Combining MATLAB & EXCEL to improve understanding in controls

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Abstract:

Control systems are modeled by using differential equations. The same control system can be stable or unstable depending on the input parameters into the system. Stability or lack of stability of a control system can theoretically be determined by solving the control system differential equation(s). The differential equation(s) can be solved numerically. A numerical solution of a differential equation produces numbers that can be plotted but not an expression. The differential equations can also be solved by classical differential equation techniques. The classical differential equation solution techniques can be supplemented by using Laplace Transform and using the MATLAB software to expedite the Laplace Transform formulations. Damping level(s) in a vibrating system greatly influence the stability level of the system. It is shown that the system can be stable or unstable depending on the system parameters, namely the type of external excitation, the system natural frequency, system damping level and the frequency of the external excitation force. The similarity and differences of the EXCEL based numerical solution and the MATLAB based theoretical solution are discussed. The use of the numerical technique and MATLAB will help students with less than perfect mathematical skills to be able to learn practical control engineering. The use of these techniques eliminates the need for physical labs which will reduce the possibility of Covid-19 virus spreading.

Introduction:

A single degree of freedom vibrating system consists of a mass, a spring and a damper. Spring force is proportional to mass displacement. Damping force is proportional to mass velocity, and the product of mass times its acceleration determines the force due to acceleration. Velocity is the first derivative of displacement with respect to time, and acceleration is the second derivative of displacement with respect to time. Because of the presence of the derivatives, the equation of motion of a single degree of freedom vibrating system is a differential equation. A single degree vibrating system is an example of an open loop control system.

A numerical technique for the solution of the differential equation is developed and implemented using EXCEL. The characteristic of a numerical solution is that the results are numbers and not in the form of expressions. The differential equation can also be solved by first putting it in Laplace Domain and then using the partial fraction and the inverse Laplace Transform capabilities of MATLAB. By using the MATLAB technique, the solution in the form of an equation is obtained.

A single degree of freedom vibrating system subjected to an external excitation at a frequency close to its resonance frequency can be unstable. This article demonstrates the solutions of a stable and an unstable vibrating system using EXCEL and MATLAB. The comparison of the

solutions will enable the students to better understand the concept of stability in controls which will prepare them for more advanced studies of controls engineering concepts.

The goal is to train students to be able to perform complicated control analysis without highly advanced mathematical skills. The approach also eliminates the need for setting up physical controls laboratories that are impractical under the Covid-19 restriction.

Technical discussion:

Figure 1 is a graphical representation for a single degree of freedom vibrating system.
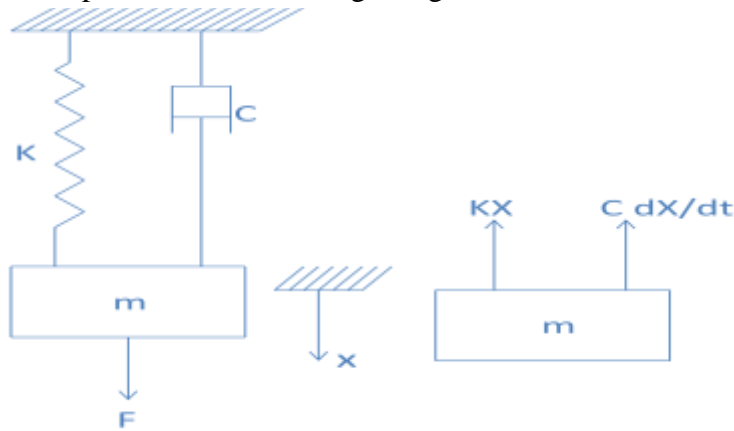

Figure 1: Graphical representation of a single degree of freedom vibrating system

The equation of a single degree of freedom vibrating system is shown in equation (1).

