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## **Closing the gap between physics and calculus: Use of models in an integrated course**

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# **Closing the gap between physics and calculus: Use of models in an integrated course**

## **Abstract**

This study focuses on bridging the gap between physics and mathematics by teaching an integrated first-year college course of physics and mathematics using Modeling Instruction (MI) and models and modeling perspective. This innovation involves redesigning the course's content, combining teaching strategies, reshaping the classroom setting, and using technology.

In this study 37 first-year engineering students were enrolled in the integrated course. The research questions were: What elements of the physical and mathematical models do students use when solving a complex real problem? How does the consistency of students' model lead to a robust understanding of the problem? To answer these questions, students worked collaboratively on a project that involved the analysis of an extreme bungee jump. The performance of the students in this project revealed: a) students' integration of physical and mathematical models, b) different representations used in the solution and c) robustness of students' models.

As conclusions we can state that as the students constructed more robust models, these were more helpful in their problem solving. Participating students constructed physical models that led them to mathematical models that required numerical methods. The use of numerical methods with technology as an aid in solving complex problems is part of the natural integration of physics, math and technology. Final presentations in a poster fair format provided students with the space and environment to present and share the models they constructed. Students were required to show their work and process to arrive to such solution to their peers. More importantly, the final project offered students the opportunity to realize how empowered they were in solving complex and non-familiar problems.

## **1 Introduction**

As Albert Einstein stated, "Physical concepts are free creations of the human mind, and are not, however it may seem, uniquely determined by the external world". Physical concepts are used to study the physical phenomena represented by models. Models that go through iterative cycles of conjecturing, testing and revising, until they satisfy constraints and provide a feasible explanation of the phenomenon under certain assumptions. In each iterative cycle, knowledge increases and a better understanding of the phenomenon is attained, and the models become more robust and connect more concepts. Models and modeling perspective reflects the physicist's way of understanding the world, so we should teach physics that way.

The importance of nurturing a scientific curiosity and motivating young students' understanding of science has been addressed for many years<sup>1</sup> and that call invites everyone<sup>2</sup>. As Barak Obama recently reinforced: "we want to make sure that those who historically have not participated in the sciences as robustly -girls, members of minority groups here in this country- that they are encouraged as well"<sup>3</sup>. In this call, physics and mathematicians become the main filters of young students' career decisions. We want them to select a program because it has physics and math,

not because it doesn't. Modeling Instruction (MI) opens the possibility to include all, provides a supportive learning environment and stimulates conceptual understanding<sup>4</sup>.

There have been efforts both in physics and mathematics not only to increase students' learning, but also to help students to see the connection between those disciplines. For instance, Physics Modeling Instruction by Hestenes<sup>5</sup> is based on students' building the concepts with activities and real life investigations rather than just receiving such concepts. The use of mathematics in this strategy comes naturally in order to try to solve a situation. In mathematics, Michelsen<sup>6</sup> presents a framework for interdisciplinary instruction involving the interplay between mathematics and science, and Lesh and Sriraman<sup>7</sup> propose mathematics education as a design science, in the sense that it provides a framework structured to promote testing, communication with relevant communities and progression.

## 2 Background

The need for bridging the gap between mathematics and sciences is well documented and has been a goal for many universities<sup>8, 9, 10, 11, 12</sup>. To that end, several universities have designed integrated courses that involve two or more areas. Some recent examples are a course that integrates calculus and introductory science<sup>9</sup>, and a science, technology, engineering and mathematics (STEM) course that integrates chemistry, biology, computer science, physics, and mathematics<sup>10</sup>. Our course, named Fis-Mat, after the name of the disciplines in Spanish, focuses on integrating physics and calculus for first-year engineering students<sup>13</sup>. The Fis-Mat course meets three times a week for a total of 5 blocks of 80 minutes each in three sessions (one block on Monday and two consecutive blocks on Wednesday and Friday). In terms of teaching load, two blocks correspond to the Physics course, two blocks to the Mathematics course and one block corresponds to the Physics Laboratory. Both professors were present and participating at all times. During the actual sessions there was no distinction between the blocks, each professor led the class depending on students' needs. The course program was structured in a coherent and articulated way without paying much attention on whose block corresponded, but rather following the interwoven sequence of concepts of both courses.

