplies: Tennis ball and basketball.

Given: Large ball has a mass that is 20 times greater than the small ball. (i.e.,  $m_a = 20m_b$ )

Find: Final ball velocities if they collide with approximately the same velocity in opposite direction.

Solution: Drop both balls with the small ball starting just slightly after the big ball. For simplicity, assume perfectly elastic collisions between basketball and ground and between basketball and tennis ball.



All impulse is internal to the system of particles so the external impulse is zero.

$$\begin{aligned} \Sigma m v_1 + \Sigma I &= \Sigma m v_2 \\ m_a v_{a1} - m_b v_{b1} + 0 &= -m_a v_{a2} + m_b v_{b2} \\ 20m_b v_1 - m_b v_1 &= -20m_b v_{a2} + m_b v_{b2} \\ 20v_1 - v_1 &= -20v_{a2} + v_{b2} \\ 19v_1 &= -20v_{a2} + v_{b2} \\ v_{b2} &= 20v_{a2} + 19v_1 \end{aligned} \tag{1}$$

$$\begin{aligned} e &= (v_{b2} - v_{a2}) / (v_{a1} - v_{b1}) \\ 1 &= (v_{b2} - (-v_{a2})) / (v_1 - (-v_1)) \\ 1 &= (v_{b2} + v_{a2}) / 2v_1 \\ 2v_1 &= (v_{b2} + v_{a2}) \\ v_{a2} &= 2v_1 - v_{b2} \end{aligned} \tag{2}$$

$$\begin{aligned} v_{b2} &= 20(2v_1 - v_{b2}) + 19v_1 && \leftarrow \text{Substitute eqn. (2) into (1)} \\ v_{b2} &= 40v_1 - 20v_{b2} + 19v_1 \\ 21v_{b2} &= 59v_1 \\ v_{b2} &= 2.81v_1 && \leftarrow \end{aligned}$$

∴ Velocity of small ball increases by almost a factor of 3!!

$$\begin{aligned} v_{a2} &= 2v_1 - 2.81v_1 && \leftarrow \text{Substitute } v_{b2} \text{ into eqn. (2)} \\ v_{a2} &= -0.81v_1 && \leftarrow \end{aligned}$$

∴ Velocity of large ball decreases slightly but continues in same direction!

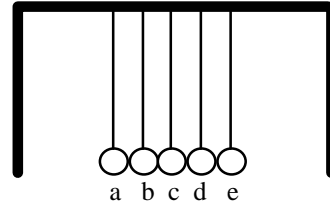
**PARTICLE: CONSERVATION OF MOMENTUM / IMPACT /  
SYSTEM OF PARTICLES - SUSPENDED STEEL BALLS**

Supplies: Suspended steel balls. Can be purchased at toy stores.

Given:

$$m_a = m_b = m_c = m_d = m_e = 1$$

Assume the coefficient of restitution ( $e$ ) = 1



Find: If 2 balls are pulled back and released, how many balls will move after impact?

Solution:

Momentum 1

$$\begin{array}{c} \circ\circ \quad \circ\circ\circ \\ \longrightarrow \\ m_a v_1 + m_b v_1 \end{array}$$

Impulse

$$\begin{array}{c} \longleftarrow \circ\circ\circ\circ \\ \int F dt \quad \int F dt \end{array}$$

Momentum 2

$$\begin{array}{c} \circ\circ\circ\circ\circ \\ \longrightarrow \\ \Sigma m v_2 \end{array}$$

All horizontal impulse is internal to the system of particles so the external impulse, in this direction, is zero.

Collisions occur as follows:

Ball "b" strikes ball "c"

$$\Sigma m v_1 + \Sigma I = \Sigma m v_2$$

$$m_b v_1 + 0 = m_b v_b + m_c v_c$$

$$v_1 = v_b + v_c$$

$$v_b = v_1 - v_c$$

(1)

$$e = (v_{R2} - v_{L2}) / (v_{L1} - v_{R1})$$

$$e = (v_c - v_b) / (v_1 - 0)$$

$$1 = (v_c - v_b) / v_1$$

$$v_1 = v_c - v_b$$

(2)

$$v_1 = v_c - (v_1 - v_c)$$

$$v_1 = v_c - v_1 + v_c$$

$$v_c = v_1 \quad \leftarrow$$

← Substitute eqn. (1) into eqn. (2).

$$v_b = v_1 - v_1$$

$$v_b = 0 \quad \leftarrow$$

← Substitute  $v_c$  into eqn. (1).

Ball “c” strikes ball “d”

$$\begin{aligned} m_c v_1 &= m_c v_c + m_d v_d \\ v_c &= v_1 - v_d \end{aligned} \quad (1)$$

$$\begin{aligned} e &= (v_d - v_c) / (v_1 - 0) \\ v_1 &= v_d - v_c \end{aligned} \quad (2)$$

$$\begin{aligned} v_d &= v_1 && \leftarrow \text{Substitute eqn. (1) into eqn. (2).} \\ v_c &= 0 && \leftarrow \text{Substitute } v_d \text{ into eqn. (1).} \end{aligned}$$

Ball “d” strikes ball “e”

$$\begin{aligned} m_d v_1 &= m_d v_d + m_e v_e \\ v_d &= v_1 - v_e \end{aligned} \quad (1)$$

$$\begin{aligned} e &= (v_e - v_d) / (v_1 - 0) \\ v_1 &= v_e - v_d \end{aligned} \quad (2)$$

$$\begin{aligned} v_e &= v_1 && \leftarrow \text{Substitute eqn. (1) into eqn. (2).} \\ v_d &= 0 && \leftarrow \text{Substitute } v_e \text{ into eqn. (1).} \end{aligned}$$

Ball “a” strikes ball “b”

$$\begin{aligned} m_a v_1 &= m_a v_a + m_b v_b \\ v_a &= v_1 - v_b \end{aligned} \quad (1)$$

$$\begin{aligned} e &= (v_b - v_a) / (v_1 - 0) \\ v_1 &= v_b - v_a \end{aligned} \quad (2)$$

$$\begin{aligned} v_b &= v_1 && \leftarrow \text{Substitute eqn. (1) into eqn. (2).} \\ v_a &= 0 && \leftarrow \text{Substitute } v_b \text{ into eqn. (1).} \end{aligned}$$

Ball “b” strikes ball “c”

$$\begin{aligned} m_b v_1 &= m_b v_b + m_c v_c \\ v_b &= v_1 - v_c \end{aligned} \quad (1)$$

$$\begin{aligned} e &= (v_c - v_b) / (v_1 - 0) \\ v_1 &= v_c - v_b \end{aligned} \quad (2)$$

$$\begin{aligned} v_c &= v_1 && \leftarrow \text{Substitute eqn. (1) into eqn. (2).} \\ v_b &= 0 && \leftarrow \text{Substitute } v_c \text{ into eqn. (1).} \end{aligned}$$

Ball “c” strikes ball “d”

$$\begin{aligned} m_c v_1 &= m_c v_c + m_d v_d \\ v_c &= v_1 - v_d \end{aligned} \quad (1)$$

$$\begin{aligned} e &= (v_d - v_c) / (v_1 - 0) \\ v_1 &= v_d - v_c \end{aligned} \quad (2)$$

$$\begin{aligned} v_d &= v_1 && \leftarrow \text{Substitute eqn. (1) into eqn. (2).} \\ v_c &= 0 && \leftarrow \text{Substitute } v_d \text{ into eqn. (1).} \end{aligned}$$

∴ Two balls will move to the right, d and e!! Ball c remains stationary.