$$m \, d^2X / dt^2 + C \, dX / dt + KX = F \tag{1}$$

Equations (2) through (9) are the numerical solution of equation (1), where "h" is the time increment and the subscripts are the steps in the numerical solution. [1]

For the approximation to produce acceptable results, h must be small enough for the results to be close to exact solutions, but not too small such that excessive numerical errors are accumulated as a result of too many solution steps. [2], [4]

$$X_1 = X_0 + h \, (dX / dt)_0 + \{(h^2 / 2) \, (d^2X / d \, t^2)_0\} \tag{2}$$

$$(dX/dt)_1 = (-2 \, m \, X_0 + 2 \, m \, X_1 + F_0 \, h^2 - k \, h^2 \, X_1) / (2 \, m \, h + C \, h^2) \tag{3}$$

$$(d^2X/ dt^2)_1 = \{F_0 - C \, (dX/dt)_1 - k \, X_1\} / m \tag{4}$$

$$X_2 = 2 \, X_1 - X_0 + h^2 \, (d^2X / d \, t^2)_1 \tag{5}$$

$$(dX / dt)_2 = -(X_1 / h) + (X_2 / h) + \{(h / 2) \, (d^2X / d \, t^2)_2\} =$$

$$-(X_1 / h) + (X_2 / h) + (h/2)\{f \, [X_2 ,(dX / dt)_2]\} \tag{6}$$

$$X_3 = 2 X_2 - X_1 + h^2 (d^2X / d t^2)_2 \tag{7}$$

$$(dX/dt)_3 = (-2 m X_2 + 2 m X_3 + F_2 h^2 - k h^2 X_3) / (2 m h + C h^2) \tag{8}$$

$$(d^2X/ dt^2)_3 = \{F_2 - C (dX/dt)_3 - k X_3\} / m \tag{9}$$

Calculations for additional time increments can be performed by incrementing the calculation indexes in equations (7), (8) and (9).

If the damping term is non zero for the system of Figure 1, the equation of motion for free damped vibration becomes as shown in equation (10).

$$m \, d^2X / dt^2 + C \, dX / dt + KX = 0 \tag{10}$$

Solution of equation (10) in Laplace domain is shown in equation (11).

$$X(S) = \{m X_0 S + m (dX/dt)_0 + C X_0\}/ \{m S^2 + CS + K\} \tag{11}$$

X(S) defines the motion of mass m in Laplace domain. Taking the inverse Laplace transform of a function defined in Laplace domain results in an equation defining the function in time domain.

If the system of Figure 1 is exposed to a sinusoidal excitation force, then the equation of motion is defined by equation (12).

$$m \, d^2X / dt^2 + C \, dX / dt + KX = A \sin \omega t \tag{12}$$

If ω (frequency of excitation force) in equation (12) is at or close to the natural frequency of the system, then the system becomes unstable unless the system damping can stabilize the vibration. [2]

Solution of equation (12) in Laplace domain is shown in equation (13).

$$X(S) = S^3 m X_0 + S^2(m V_0 + C X_0) + S (m \omega^2 X_0) + (m \omega^2 V_0 + C \omega^2 X_0 + A\omega) \, /$$

$$S^4 m + S^3 C + S^2 (K + m \omega^2) + S C \omega^2 + K \omega^2 \tag{13}$$

Examples of numerical solutions of vibration of a single degree of freedom system using EXCEL based numerical techniques and using MATLAB based Laplace transform techniques:

The numerical formulation can be programmed into an EXCEL spread sheet and the Laplace transform formulation can be programmed in MATLAB and different scenarios of a single degree of freedom vibrating system can be studied using both techniques.

Scenario 1:

Table 1 summarizes the vibrating system parameters of scenario 1.

Table 1 (Summary of Parameters of Scenario 1)

Mass (lb.sec²/inch)	Damping (lb.sec/inch)	Siffness (lb/inch)	Time increment (sec)	External force (lbf)	Initial displacement (inch)	Initial velocity (inch/sec)
1	3	2	.2	0	.1	.05

Figure 2 is a plot of the EXCEL implementation.