The effort of integrating disciplines aims to provide a more connected view of knowledge, as well as to teach what is needed when it is needed; that is, to ensure that the requirements between courses are met at the right time. Often, when physics and mathematics courses are taught separately, some mathematics' concepts that had yet to be taught in the calculus courses are already needed for application in physics courses. Fis-Mat fully integrates the first semester physics course and its corresponding physics laboratory, with the first semester calculus course.

The basic building blocks in science are the models, and science is a modeling process. The main pedagogical approach of Fis-Mat is Modeling Instruction, which focuses on qualitative and quantitative model development in an explicit cycle that allows students to conjecture, test and refine their model<sup>14, 15, 16, 17, 18</sup>. Modeling Instruction argues that science instruction should teach students the basic rules of modeling and should organize the course content around a small set of scientific conceptual models<sup>17</sup>. Fis-Mat uses mathematical laws to study physical phenomena so students can make strong predictions and construct complete models using a variety of representations<sup>15, 16, 18</sup>. Fis-Mat presents physics as a science in which all topics are

interconnected with a small number of physical laws<sup>19</sup>, in which mathematics plays an important role<sup>20</sup>. The conceptual models that structure the course content in MI are shared among members of the learning environment, and the validity, deployment and interpretation of the models are established through classroom activities and discourse. In this approach, models serve as conceptual resources that can be used to develop understanding of a variety of phenomena. MI “helps students develop model-centered knowledge foundations that resemble those of practicing [sic] scientists”<sup>14</sup>. This paper provides a description of the course, its teaching strategies, the classroom setting, the characteristics of the participants and the academic results. It also offers some concluding remarks and proposes steps for the future.

### 3 Method

#### 3.1. Setting and Participants

The setting for this study was a first semester integrated physics and calculus course for engineering majors. This was the second iteration of the implementation of the Fis-Mat course, and modifications had been implemented based on the experience, feedback and reflections gained from the previous implementation<sup>13</sup>. All 37 participants were freshmen enrolled in a new program (second year of implementation) that aimed to train students to develop comprehensive sustainable solutions in emerging fields of engineering, named “Innovation and Development Engineering”. This implementation attempted to answer two questions: What elements of the physical and mathematical models do students use in solving a complex real problem? How does the consistency of a students’ model lead to a robust understanding of the problem?

#### 3.2 Course Description

Fis-Mat uses the physics curriculum as its backbone, while the mathematics brings added support for idea-building and operations. In developing this course, we considered previous research<sup>21, 22</sup> and included modeling as a principal teaching strategy<sup>14, 23</sup>, along with an innovative classroom design that also serves as a physics laboratory to conduct experiments<sup>24</sup>. The primary goals of the Fis-Mat Project are: a) to improve students’ abilities to make connections between physics and mathematics, b) to increase students’ motivation to advance in their engineering studies, and c) to develop diverse competencies, such as critical thinking and the ability to work collaboratively.

The course was taught in a SCALE-UP classroom, where students also took the lab session as part of the whole class<sup>24, 25</sup>. Each topic starts with an investigation to find relationships that helps to explain certain phenomena. The findings are written on a whiteboard and the students present their conclusions to their peers in a discussion circle. Instructors lead the discussion and promote student discussion (Fig. 1). This work pattern was followed during the whole semester, always working in teams of three students. In Figure 1, the first picture shows a group of students investigating how a spring reacts to different amounts of hanging masses. The second photo corresponds to a whiteboard with the solution to a problem relating the analysis of a box held up against a wall by a person. The third frame shows a discussion circle presentation of teams’ whiteboards to share ideas and discuss their models with other teams. In that occasion, teams were discussing problems of constructions of functions (level of water in relation to time) knowing the behavior of the rate of change function.

