Abstract

Stress concentrations in thin tensile plates with circular holes have been of interest to engineers for a long time. Many investigations in regard to this subject, experimental and theoretical, have taken place. This paper deals with the investigation of assembly stresses and its effects on stresses around holes. Combined contact, bearing, and axial stresses laboratory experiments were designed and constructed for the use of mechanical engineering students. Two design projects are suggested in conjunction with the proposed experiments. The goal is that these experiments is to improve the students’ comprehension in advanced mechanics of materials, experimental mechanics, or machine design courses.

Introduction

In most undergraduate Mechanical Engineering (ME) courses, the discussion of assembly stresses is relatively brief. Stress concentrations have been of interest to engineers and mathematicians for a long time. Kirsch developed the theoretical stress concentration for an infinite elastic isotropic plate containing a circular hole subjected to a tensile axial load\(^1\). This theory predicts a stress-concentration factor (SCF) of 3.0 for the hole with the maximum tensile and compressive stresses being 0 and 90 degrees from the horizontal axis of the hole, respectively. The validity of Kirsch’s solution was examined utilizing optical experimental techniques\(^2\). The solution for the circular hole in a finite-width plate under uniaxial tension was published by Howland\(^3\) in 1930. In more recent years, experimental solutions have been obtained for a wide variety of hole shapes under different loading conditions\(^4\).

The increasing industrial demand for more sophisticated structural and machine components requires a solid understanding of the concepts of stresses in members and stress concentrations around discontinuities. Students in a mechanical engineering program are introduced to the concepts of stress and strain in a solid body through the Introduction to Mechanics of Materials course. The contact stresses are then discussed in senior level courses due to the added complexity of the theoretical derivations. In the first Machine Design course, junior mechanical engineering students, learn to calculate the bolt/rivet and joint members stresses. However, in a first ME course, there is generally insufficient time to carefully consider the assumptions that are made in developing the theories and to delve into the corresponding approximate nature of the
theories of stress concentrations. The combined effects of assembly stresses in the Machine
Design course are not covered. In fact, the literature review showed that many articles have been
written about SCF around holes in members in tension; the literature is bereft when describing
the effect of bearing and contact stresses. In 2003, utilizing the methods of reflected and
transmitted caustics, the effects of assembly stresses were indirectly mentioned in regard to the
stress field in a plate\textsuperscript{5}.

Experimental stress analysis laboratory practice improves the students’ comprehension in
understanding the stress theory learned in lecture. There are many articles in the literature in
regard to the use of different experimental stress analysis methods and experiments designed to
enhance the learning of mechanics concepts\textsuperscript{6-8}.

A combined loading topic in a basic mechanics of materials course usually deals with the
analysis of stresses and strains produced by three fundamental types of loads: axial, torsional, and
flexural. Many machine and structural elements are subjected to a combination of any two or all
three of these types of loads. The joint connections are usually formed by bolting the ends of
two-force members. Combined axial, contact, and bearing stresses laboratory experiments were
designed and constructed for the use of mechanical engineering students\textsuperscript{9}.

Reflected photoelasticity

Photoelasticity is a full-field technique used in stress analysis testing. Initial observation of a
photoelastic pattern provides quick qualitative analysis of the overall stress distribution. The
photoelastic method of stress analysis has been widely used for stress concentration studies. It is
important that the students visualize the nature of the quantities being computed. Therefore, to
enhance the students’ overall understanding of the multiple aspects of structural strength, the
behavior of stressed members is discussed in this paper. This is accomplished by utilizing the
experimental method of reflected photoelasticity. The accuracy of the proposed experiments
derives from the fact that the physical stress models must obey the practical laws of physics
rather than the theoretical laws of numerical mathematics which may be disobeyed. This can be
accomplished because the method gives direct strain measurements on real parts under static or
dynamic loads; no modeling is required. Mechanical engineers and Mechanical Engineering
students always look for practical problems and practical experimental techniques to learn. The
reflected photoelasticity method, also known as photostress, is a practical and versatile technique
for experimental stress analysis. The photoelastic fringe pattern is rich with information and
insights for mechanical engineering students. The method allows testing of the actual product or
structure under real working loads at room temperature. Therefore, the applicability extends over
a range of practical engineering areas such as military track shoes\textsuperscript{10}, aircraft design\textsuperscript{11}, and stress
concentrations in bone\textsuperscript{12}.

Setting

The proposed experiments introduce the students to a practical experimental technique of

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reflected photoelasticity for the evaluation of stresses in a member. The objectives of the experiments that uses a specimen in uniaxial tension are to show the student the following:

1. The development of stress in a member.
2. The region where the theoretical axial stress equation is valid.
3. The determination of the stress concentration factor at the boundary of a circular hole.
4. The effects of assembly stress on stress concentration.