Inputting the parameters defined in scenario (1) into equation (11) results in equation (14).

$$X(S) = \{.1\ S + .35\}/\ \{S^2 + 3S + 2\} \tag{14}$$

Rearranging equation (14) results in equation (15).

$$X(S) = [\{.1\ S^2 + .35S\}/\ \{S^2 + 3S + 2\}]\ (1/S) \tag{15}$$

Therefore, the motion of mass m can be obtained as a unit-step response of the transfer function defined in equation (16).[4]. This response is shown in Figure 3.

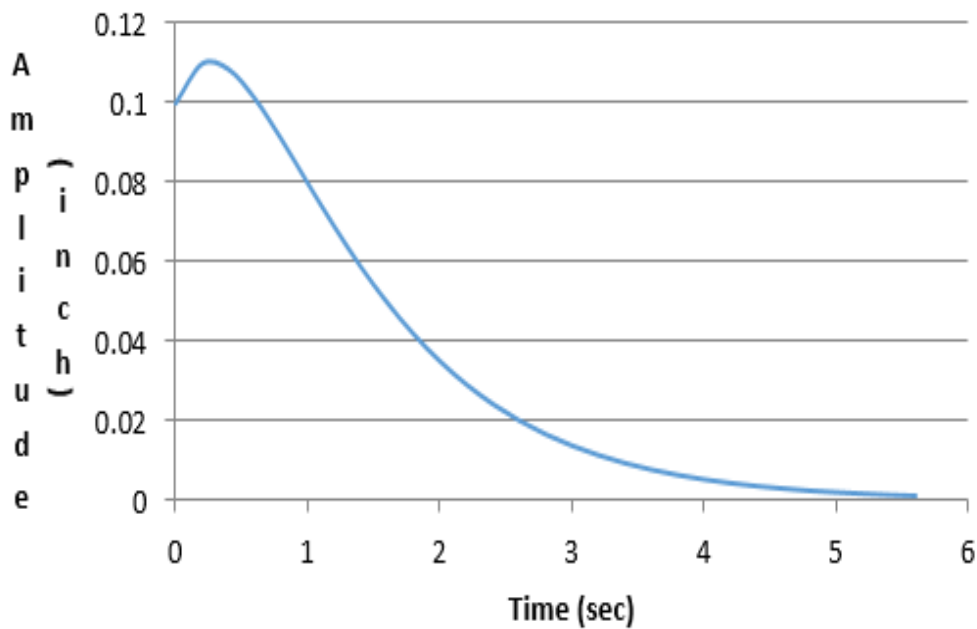$$G(S) = X(S) = \{.1\ S^2 + .35S\}/\ \{S^2 + 3S + 2\} \tag{16}$$

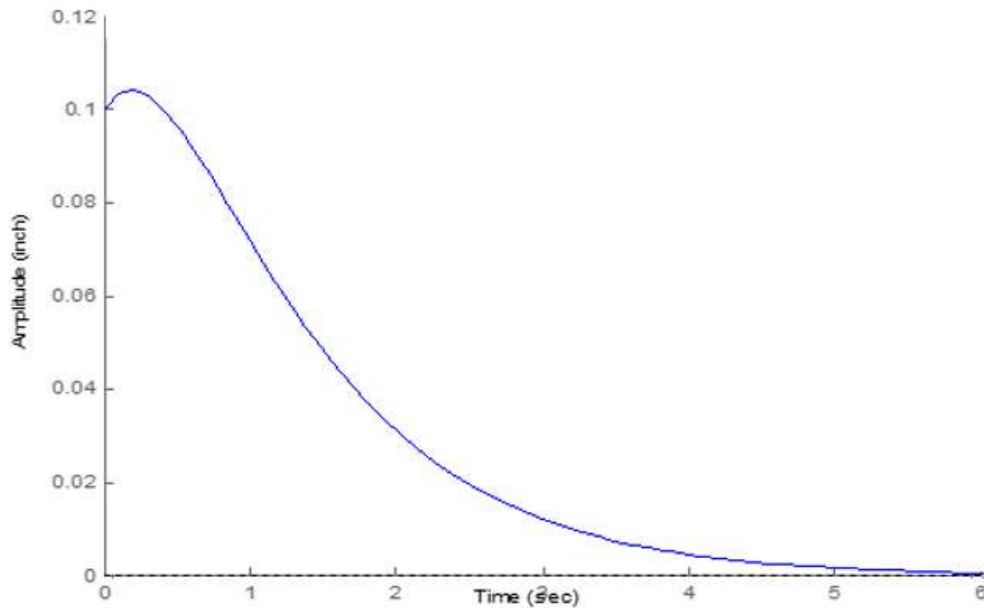Figure 2: Result of vibration analysis of scenario 1 using EXCEL