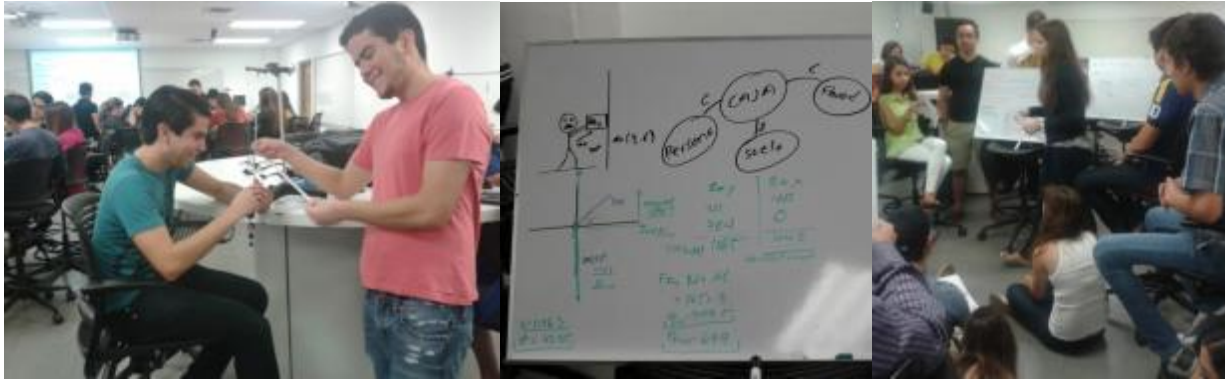


Figure 1. Investigating, creating whiteboards, presenting and discussing.

There were four faculty members from the School of Engineering involved in the development of Fis-Mat course: two from each Physics and Mathematics Department. All four met regularly to discuss and make decisions regarding the course implementation. Joint teaching of the course began in the fall of 2013, with one Physics and one Mathematics faculty member. The major topics covered in the Fis-Mat course included at least all the topics of a regular first year physics course and calculus course for engineering majors<sup>13</sup>.

### 3.3 Final Project of the Course

The implementation strategy of using projects in engineering education has proven successful, as it fosters individual responsibility in students and creates an environment similar to that of professional engineering<sup>26</sup>. Based on that, students had four weeks to work on the *Bungee jump Project* as a final assignment, in which they had to design, implement, document, and make a poster in time for a presentation fair open to visitors. Students worked in groups of three, with eleven teams in total of three students each, and one team of four students (37 students). Each team analyzed the situation, constructed their models, made the appropriate calculations, discussed the results and reached their conclusions. With that information, each team designed a poster (90 x 120 cm) to share their models, method and solution to the problem.

The project was divided in two parts. The first one centered on how the concept of air friction could be included in the representations they constructed during the semester and apply it in their calculations, no numerical data was provided (see Fig. 2). The second part included the development of the complete model, the analysis of the data, and the justified decision of whether, based on their variables, the students considered that the being would be able to perform the jump. The second part of the project included the first part plus the numerical information such as height of the UFO, length and elasticity constant of the cord, air density, and data regarding the jumping person.

In the first part, all teams presented their analysis orally supported by some visual aids (Power Point presentations, pictures, drawings, and/or Word and Excel documents) that showed their models and different representations. All teams concluded that air friction is just another force and they must simply take it into account in the force diagram and their sum of forces to get the actual acceleration on an object. More importantly, all teams presented different representation

models, which provided an opportunity to discuss how, studying the same phenomena, the variety of representations act as pieces of a puzzle that match perfectly providing a complete and consistent model. The statement of the first part of the Bungee Jump problem is shown in Figure 2. Notice that to motivate the analysis and promote the construction of models, no numerical data was provided with the problem. That information was released after the teams presented at the end of the first part of the project.

### **Bungee jump from an UFO problem**

Consider the following situation. The most extreme bungee jump in history of mankind consists of a cord of length  $L$  with an elasticity constant  $k$  that is tied to a UFO located at a height  $h$  from the ground. Then, a being jumps from the UFO. Would you dare?



In teams of three create a full model that explains what happens after the being jumps from the UFO until it reaches the point closest to the ground. Consider the following two scenarios:

- 1) There is no air friction.
- 2) There is air friction.

*Figure 2.* First part of the final project assignment.

