# Co-simulation of Electric and Magnetic Circuits

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Abstract: This paper reviews magnetic circuit models of magnetic structures, developed as analogs of electric resistor networks. It demonstrates magnetic simulation by circuit simulation of a magnetic circuit representing a three-winding magnetic structure, using known winding currents to calculate magnetic fluxes. Simultaneous simulation of both a magnetic circuit representing a magnetic structure and electric circuits connected to the windings eliminates the need to specify winding currents as known independent sources. This technique is here termed "co-simulation" for brevity. The controlled sources and auxiliary circuits necessary to implement co-simulation are arranged with winding turns as factors in controlled sources, to allow easy adaptation for different windings. Co-simulation is illustrated using the same three-winding structure used in the magnetic simulation, with a pulse input to one winding and a resistive load on the other two windings. The results of this co-simulation are compared to an electric circuit simulation in which the magnetic structure is described with three self inductances and three coupling coefficients. Extensions of co-simulation to account for nonlinear magnetic core behavior and core loss are described.

## I. Introduction

A convenient point of introduction of magnetic circuit analysis is to note an analogy[1,2] which can be established between magnetic and electric quantities. Specifically, magnetic potential or magnetomotive force $\mathfrak{I}$ (mmf) is represented by electric potential v or electromotive force (emf), magnetic flux $\phi$ is represented by electric current i, and magnetic reluctance $\mathfrak{R}$ is represented by electrical resistance R. This analogy has been used to model magnetic flux paths in various magnetic structures by calculating the reluctance (or permeance, the inverse of reluctance) of paths of various shapes in both soft magnetic materials, such as iron, and in air[3]. The use of this analogy has been criticized because the product of the "effort" variable mmf, and the "flow" variable flux is energy with SI units of watt times seconds or Joules, not power in watts[4]. In spite of this objection, the analogy retains its intuitive attraction, with the ampere-turns of mmf "forcing" a "flow" of flux through reluctance. Magnetic circuit models developed using this analogy are useful and suitable for solution using circuit simulation.

Other physical problems, modeled as networks, have been formulated for solution using circuit simulators[5]. While some of these have an "effort" variable times "flow" variable product which is power, this result is not always true. In thermal conduction problems, for example, the product of "effort" variable and "flow" variable is again not power, but power times temperature. In these cases, including magnetic reluctance networks, an electric circuit simulator is used as a generalized, versatile numerical solver of network equations, without any necessity for physical correspondence between electrical circuit elements in a simulation and the physical elements they represent.

Since the windings of magnetic structures are naturally connected to electric circuits, then magnetic circuit models provide an opportunity to simulate both a magnetic circuit and the electric circuits that drive them simultaneously. Co-simulation is used here as a convenient descriptive term for this simultaneous simulation. This technique has been used to analyze circuit behavior of saturating magnetic structures, such as ferroresonant transformers[6], and structures with unusual current and flux path geometries. However, its description in engineering literature is generally limited to specialist publications[7], sometimes with a suggestion that it is a necessary replacement for inadequate transformer models in circuit simulation[8].

In this paper, co-simulation is presented as complementing and extending circuit simulation, using an analysis tool familiar to electrical engineers and electrical engineering students to solve problems involving magnetic structures as well as electrical circuits. Co-simulation can provide fluxes and mmf drops as simulation outputs in addition to more familiar currents and voltages, demonstrating for students the general power of a tool they have mastered. Of course, there is never sufficient time to expose students to all the delightful extensions of what they have learned. At Indiana Institute of Technology, co-simulation is used as one of several "challenge" topics that vary from year to year in an Electrical Machines course in Electrical Engineering. Students in the course have successfully sorted out the magnetic and electric simulations, and the technique is being used as part of the simulation phase of an Electrical Engineering Senior Project.

To illustrate the technique in this paper, a magnetic circuit model is derived for a three-winding structure. It is simulated using specified winding currents to obtain the flux waveforms in each path. The magnetic circuit alone is then exercised to obtain the three self and three mutual inductances of the structure. Co-simulation is illustrated for one winding driven and two loaded. Electrical results are compared to those obtained from simulation using self inductance and coupling values obtained from exercising the magnetic circuit. Although the term "magnetic circuit" may be used for a physical structure[9], it is used here to refer to the reluctance network and mmf sources used to model the flux paths and windings of a physical structure.

## II. Magnetic Circuit Elements

A magnetic circuit model contains a reluctance, represented by a resistor, for each piece of the flux path. Each reluctance is calculated as

$$\Re = \frac{l}{\mu\, A},$$

where $\Re$ is the reluctance, l is length of the piece of the flux path, $\mu$ is the magnetic permeability of the material, and A is the cross sectional area of the flux path. Typically, reasonable approximations of length and area are adequate for cores. If a consistent formal approximation is required, an effective length and effective area calculated from core constants used to describe commercial core shapes may be used[10]. For an air gap, the flux may spread beyond the area of the core faces defining the gap. Details of various techniques that account for this reduced reluctance of air gaps can be found[3,10]. In order to focus on co-simulation rather than details of reluctance modeling, this paper uses a gapless core in its examples.