Figure 3: Result of vibration analysis of scenario 1 using MATLAB

A comparison of Figures 2 and 3 shows that there is a slight difference between the results obtained through EXCEL and MATLAB. However, the closeness of results indicates that the difference is due to numerical errors associated with all numerical techniques. It can be concluded that both sets of numerical results are correct.

Scenario 2:

Table 2 summarizes the vibrating system parameters of scenario 2.

Table 2 ((Summary of Parameters of Scenario 2)

Mass (lb.sec^2/inch)	Damping (lb.sec/inch)	Siffness (lb/inch)	Time increment (sec)	External force (lbf); t is sec	Initial displacement (inch)	Initial velocity (inch/sec)
1	0	2	.8	Sin 1.4 t	0	0

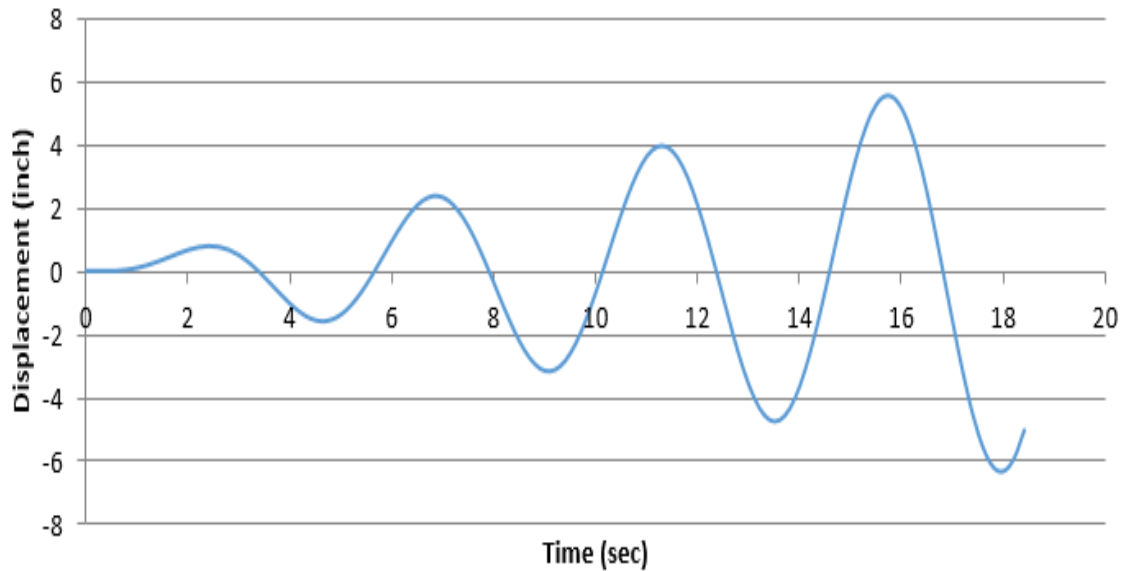Figure 4 is a plot of the EXCEL implementation.