In light of the course content and teaching methodology used (strong emphasis on models and modeling), the final course project, *Bungee Jump Problem*, was designed to serve as a complex situation that involved different concepts seen during the course, such as friction and forces on a spring. It also included concepts not explicitly covered in class, like forces due to air friction. In the second part of the project, students needed to take what they had learned, use knowledge from their specific fields and broaden their understanding of these concepts using numerical methods to construct their models and solve the problem.

A complete model of the situation involves a combination of diagrams, drawings, principles, and relationships that complement one another (see Fig. 3). The more representations used in the model, the more robust the model is in terms of completeness and consistency. To make things more interesting, all teams worked on the same problem, but the length of the bungee cord was different for each team. Therefore, the numerical answer for each team was different. The length depended on the average of their student ID numbers.

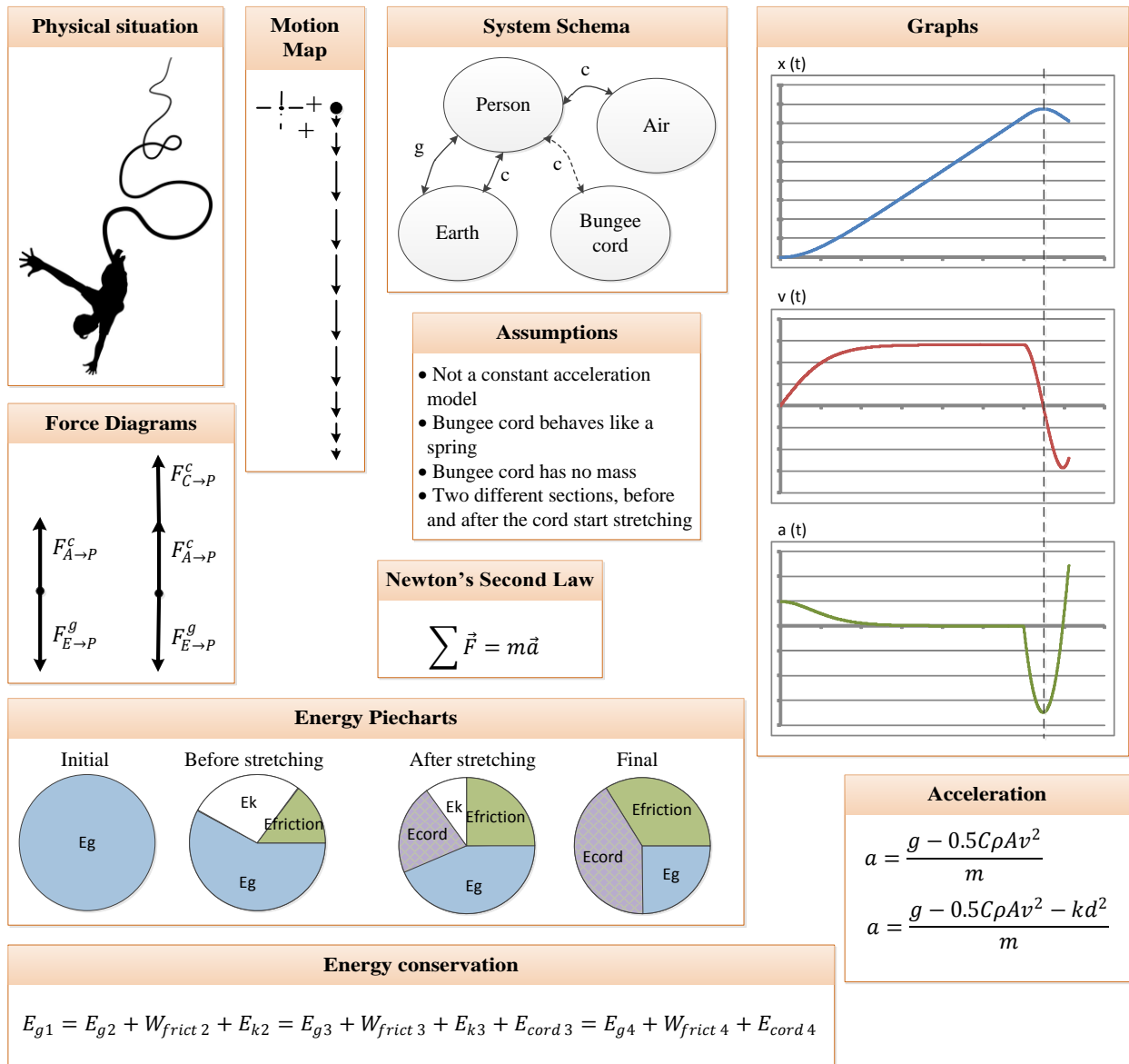


Figure 3. Complete model for the second part of the bungee jump problem.

The criteria that students had to meet were: 1) establish the connections between physical and mathematical concepts and procedures; 2) document their work, and prepare a poster presentation accompanied with an oral explanation of their reasoning. The poster presentation was evaluated on the numerical results, the consistency of their models, creativity of the design and the oral presentation (explanation). The model led to a solution of differential equations: since this was a Calculus 1 course, students found an approximate answer using numeric methods. Euler's method was part of the calculus content of this course as a way to understand cumulative change. Thus, students applied this method to find an approximation of the numerical answer. All teams used an Excel spreadsheet to perform the numerical approximation. We had previously used them in class and as homework to analyze some other situations (simple models and recursive ones).



Figure 4. Students presenting their work during poster presentation fair.

All twelve posters were presented on a hallway. Each team presented their findings as visitors stopped by their stands. All students' explanations were video recorded, and files of the poster, spreadsheet of the numerical analysis, as well as any other document they used for their presentation, were collected. An example of one team's poster during the presentation is shown in Figure 4.

During the poster fair, both professors were taking notes and asking questions regarding the solution. The team grades were based on the poster design (clarity, number of representations, consistency of the complete model), the provided solution (numerical approximation and conclusion), the oral explanation (fluent and coherent speech), and team work.