Equipment and specimen

The experiment setup is shown in figure 1; equipment, specimen, and parts needed are:
1. Tensile machine
2. Reflection polariscope
3. Manual null-balance compensator or photoelasticity digital strain indicator
4. Aluminum specimen with circular holes coated with a photoelastic coating material
5. Small end plates
6. Bolts, nuts, and washers
7. Torque wrench with an adjustable setting
Experiments

This paper aims introduce students to photoelasticity and its practical applications. Many laboratory experiments were suggested to enhance the students’ learning of the Machine Design course. In 1997, the use of assembly stresses in reducing the stresses around holes was discussed. With the method of reflected photoelasticity, accurate quantitative data at selected points is easily obtained using straightforward measurement techniques and modern optical instrumentation. Two experiments are suggested to improve the students’ comprehension regarding what they learned in both a sophomore mechanics of materials and upper level machine design course or experimental mechanics course. For non experimental mechanics course, the students may use coated specimens and concentrate on collecting the data. Both the null-balance and tardy compensation compensations should be used, where possible. Calibration of photoelastic coating is essential and can be done following the procedures outlined in the literatures. The dimensions of the specimens used are shown in Fig. 2. Of course, different thickness and length can be used.

Fig. 2. Sketch of specimen
Experiment #1:

Utilizing the suggested method, the overall stress distribution can be easily studied by recognizing the fringes, their numerical color, and spacing with respect to one another. The purpose of this experiment is to determine the value of the stress concentration factor for a finite specimen with a central circular hole. The objectives of this exercise are for the student to:

1. Visualize the development of stresses in the vicinity of a circular hole loaded in uniaxial tension.
2. Establish the region where the strain is uniform; thus determine the zone that is not affected by drilling the hole.
3. Calculate the SCF and locate the points with the maximum tensile and compressive stresses.
4. Compare the experimental results with the published ones learned in lecture.
5. Determine the SCF for a given load and different torques on the bolts.
6. Determine the directions of the principal strains and stresses.

A specimen that is entirely coated with photoelastic material can be used. The coating must match and be perpendicular to the boundary of the plate; this is especially important around any discontinuity. Therefore, the holes were drilled through the coating and the plate. The author’s experience indicates that this procedure provides better matching of the edges of the coating and the boundary of the test part at the hole than to precut a hole in the plastic before bonding. No residual birefringence was noticed during the experiments.

When a photoelastic material is subjected to a load and viewed with polarized light, colorful patterns are seen which are directly proportional to the stresses and strains in the material. When the stress gradient is steep, these color bands (fringes) are closely spaced. Thus, the students will be able to identify the points with the maximum stress and calculate the stress concentration factor. The appearance of a uniform color represents a uniformly stressed area. Therefore, the students learn to locate the zone that is not affected by the geometric discontinuity.

For a reference, torque equals to zero is when the bolts and nuts barely touch the stiffeners. A typical optical image obtained for this experiment is in Fig. 3. The coating displays the strains in a colorful, informative pattern that reveals the strain distribution. In addition, the fringes colors and patterns pinpoint the highly strained points as well as the unstressed areas.

The students can plot the strain distribution at different distances from the fixtures. In order for the students to learn the effects of contact stress as well as bearing stresses from the bolts, a torque wrench with an adjustable setting is used. Several bolt’s preload can be achieved by varying the torque on the bolts and the students can visualize the effects of assembly stresses.

Design Project 1

Current technical advances and radical designs make the use of reflected photoelasticity ideal in the establishment of design criteria as well as the reduction of weight. This is because the fringe...
pattern is rich with information and insights for the design engineer. Therefore, a design project can be assigned using the plate utilized in experiment #1. The purpose of the project is simply to verify the design stresses or to further refine the design.

There are growing concerns for product reliability, weight reduction, and material conservation. Mechanical engineers seek valuable design information on how to modify a part to make it lighter, especially in the aerospace industry, and at the same time have less stresses. Stress analysis of a part utilizing photoelastic coating leads to corrective measures for preventing failure which usually involve material removal.

Project: The weight of a structural component used in the aerospace industry that is subjected to an axial load, like the one shown in Fig. 2, is to be reduced by ten percent. Suggest a method to reduce the weight. By examining the strain distribution in Fig. 3, the students should recognize the area of low stress that is indicated by the black fringes which a distance from the hole. Also, the student may note that the over stressed zone is usually surrounded by an area of near zero stress. However, removing this section gradually may lead to a stress concentration. When the test part extended beyond the edge of the photoelastic coating, the edges should be beveled at $45^\circ$ to eliminate any undesired stress concentrations at the edges.

Fig. 3 Photoelastic fringe patterns from a model with central hole

Load = 11880 N

Load = 16720 N
Experiment #2: Stress Concentration Reduction

Significant improvement in fatigue life of a part can be achieved with reduction in maximum stress level. The problem of reducing stress concentrations around holes in plates occurs in many design situations. The students learn how to calculate the bearing stress resulting from bolts and rivets in the undergraduate courses. In addition, they are introduced to the contact stresses and stress concentrations but the literature is bereft when describing the combined effect of bearing and contact stresses on stress concentrations.

One method for reducing the stress concentration around a circular hole in uniaxially loaded plate was demonstrated by Heywood, as well as Erickson and Riley, in which smaller holes are introduced on either side of the original hole.