Figure 4: Result of vibration analysis of scenario 2 using EXCEL

The natural frequency of the system is 1.41 rad/sec. The frequency of the input force is also 1.41 rad/sec. As expected, the vibration amplitude in the absence of a damping mechanism will grow indefinitely (the system is unstable).

Inputting the parameters defined in scenario 2 into equation (13) results in the transfer function shown in equation (17).

$$X(S) = (S^3 + 1.41) / (S^4 + 3 S^2 + 6) \qquad (17)$$

Using MATLAB, equation (17) can be converted into equation (18).

$$X(S) = \{(0.29S - 0.4) / [(S - 0.69)^2 + 1.4^2]\} + \{(0.71S + 1.79) / [(S + 0.69)^2 + 1.4^2]\} \quad (18)$$

Laplace Transform formulas are used to find the Inverse Laplace transform of equation (18) shown in equation (19).

$$X(t) = e^{0.69t} (-0.42 \sin 1.4t + 0.2 \cos 1.4t) + e^{-0.69t} (0.92 \sin 1.4t + 0.71 \cos 1.4t) \qquad (19)$$

Equation (19) can be put in the format of equation (20).

$$X(t) = 0.46 \, e^{0.69t} \sin(1.4t - 2.67) + 1.16 \, e^{-0.69t} \sin(1.4t+0.785) \qquad (20)$$

The portion of the solution of equation (20) containing the $e^{-0.69t}$ will die down as time is increased. However, the portion of the solution containing the $e^{0.69t}$ will increase as time is increased. This observation is consistent with the increasing amplitude of Figure 4.

Figure 5 is a plot of equation (20).

Displacement calculated by finding the exact response by
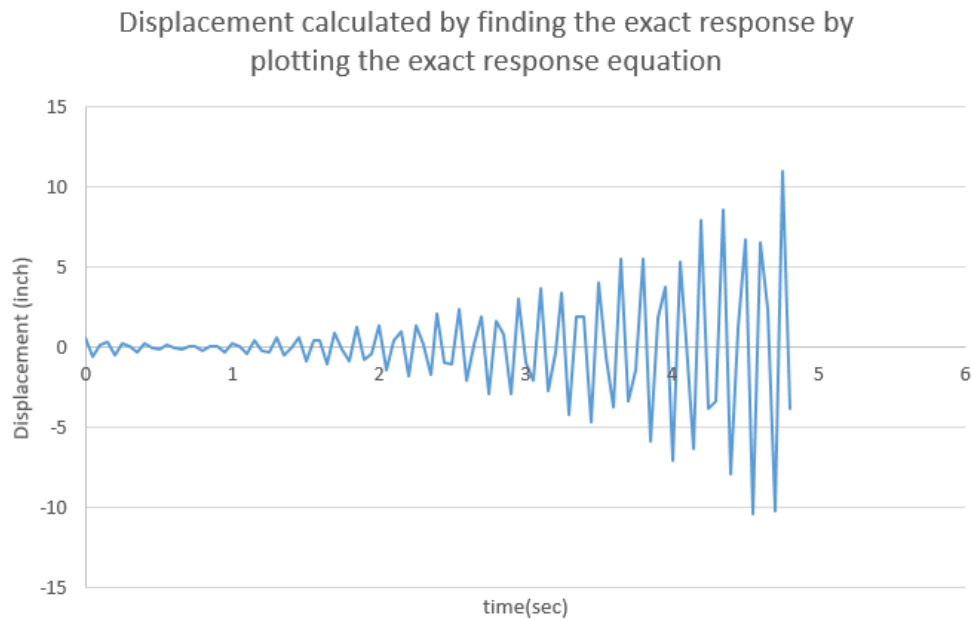plotting the exact response equation