#### 4 Results

All twelve teams participated in a Poster Fair, at which other students, professors and directors attended. Each team presented their work and explained the two different scenarios they were asked to work on in the *Bungee Jump Project*. The analysis of students work focused on the number of representations and consistency among the different representations. Table 1 summarizes the collaborative work presented by the students. There are two forms of evidence for each task: we designated the word "yes" when students showed evidence in the oral presentation, and the word "implicit" when students described the schema or situation but they did not show the schema or drawing in the Presentation. We also marked as "implicit" when a representation was needed to show correctly another one not present (i.e., giving right distance from the floor but not showing or saying they used a numerical approximation to compute it).

In Table 1, the first column is the name of the team, the second column shows the assumptions made explicitly by the group in their model, and the third column the physical principle students



used for their model (Newton’s Second Law or Energy Conservation Principle). The following columns indicate whether students made explicit drawings of the physical experiment, the abstraction of the system (system schema) and the force diagrams. The last column indicates whether the students were able to construct the numeric approximation using the acceleration equation and Euler’s method (numerical approximation to calculate the distance traveled).

**Table 1** Model representations in the poster.

Team	Assumptions	System schema	Physical situation	Graphs	Energy pie charts	Energy	Motion Map	Force Diagram	Acceleration Eq.	Num. Approx.
A	Imp	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Imp
B	Imp	No	Given	Inc	Yes	No	No	Inc	No	No
C	Yes	Imp	Yes	No	Yes	Yes	No	Yes	Imp	Yes
D	Imp	Yes	No	Yes	Yes	No	Yes	Yes	Imp	Yes
E	Imp	Yes	Yes	Yes	Yes	No	Yes	Yes	Imp	Imp
F	Imp	Yes	Yes	Yes	Yes	No	Yes	Yes	Imp	Yes
G	Imp	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Imp	Yes
H	Imp	Yes	Yes	Yes	Yes	Yes	No	Yes	Imp	Yes
I	Imp	Yes	Verbal	Inc	Yes	No	No	Yes	No	No
J	Imp	Yes	Yes	Inc	Yes	No	Inc	Yes	Imp	Yes
K	Imp	Yes	Yes	Yes	Yes	No	Yes	No	Imp	Imp
L	Imp	Yes	Yes	Hand made	Inc	No	Yes	Yes	No	No

Note: Inc = Inconsistent model, Imp = Implicit

The Bungee Jump Problem had to be solved using forces, since the students didn’t have a way to get exactly the energy lost due to air friction. Once getting a numerical solution by force, you can use the Work-energy theorem to calculate energies, but it is not possible for students at this moment to do it the other way around.