A magnetic circuit model contains an mmf source, represented by a voltage source, for each winding. The magnitude of each voltage source is calculated as

$$\Im = N i,$$

where $\Im$ is mmf, N is the number of turns on the winding, and i is the current in the winding. The polarity of each voltage source is determined by application of the right-hand rule: fingers follow the assumed direction of current in the winding and the thumb points in the direction of positive polarity of the voltage source.

III. Example of Magnetic Circuit

Figure 1 shows a magnetic structure which to be modeled with a magnetic circuit as an example. The core material has a relative permeability of 2000. Each of the three vertical legs of the core has a 100-turn winding labeled 1, 2, or 3, carrying current in the direction indicated by the arrows.

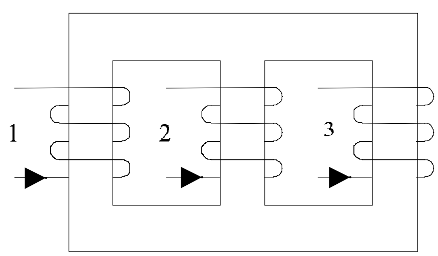


Figure 1 Magnetic Structure with Three Windings.

Each of the three vertical legs has an effective length of $1.0 \times 10^{-1}$ meters with an effective area of $4.0 \times 10^{-4}$ meter$^2$. Each of the four horizontal pieces has an effective length of $7.0 \times 10^{-2}$ meters, with an effective area of $8.0 \times 10^{-4}$ meters$^2$. These effective lengths and areas include the effects of corners.

The windings are excited with sinusoidal currents, in phase, with values $i_1$ = 10 mA in winding #1, $i_2$ = 30 mA in winding #2, and $i_3$ = 50 mA in winding #3. The magnetic circuit for simulation then looks like the schematic input to a simulator[11] shown in Figure 2. The reluctance values may be read directly in $10^3$/Henry, rather than the kΩ in the figure. The magnitude of the mmf sources in ampere-turns, rather than volts, is determined by the current in each winding times 100, the number of turns in each winding. The polarity of the sources is positive downward because positive current in a winding produces flux flow downward in Figure 1. The circuit could certainly be simplified by combining the reluctances that appear in series. However, a direct the mapping from magnetic structure in Figure 1 to magnetic circuit in Figure 2 is best illustrated by the circuit before any simplification. The 1.0/H reluctances from nodes 8, 10, and 11 to ground serve a flux sensors, permitting direct display of flux in each path as a mmf at a node without significantly changing the circuit.  The location of the ground is arbitrary, and was chosen for ease of monitoring the flux.

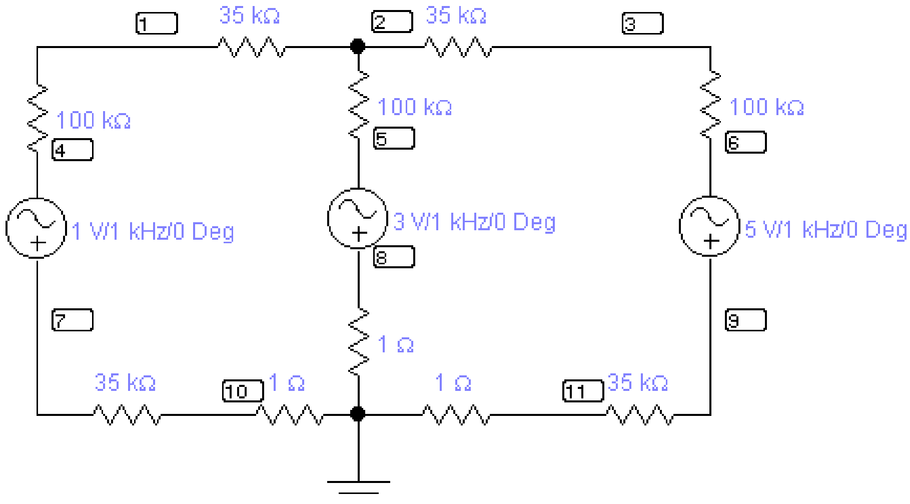


Figure 2.  Magnetic Circuit in Electric Circuit Simulator.

With the magnetic circuit model established, the flux in each leg of the core may then be obtained from simulation.  Figure 3 shows a waveform of flux in the right-hand leg of the core as an example obtained by transient analysis[11].  The horizontal axis is in seconds, as labeled, but the vertical axes may be read directly in Webers, since voltage at node 11 represents the mmf drop across a 1.0/Henry reluctance in the flux path.