Figure 5: Result of vibration analysis of scenario 2 using exact solution

A comparison of Figures 4 and 5 shows that both Figures indicate an unstable system as expected. However, the numerical responses are different. This is due to the large time interval used in the numerical solution of scenario 2 which results in numerical errors for solution of scenario 2. Recall that the ideal choice for h is not obvious because h must be small enough for the results to be close to exact solutions, but not too small such that excessive numerical errors are accumulated as a result of too many solution steps. [3], [5]

The plot of Figure 5 is the accurate plot because MATLAB tools along with inverse Laplace Transform formulas were used to find an exact solution. The numerical technique gives a smooth plot as shown in Figure 4 that is not the ideal plot for an accurate analysis of a control system. The exact solution gives a complete picture of system behavior including the decaying portion of the solution and phase angles as indicated in equation 20 and Figure 5. For these reasons, the choice of obtaining an exact solution is the preferred method for control system analyses.

Classroom use and lab use of technique:

The lab portion of a controls course the author has taught consisted of some physical controls experiments. However, the physical experiments were time consuming. By using the type of computer simulations demonstrated in this article, various changed scenarios related to the physical experiments were quickly implemented. The speed by which the computer simulations

were implemented greatly enhanced the student learning process regarding stability of control systems.

The end of the semester comments by students showed that the majority of the students benefited from the approach. The instructor's observation was that the introduction of the technique reduced the mathematics burden on the part of the students. This especially helps Engineering Technology students which are generally weaker than the engineering students in theoretical aspects including mathematics.

The technique can be expanded to more advanced controls courses by extending it to state-space analysis technique. A more complex system can and should be represented by higher-order differential equations. The higher order differential equations can be solved by classical mathematical or numerical techniques which is time consuming and difficult for most students (especially Engineering Technology students). Or, a higher order differential equation can be put in a matrix form consisting of only first order differential equations (MATLAB has the capability to do this automatically). Putting higher order differential equations in a matrix form consisting of only first degree differential equations is the basis of the state-space analysis technique. The matrix equations can then be solved by the extensive matrix solution capabilities of MATLAB (MATLAB stands for Matrix Laboratory).

Summary and Conclusion:

A control system can be mathematically described by developing a differential equation of the system. There are techniques for analyzing a differential equation. One technique would be to find an exact solution for the differential equation. However, most differential equations don't have known exact solutions. All differential equations can be solved by numerical techniques. However, the numerical techniques do not provide expressions that can be used to gain insight into the system, and the numerical techniques have the potential of being inaccurate. Numerical techniques produce results that can be plotted and the plots give some level of insight into the system behavior.

The control system can also be modeled in Laplace domain and the MATLAB partial fraction features can be used to put the output in a format that can readily be put in time domain by using the Inverse Laplace Transform formulas. This technique provides an expression that defines the control system behavior that will be more useful than only a plot in fine tuning a control system.

The techniques discussed in this article were used in a controls course and its accompanying lab the author has taught. The demonstrated techniques reduced the mathematical effort necessary on the part of the students. The demonstrated techniques can be expanded in more advanced controls courses to involve more complex control systems that would have higher order differential equations. The use of the state-space technique for mathematically solving the higher order differential equations is discussed but details of this technique are outside the scope of this article and not presented.

References:

[1] Solving Vibration Analysis Differential Equations by Numerical Techniques, Computers in Education Journal, Vol. 4, No. 3, July – September 2013, by C.Hagigat.

[2] Theory of vibration with applications, third edition by William T. Thompson, section 2.2.

[3] Technical calculus, special edition for the University of Toledo, section 10.7.

[4] Numerical Methods in Engineering Practice by Amir Wadi Al-Khafaji and John R. Tooley.

[5] Modern Control Engineering, Fifth Edition by Katsuhiko Ogata.

[6] Feedback Control Problems using MATLAB and the Control System Toolbox by Dean Frederick and Joe Chow.