It is important to mention that 9 of the 12 teams arrived to a correct result in their Posters. All the teams that used Newton’s Second Law and got an equation for acceleration arrived at the correct answer by using Euler’s method. Not getting an equation for acceleration makes students use the model corresponding to constant acceleration, which in a case with air friction is false. Finally, no team that explicitly used Euler’s method got their answer wrong. This means that students were able to correctly calculate the numerical approximation to the travelled distance. This is particularly important given that the system of equations that the model arrives to is a second order differential equation that requires a recursive numerical method to be solved.

The length of the cord was calculated so that most teams would face the case that their jumping being would die if air friction were ignored, but able to live when air friction was taken into consideration. Only one of the teams had large student ID numbers and using their calculations the being would have died if they hadn't had incorporated the constant acceleration principle.

When a group arrived at a correct relationship for calculating the kinetic friction coefficient, we are assuming that they had at least implicitly considered force diagrams and system schema. In that case, we also assumed that since all groups designed and constructed their experiments, they had a clear idea of the physical arrangement of the objects.

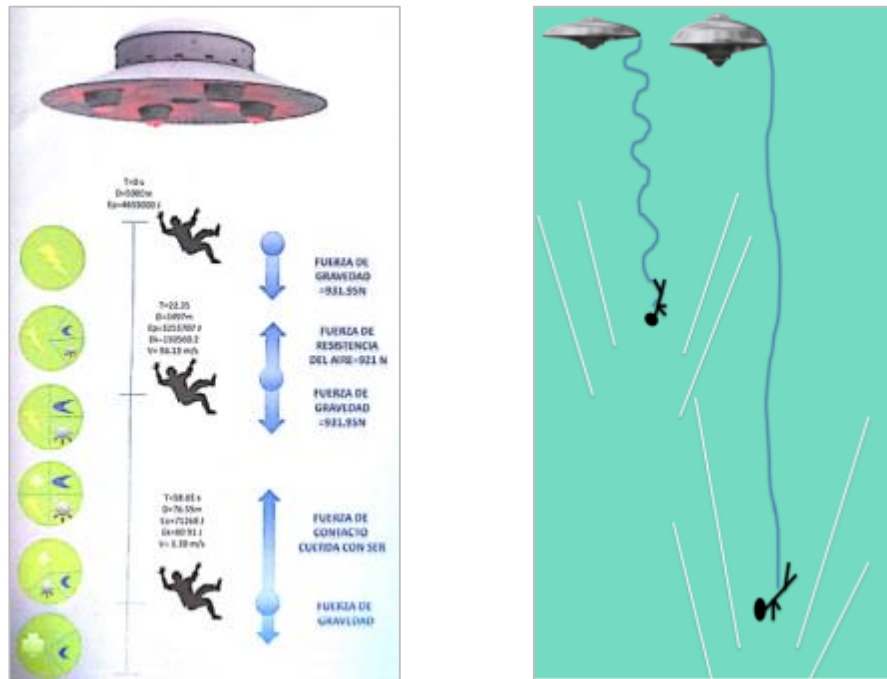


Figure 5. Physical situation, force diagrams and energy pie charts from two teams.

To better understand what makes a representation consistent or inconsistent, a couple of examples from the actual posters are explained. Figure 5 shows two examples extracted from two posters. The models show the physical description from teams C and G, respectively. Team C drew together to the physical description the energy pie charts to better relate how energy was changing in each phase of the problem, while on the right adding how the force diagrams changed depending on the height. They only showed information for forces at the moment the being jumped, when it had achieved terminal speed and when the being was not moving closest to the ground before going back up because of the spring. Team G made a simpler drawing and added the idea of air friction by adding straight lines to the sides of the falling person. It is creative, but it did not provide much information regarding energy or force changes.