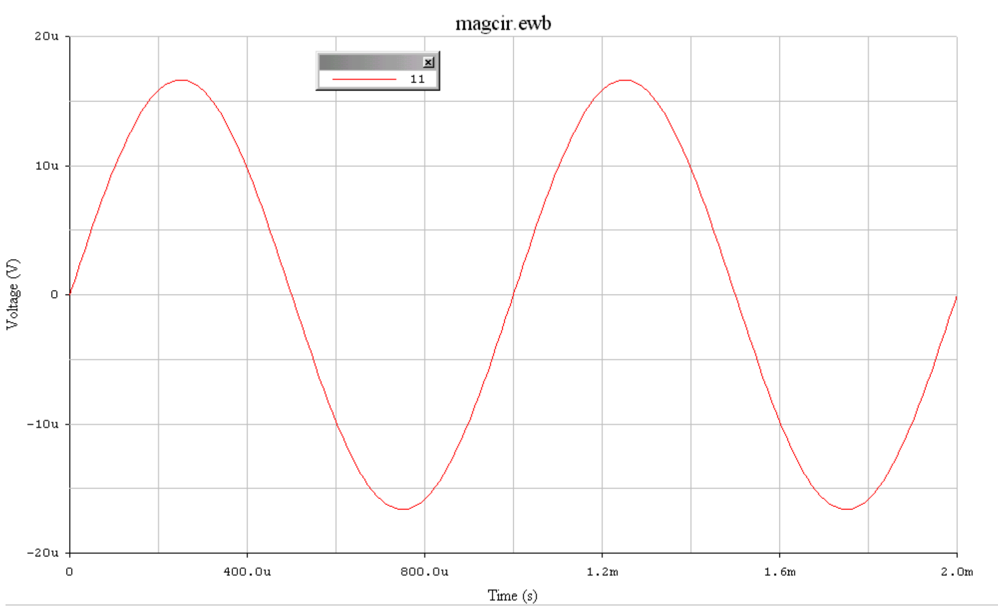


Figure 3.  Magnetic Circuit Simulation Solution showing flux vs. time in the right-hand leg of the core.

This simple magnetic circuit in the example could readily be solved analytically using superposition of the mmf sources, for example. Circuit simulation applied to even this circuit, however, provides all the advantages of electric circuit simulation, including rapid examination of the effects of more complex current waveforms.

## IV. Short Circuit and Open Circuit descriptions

Two common descriptions of electrical networks are the open circuit impedance matrix and the short circuit admittance matrix[12]. Since a magnetic circuit is constructed as an analog of an electric circuit, and similar techniques such as superposition may be used for analysis, it is worthwhile to consider the nature of open and short circuits in magnetic circuits. If a winding carries no current, i.e., the winding electrical circuit is open, then the mmf source representing the winding is zero, i.e., the source can be replaced by a short in the magnetic circuit. If a winding has zero voltage, i.e., the winding circuit is electrically shorted, then by Faraday's Law, there can be no change in the flux through the winding. This constraint combined with a zero-flux initial condition means that no flux can flow through the mmf source representing the winding, i.e., the source can be replaced by an open magnetic circuit. Thus, in general, an open-circuited winding corresponds to a short in the magnetic circuit, and a short-circuited winding corresponds to an open in the magnetic circuit.

## V. Limitation of Magnetic Circuit Simulation

An obvious limitation of magnetic circuit simulation of a magnetic structure is that the current in each winding must be known in order to determine the independent sources. In order to find these currents, the magnetic structure must be modeled in some manner that provides an electrical solution. Since a magnetic circuit is intended to model both the windings and flux paths of a magnetic structure, it is reasonable to expect that it can be used to provide such a solution. If a magnetic circuit is exercised to provide either an equivalent circuit or a set of self inductances and coupling coefficients, then naturally there is no direct information about the magnetic variables, in particular flux and flux density in a path. In addition, there are limitations to an equivalent circuit approach[4]. However, a co-simulation may be performed, in which both an electric circuit and magnetic circuit are simulated simultaneously, providing direct information about both the electric and magnetic variables.

## VI. Coupling Coefficients from Magnetic Circuit

If the focus of a problem is on solution for electric variables only, then a magnetic circuit model of a structure may be exercised to determine a set of values of self inductances and coupling coefficients, just as the windings of an actual structure may be measured to determine these parameters. The self inductances and couplings may then be used in a circuit simulation. For these exercises, it is convenient to introduce a new magnetic variable that includes both the flux in a winding and the number of turns in the winding. The flux linkage $\lambda$ for a winding is defined simply as

$$\lambda = N\phi \quad ,$$

where N is the number of turns and $\phi$ the flux. In an electrically linear system, each flux linkage is linearly dependent on the winding currents[13], with the coefficients being the self and mutual inductances. As an example, the relations for a three-winding structure are

$$\lambda_1 = L_{11}\, i_1 + L_{12}\, i_2 + L_{13}\, i_3$$
$$\lambda_2 = L_{12}\, i_1 + L_{22}\, i_2 + L_{23}\, i_3$$
$$\lambda_3 = L_{13}\, i_1 + L_{23}\, i_2 + L_{33}\, i_3$$

There are six parameters needed to describe a three winding structure: three self-inductances $L_{11}$, $L_{22}$, and $L_{33}$; and three mutual inductances $L_{12}$, $L_{13}$, and $L_{23}$. Thus, it takes six independent "measurements" of the magnetic circuit model to characterize the structure.

The self and mutual inductances can be determined with a direct current simulation of the magnetic circuit. If each winding is, in turn, excited with 1.0 ampere while the others are open (represented by a magnetic short or a zero-volt voltage source in the magnetic circuit), then a determination of the flux linkage of each winding source provides a numerical value of an inductance. Numerically, the value of the flux in Webers, as determined from a simulation, must be multiplied by the number of turns (100 for all windings in the example) to obtain the flux linkage. Since only dc sources are needed, flux values may be readily determined using a multimeter in the magnetic circuit, as shown in Figure 4[11]. The multimeter has been enlarged to show the flux reading. Note that these exercises or "measurements" may be performed with any convenient winding current value, such as one ampere – the winding currents need not be related to any currents expected in operation.