A partially coated specimen can be used in this experiment. In general, the purpose of this experiment is to study the effects of assembly stresses on the stress concentration factor around a circular hole. The objectives of this experiment are for the students to:

1. Learn measurements techniques of both tensile and compressive stresses from the resulting fringe pattern.
2. Calculate the magnitude of assembly stresses.
3. Determine the effect of the bolt’s preload on the stresses around the holes.
4. Learn the methods of reducing stresses around discontinuities.
5. Recognize the need for life-long learning.

In addition to the reflected photoelasticity equipment and materials, objectives one and two can be accomplished by using a torque wrench with adjustable setting in order to ensure a specified torque on the bolts with high accuracy. The torque can be varied at intervals of 2 to 4 N.m. The students can determine stress concentration factors in the vicinity of a circular hole that was displaced from the center in a nonlinear stress field by boring holes at different distance from the plate’s center. For example, the first plate has the hole one diameter below the center while the second plate has the hole located two diameters below the center, etc.

Using the null-balance compensator or photoelasticity digital strain indicator, the students can measure the maximum difference in principal strains at the edge of the hole. Figure 4 shows the difference in principal strains of two plates for different loads. The dimensions of both plates are exactly the same as the one shown in Fig. 2. The diameter (d) of the holes in both plates is also equal.

When readings are made at free boundaries, such as circular holes in this study, the calculated stress represents the stress tangent to the free surface because the stress perpendicular to the edge is known to be zero. Thus, the SCF can be calculated easily and compared with the published results. Students will understand that skill training is a part of life long learning. Traditionally, the students calculate the ratio of the hole’s radius divided by the plate’s width as a first step in calculating the maximum stress at the hole. The SCF can be calculated utilizing the SCF graphs.
available in the mechanics textbooks, machine design textbooks, or SCF handbooks. However, according to the proposed experiments, the same ratio has different SCF values. The student can compare the experimental result for the plate with a central hole with the published results. On the other hand, for the hole displaced from the center and approaching the fixtures, there is no result available in the literature. The students at this stage understand that life-long education is a responsibility for every engineer and demonstrates the ability to go beyond the course expectations. Indeed, it enhances the interest in pursuing advanced degrees.

It is very beneficial to the students to observe the photoelastic fringe development stages at different loads for the coated part. Aside from its esthetically pleasing character, the development of a fringe pattern is rich with information for students. Full-field interpretation of fringe patterns, aids the students in the overall assessment of nominal strain magnitudes and gradients. Examining the intensity and the patterns of fringes for 11880 N and 16720 N loads shown in Fig. 5, the students can distinguish the difference in the localized high stress zone compared to the

Fig. 4 Difference in principal strains versus applied load

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same loads that are shown in Fig. 6. It is important to remember that the hole size and the plate width are the same in the figures, yet there is a difference in the stress pattern.

![Fringe patterns from a specimen with L/d = 4.2](image1)

Fig. 5 Fringe patterns from a specimen with L/d = 4.2

![Results from a plate close to the bolts](image2)

Fig. 6 Results from a plate close to the bolts

The effects of combined contact, bearing, and axial stresses are clear from the fringe pattern in figure 6. The students’ learning of calculating the tensile and compressive stresses can be further enhanced by taking measurements at several points at the boundary of the hole as well as close to the fixtures. The difference in the principal strains versus the location along the hole’s circumference is plotted in figure 7.

Design project 2

In this design project, the students are introduced to the reduction of stress concentration around
a circular hole, experimentally. The reflected photoelasticity is a powerful method for the visualization of the development of stress concentration. By using the method, the difference in principal strains (maximum shear strain) can be measured. However, in uniaxial applications and edges of holes, one of the principal stresses is zero. Hence, one measurement is needed to calculate the maximum stress.

![Graph showing difference in principal strains vs. angle (degrees)](image)

Fig. 7 Normal strain distribution around the hole

Project: In many design situations or practical repairs the circular holes are not central nor drilled in places where the stress distribution is uniform. One of the design requirements for the member shown in Fig. 2 is to drill a hole to pass small cable through the plate when it is subjected to a 22000 N load. Suggest a location where the hole should be bored without exceeding the value of axial stress significantly.

The hole should be drilled around the center of the near zero stress area. The students have learned from the above experiment that contact and bearing stresses contribute to reducing the stresses around the hole. Therefore, The hole with the size that is slightly larger than the cable diameter should be drilled close to the fixtures. However, studying the fringe patterns and intensity, the hole should not be drilled very close to the fixtures. A group of three students (three plates) is recommended for optimizing the reduction of stress in the vicinity of the hole.
Conclusion

Stress analysis experiments and design projects are proposed for the enhancement of learning some of the mechanics of materials and machine design fundamentals. The experiments described in this paper are a valuable addition to an undergraduate mechanical engineering laboratory content. It has been established that by using the method of reflected photoelasticity, it is possible to determine the effect of assembly stress on the stress concentration in the vicinity of a hole. The objectives of the proposed experiments will help the students to recognize the need for lifelong learning.

Bibliography


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