An example of an inconsistent motion map is shown in Figure 6. The motion diagram stated that the being was falling slower every time, then it stopped and immediately moved up at a high speed that decreased every time. While this is true for certain parts of the whole movement, both are not one after the other. When you also take into account the graphs presented in the same

poster more problems become evident. The first one being that the graphs weren't actually labeled. Secondly, the position graph (blue) stated that the whole 5000 m took 2.4 seconds, thus, the velocity graph should be decreasing and negative, and that the acceleration was decreasing. Actually the velocity graph was plotted using the wrong axis by the students. Looking more carefully to the Excel file provided by the students we can see that while the velocity and acceleration were correctly calculated in the Euler method, the way they calculated actual position was not right, producing a graph that didn't relate to the movement. More importantly the motion map and the graphs didn't tell the same story, so there were consistency issues. Since this specific team was able to get a right answer we can assume they just added the graphs at the end by grabbing numbers from their Excel worksheet and were not careful with labels. This assumption is supported by the video tape of their explanation given during the poster fair.

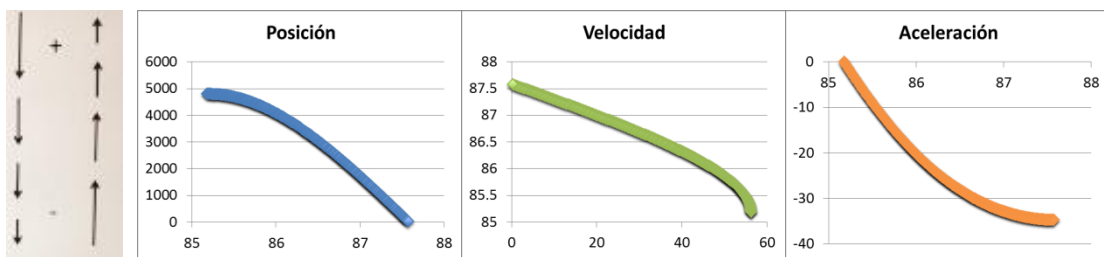


Figure 6. Inconsistent motion map and graphs from Team J.

Team E showed fairly consistent graphs. This team decided to run the numerical model all the way until it stopped moving and only made a small mistake. Since the equation used when the string is stretching and the equation used when it is not is different they used the wrong one after the first bounce, which is why the acceleration graph had a strange behavior at the peak that disappeared as it reached less height. Since the problem actually ends when the being gets to the closest point to the ground, this is not considered an issue. The graphs of velocity, position and acceleration of team E can be found in Figure 7. It is interesting that this was the only team that showed both the whole graph and a zoomed in view of the important part when needed.

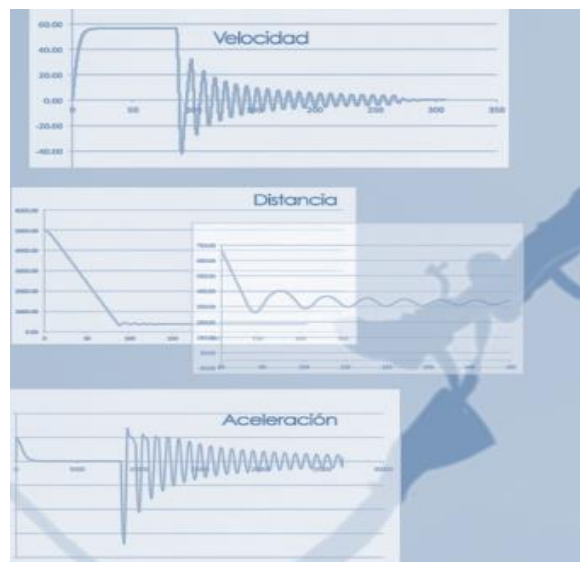


Figure 7. Consistent graphs from team E.

Finally, inconsistent graphs and energy pie charts from team L are shown in Figure 8. The graphs were actually made by hand since this team didn't use numerical methods to solve the problem. Although the graphs are mostly consistent between them, the problem arises when analyzing the problem and making comparisons with the energy pie charts. The velocity graph stated that the person at one moment had a constant velocity at which point the energy pie charts indicated only kinetic energy. The problem is that energy was being lost by friction due to the air and you will still have gravitational energy, since the position graph clearly pointed that the object was still falling.