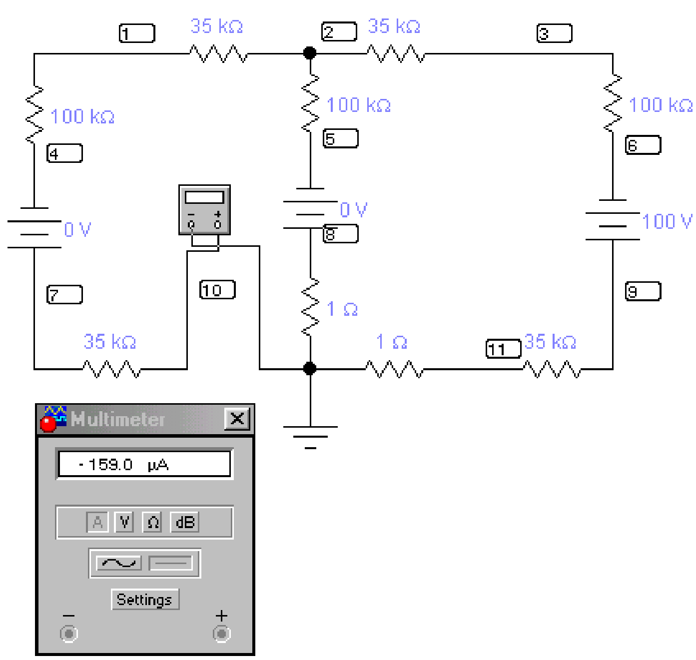


Figure 4. Magnetic Circuit Simulation showing $L_{13}$ = -15.9 milliHenry.

The inductance values determined in this way for the example three-winding structure are shown in Table 1. The corresponding values of coupling coefficients are shown in Table 2. These values were calculated using[14]

$$k_{ij} = \frac{L_{ij}}{\sqrt{L_{ii}\, L_{jj}}} \quad \text{where i, j} = 1, 2, 3.$$

| Inductance | $L_{11}$ | $L_{22}$ | $L_{33}$ | $L_{12}$ | $L_{13}$ | $L_{23}$ |
|---|---|---|---|---|---|---|
| Value(mH) | 42.9 | 54.1 | 42.9 | -27.0 | -15.9 | -27.0 |

Table 1. Inductance Values from Magnetic Circuit Simulations.

All three mutual inductances are all negative. This result is consistent with the core structure and the winding current directions. Any of the three mmf sources, excited individually, causes flux downward in its own leg, but upward in each of the other mmf sources. Thus, for example, in the flux linkage equations, a positive $i_2$, with $i_1$ and $i_3$ zero, produces and negative $\lambda_1$ and $\lambda_3$, but a positive $\lambda_2$.

| Coupling Coefficient | $k_{12}$ | $k_{13}$ | $k_{23}$ |
|---|---|---|---|
| value | -0.560 | -0.371 | -0.560 |

Table 2. Coupling Coefficient values calculated from inductance values.

## VII. Co-simulation

With the self inductance and coupling coefficient values from simulation of the magnetic circuit, it is possible to include a model of the magnetic component in any circuit simulation, provided the simulator permits positive and negative coupling coefficients. In order to determine the behavior of the magnetic variables, such as peak flux, it would then be necessary to apply the winding current waveforms to a simulation of the magnetic circuit. Instead, with co-simulation, both the electric and magnetic circuits are simulated together, so all electrical and magnetic variables are available. However, it is necessary to add "translation" circuits to link the voltage and the flux linkage.

One possible translation circuit is shown in Figure 5, implemented using controlled sources in a circuit simulator[11]. The connections to the left are connections to the electrical circuit, with positive current assumed to enter the top left connection. The connections top and bottom are connections to the magnetic circuit, with the bottom connection being positive mmf.

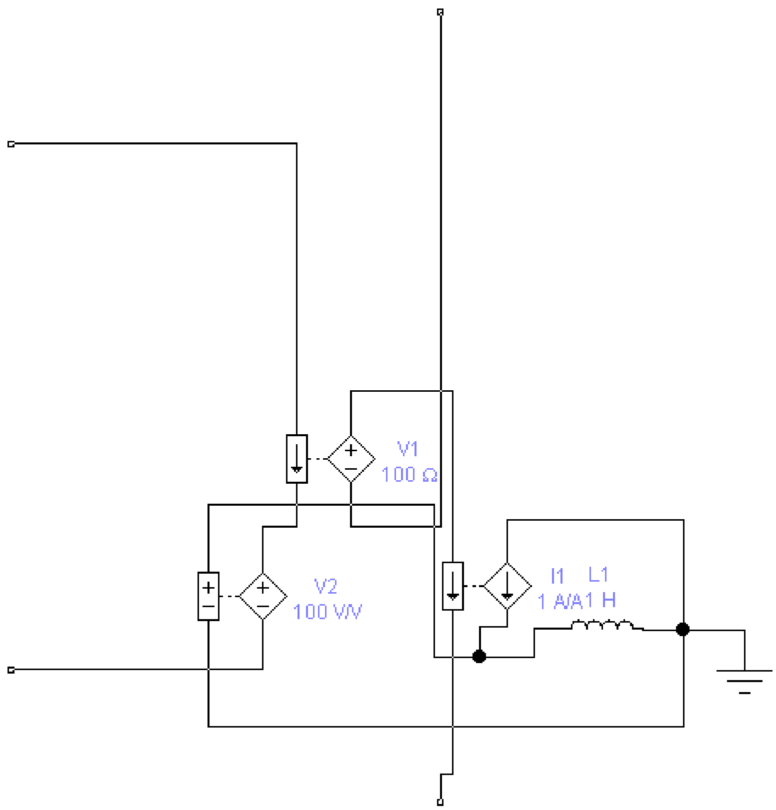


Figure 5 Subcircuit implementing electric–magnetic circuit link for 100-turn winding.

The source V1 is a current-controlled voltage source that generates mmf equal to 100 turns times the current in the winding. The source I1 is a current-controlled current source that drives a current equal to the magnetic flux through inductor L1. Since L1 has a value of 1 Henry, the voltage across the inductor is numerically equal to the derivative of current I1, which, in turn, is the flux. Source V2 is a voltage-controlled voltage source that multiplies the derivative of the flux by 100 turns to obtain the voltage of the winding in the electrical circuit. In general, a different subcircuit is needed to represent each winding, since the number of turns is built into each subciruit. For the example considered here, the same subcircuit may be used since each of the three windings has 100 turns.

Figure 6 shows the schematic simulator[11] input for co-simulation of the structure of Figure 1 with winding #1 driven with a 20 volt-peak, 1 KHz square wave, 50 % duty cycle, offset to produce only positive pulses, supplied by a function generator with a 50-ohm output impedance. Windings #2 and #3 are each loaded with 50 ohms. The subcircuit W100 is the circuit shown in Figure 5, representing a winding with 100 turns. The function generator has been enlarged to show the settings.

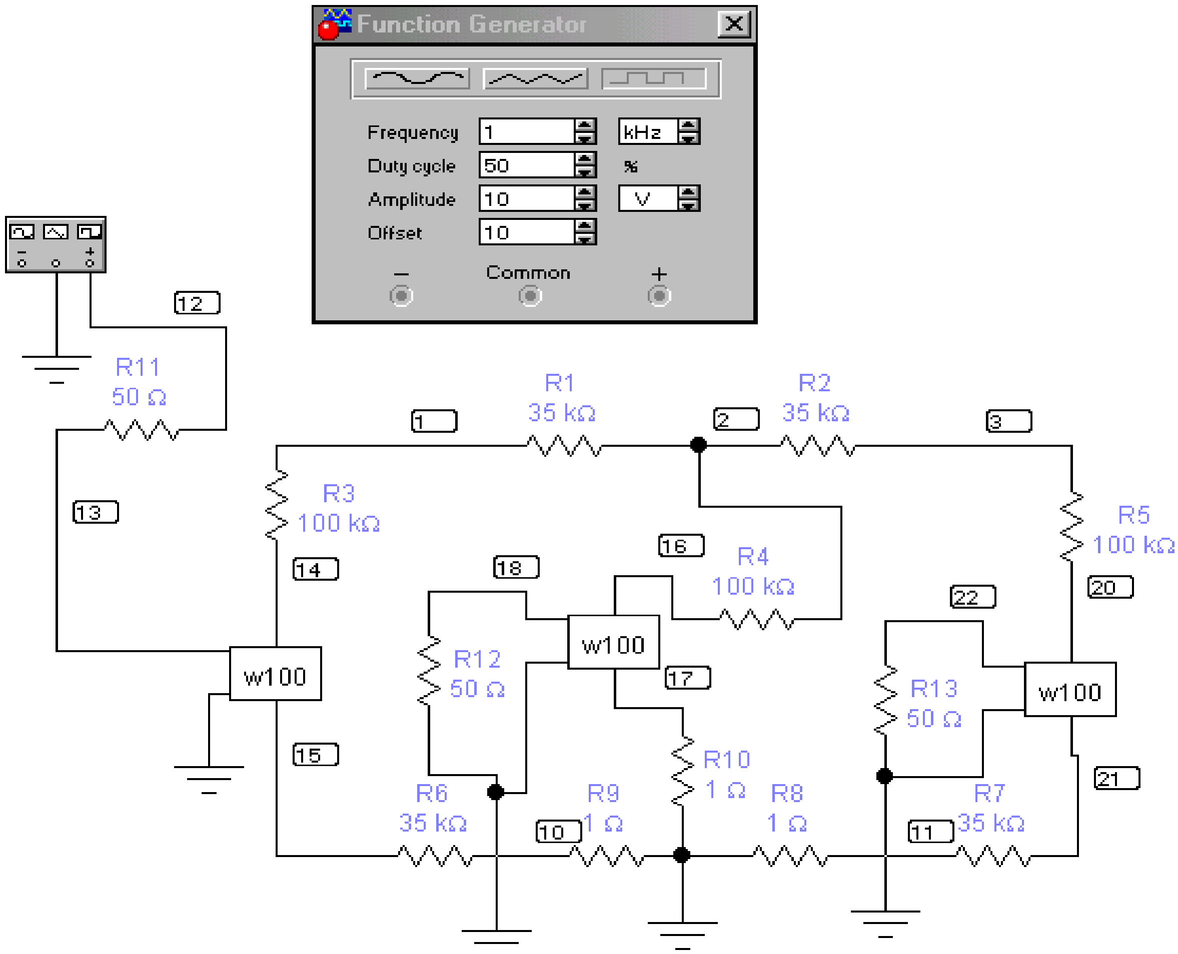


Figure 6.  Co-Simulation schematic input.