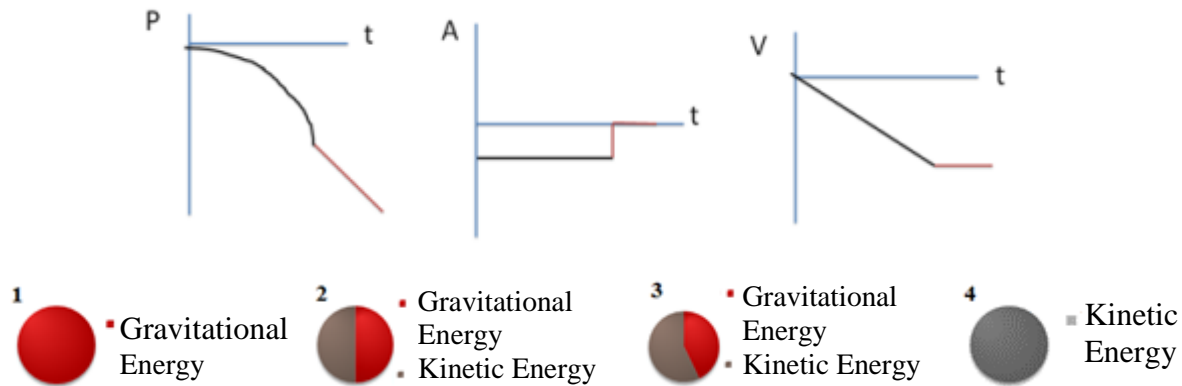


Figure 8. Inconsistent graphs and energy pie charts from team L.

Even with some inconsistencies in some of the representations, most of the groups were able to solve the problem in an accurate way. Authors consider that students may have not had experience designing a poster, since it is not a common practice for first year university students. This inexperience may have led them to design their posters in a rush, opening the possibility to make mistakes when copying the wrong graph, missing some labels, or even in the arrangement of the elements of the poster, such as selecting the appropriate size of a figure or picture.

## 5 Conclusions

Having two professors from different areas teaching the same course could be a nightmare, especially so when they have different views of the universe. However, Modeling Instruction as well as Models and Modeling perspective provide a common platform to integrate Physics and Mathematics courses. In that case, the center of the course's content is building models that explain the physical world using mathematics.

Only a few models can be built during a semester and some models constructed may have a limited scope. But this investigation shows that once students start organizing their reasoning in terms of models they are capable of incorporating concepts not seen in class to their models to solve different and challenging problems. The model-building approach goes to the conceptual and fundamental ideas on both disciplines, physics and mathematics. This teaching strategy takes some more time in class as well as in preparation, but the end results show students that they are more capable to react to whichever problem appears in their path.

The presentations of the first part of the Bungee Jump Problem showed that students had a hard time making the connections between representations that ensured a consistent and complete model. Only when teams presented their models to other students were they able to point out the inconsistencies. This also demonstrates that students are able to not only complement their models from outside sources, but to compare them with what their peers found and enrich their perspectives.

Poster presentation fair has been an interesting proposal to encourage students to reflect on the power of the model building approach as they worked and solved a challenging final problem. This turned out to be a very illuminating experience for the students, as they commented that friends who enrolled in separated sections of the first year physics and mathematics courses told them that they wouldn't have known how to approach a problem like the Bungee Jump Project, simply because it is not similar to any problem seen in class. The Bungee Jump Project allowed them to appreciate the power and potential of model construction based on analysis and fundamental concepts.

It is observed that the more robust the models are, the more likely the students are to solve the problem and reach a correct solution. Also, it is noticed that students were able to construct physical models that led them to mathematical models that required numerical methods. Involving situations with concepts from both physics and mathematics help students to better understand the relationship between both. In real life, problems tend to be harder and it is our responsibility and commitment to prepare students to solve problems that haven't yet been defined or stated.

## Acknowledgements

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