In Figure 6, the resistors R11, R12, and R13 represent electrical resistors.  The other resistors represent reluctances in the magnetic circuit of Figure 2.  Some node numbers in the magnetic circuit of Figure 6 differ from those in Figures 2 and 4 because of automatic re-numbering during modifications to add the subcircuit and electrical connections.

Figure 7 shows the input (node 13, lighter red trace) voltage and one output voltage (node 18, heavier blue trace) obtained from this co-simulation.

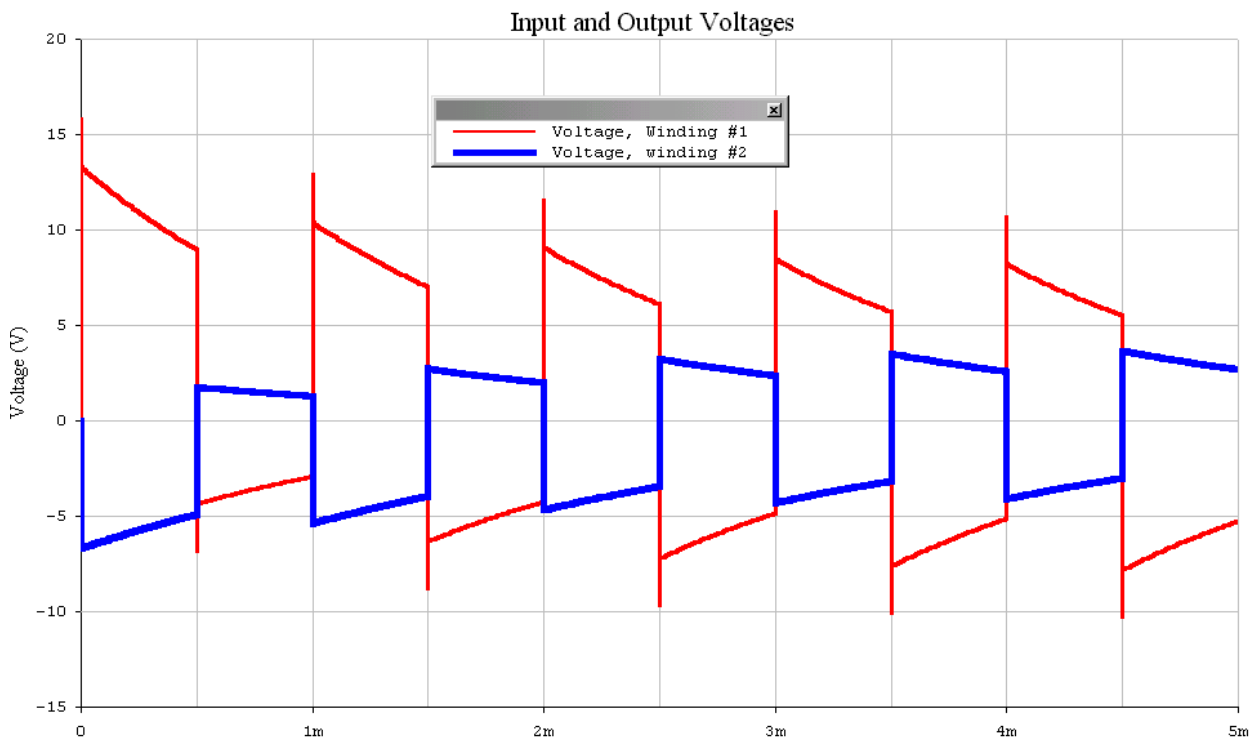


Figure 7. Voltages obtained from Co-simulation.

Figure 8 shows flux waveforms obtained from the same co-simulation. Again, using the sense reluctances of 1.0 /Henry, the calculated mmf values at nodes 10, 17, and 11 are equal to values of flux in Webers for the left flux path (top trace in red), center flux path (bottom trace in green), and right flux path (center trace in blue), respectively. The vertical axis label has been changed to "flux (Weber)" to reflect this equivalence.

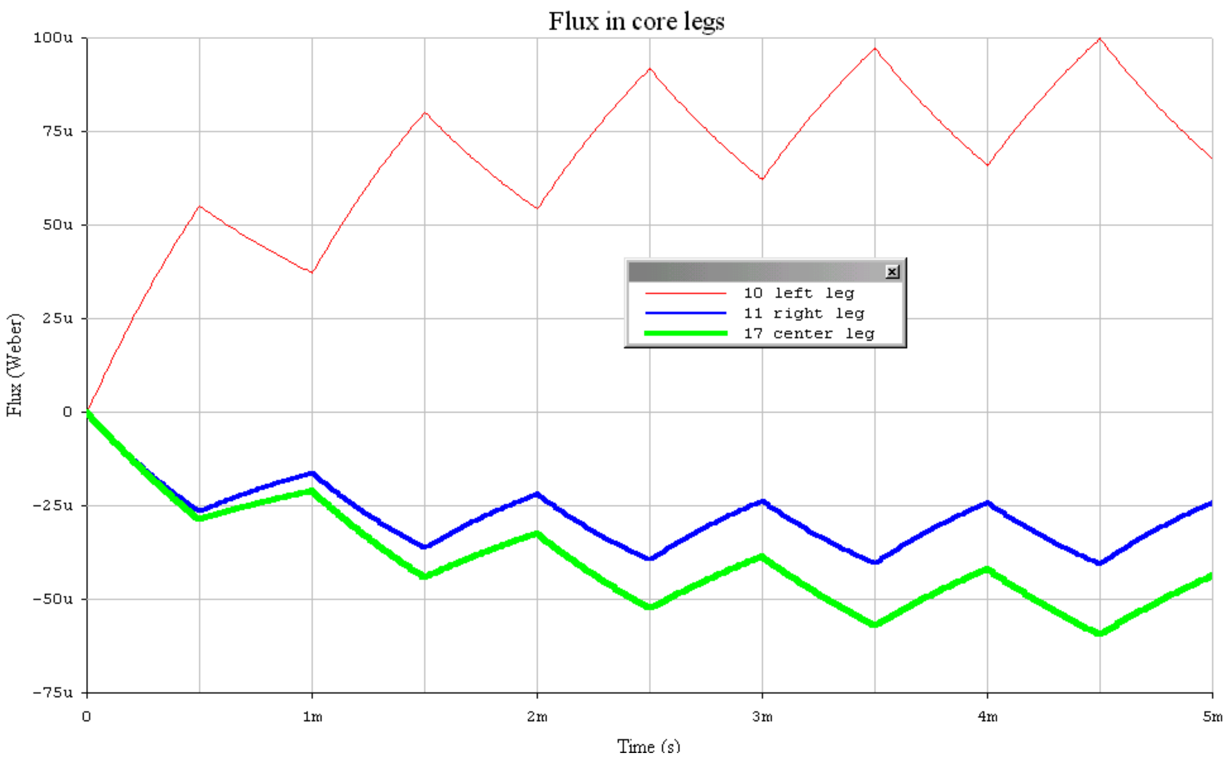


Figure 8. Fluxes obtained from Co-simulation.

The voltage waveforms in Figure 7 illustrate the transients at the start of the square wave drive, with the shift in voltage offset approaching zero for periodic steady state. The flux waveforms in Figure 8 illustrate the transients more clearly, with the average flux in each leg approaching a steady-state value.

## VIII. Coupling Coefficient model

If the focus of a problem is electrical variables, then the electrical part of the analysis can be performed very simply using the self inductances and coupling coefficients derived from direct current simulations of the magnetic circuit, as described in section VI. The values for the example are given in Tables 1 and 2. A netlist input to a simulator[15] was used in this case to further illustrate the simplicity of this simulation.

The entire netlist with simulation commands is shown in Figure 9. Rsource is the 50 ohm source impedance of the function generator, R22 is the 50 ohm load on winding #2, and R33 is the 50 ohm load on winding #3. Although simulation instructions limit the value of coupling coefficient k to be between zero and one[16], the simulation program actually accepts negative coupling coefficients, as illustrated.

```
V1  20 0 PULSE(0 20v 0 1us 1us 0.5ms 1.0ms)
L11 40 0 42.9mH
L22 60 0 54.1mH
L33 80 0 42.9mH
K12 L11 L22 -0.560
K13 L11 L33 -0.371
K23 L22 L33 -0.560
Rsource 20  40  50
R22  60  0  50
R33  80  0  50
.PROBE
.TRAN 0.01ms 5ms
```

Figure 9.  Netlist and Simulation Commands for Coupling Model.

The results of the simulation[15] are shown in Figure 10, waveforms of the voltage across winding #1 (green trace with squares) and the voltage across the 50-ohm load resistor on winding #2 (red trace with diamonds).  These same voltages, obtained from co-simulation, are shown in Figure 7. Comparison of the waveforms, Figure 7 and Figure 10, shows that the two approaches, co-simulation and coupled windings, provided the same electrical results.  Of course, only co-simulation provides results for magnetic variables.

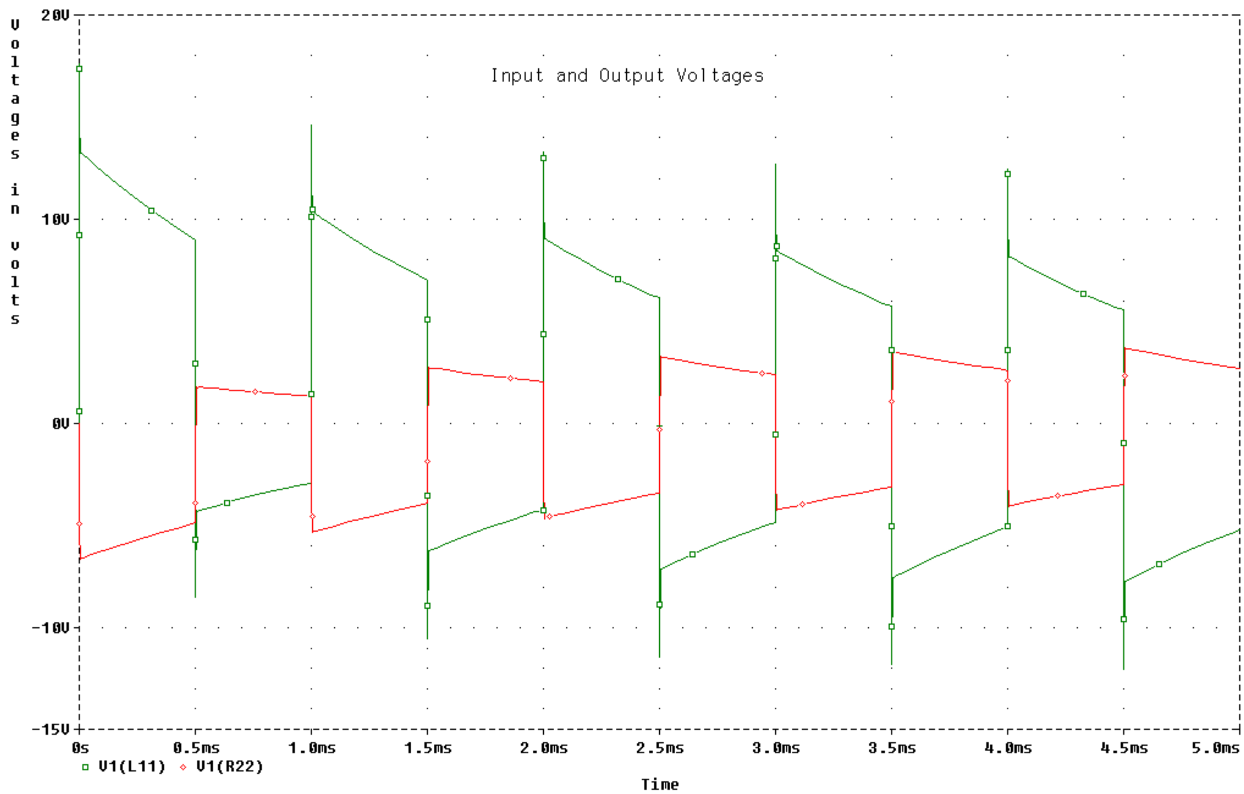


Figure 10.  Input and Output Voltages from Coupling Model Simulation.

Although a coupling model of the magnetic structure is rigorous and Figure 9 shows its description for simulation to be straightforward, keeping track of multiple coupling coefficients, with signs, provides opportunity for confusion[8]. The magnetic circuit model must be carefully exercised to obtain values of all self inductances and values, with signs, of all mutual inductances from the flux linkage equations. The signs of the mutual inductances must be assigned to the coupling coefficent values, also, during simulation.

## IX. Modifications and Extensions

The subcircuit in Figure 5 calculates winding voltage by taking the derivative of winding flux. An equally valid subcircuit could be constructed to calculate flux based on the integral of winding voltage[6]. Then an mmf drop across a flux (current) source could be used to calculate the winding current. Experience suggests that the integration form of the subcircuit may be preferred if a simulation requires non-zero initial winding current in windings: even a small difference between an initial condition and the first calculated value at the start of a simulation may produce large flux derivatives which prevent subsequent numerical convergence. On the other hand, the derivative form may be preferred if a precise periodic steady state solution is required: accumulating small integration errors tend to cause periodic steady state solutions to drift.

Nonlinear magnetic core behavior may be included in co-simulation by making a reluctance with a value that depends on the flux through it. The equivalent is a current-dependent resistor, which can be accomplished using a controlled source[17]. Thus, co-simulation can be used to simulate operation of devices in which linear behavior is desired but which may approach saturation, as well as devices, such as ferroresonant transformers, which depend on nonlinear behavior for correct operation.

Core losses may be introduced directly into a reluctance network. However, core loss is introduced by adding a series inductor in a magnetic circuit, which reduces the intuitive value of a magnetic circuit model. Thus, a magnetic circuit model with inductors for loss may be a useful simulation model, but it approaches a behavior model, weakening its value as a tool for learning about magnetic structures.

## X. Conclusions

Through co-simulation, electrical engineers and engineering students can use their skill in electric circuit simulation to model and analyze the in-circuit operation of magnetic structures. The necessary simulation overhead to link the electric and magnetic circuits -- three controlled sources for each winding -- may be conveniently placed in subcircuits, offering less clutter in the combined circuit schematic. If only voltages and currents, not fluxes and mmf values, are required from a simulation, then a coupling model of a magnetic component, offering simpler simulation input, may be used. Self and mutual inductances needed for the coupling model may be obtained by separately simulating a magnetic circuit model with dc mmf sources, repeating the simulation with different source combinations to obtain all values. Thus, the two approaches are complementary. The electric circuit is easier to simulate separately using coupling coefficients, but requires repeated simulations of a separate magnetic circuit, and provides no magnetic variable values during operation. In co-simulation, only one simulation is required to produce both electric and magnetic variables during operation, but the combined magnetic and electric circuit schematic is more complex, with non-physical components necessary to link the two types of circuits.

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