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Comparison of Two Teaching Methods for Analyzing Fourbar Linkages

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Abstract

The fourbar linkage is one of the first mechanisms that a student encounters in a machine kinematics or mechanism design course and teaching the position analysis of the fourbar has always presented a challenge to instructors. The goal of this research study is to compare the effectiveness of teaching fourbar linkage analysis to engineering students with two different methods. An experiment with undergraduate engineering students is conducted to methodically establish the teaching effectiveness of the projection method in comparison with the traditional half-angle method.

In this study, we seek to quantify student performance in solving a fourbar linkage mechanism based on the time to solve and the correctness of the solution itself. In addition, we are also collecting students' self-reported perception of each method for comprehensibility, effort to solve, and ease of implementation with calculation tools. The goal is to test the research hypothesis that the projection method for fourbar linkages is easier to comprehend and easier to apply for solving problems.

The study is conducted with twenty-seven participants who are randomly divided into two nearly equal groups A and B. Two different fourbar lectures are given, and two different problems are used in this experiment. The first lecture is about the half-angle method and the second lecture is about the projection method. After the first lecture, participants in group A receive Problem 1, and participants in group B receive Problem 2. After the second lecture, the distribution is reversed. For problem 1, group A is the control condition and group B is the experimental condition. For problem 2, group B is the control condition and group A is the experimental condition.

From this study, we have found that the time required to complete problems using the projection method is significantly lower than the half-angle method even if the participants perceive both methods to be equally useful. We have also found that the student performance is significantly better with the projection method on one of the problems, although there is no observed statistical significance on the total score for the other problem. Based on our observations from this study, we conclude that the projection method is at a minimum similar to, if not easier than, the half-angle method for teaching fourbar linkage analysis to undergraduate students.

Introduction and Related Work

Position analysis of fourbar linkages has a long history, from the nineteenth century [1]-[2] until the present day [3]. Researchers have developed a variety of methods for conducting position analysis, but the solutions presented in the literature fall under two general categories:

Method 1: The angle between the coupler and the rocker (angle *BCD* in Figure 1) is found using the law of cosines. Once this is known, the coupler and rocker angles are found using some combination of the laws of sines and cosines.

Method 2: A vector loop equation is written around the linkage, and then half-angle tangent identities are used to solve for the two unknown angles.

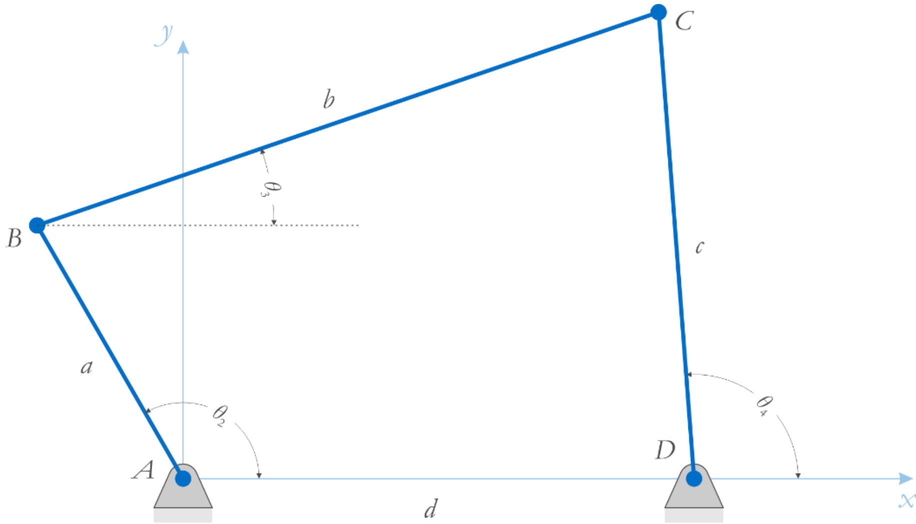


Figure 1. The classic fourbar linkage, with all angles defined from the horizontal. Here we assume that θ_2 is known, and we wish to solve for θ_3 and θ_4.

Both Norton [4] and Waldron [5] utilize *Method 2*, whose derivation is lengthy and whose final results permit no simple geometric interpretation. *Method 1* has a much simpler derivation and is used by Martin [6], Myszka [7], and Bulatović and Đorđević [8]. The dot product method presented by Wilson and Sadler [9] obtains essentially the same results, but in a more complicated fashion. Prior work by two of the co-authors [10] summarizes the new method for teaching fourbar linkages to engineering students. This method is referred to as the projection method. In the prior paper [10], the comparisons with other fourbar methods are established. Prior work has also verified and discussed the computational efficiency of the projection method in comparison to others [11].

There have been few studies exploring the effectiveness of different methods for teaching fourbar linkage analysis to undergraduate students. One relevant paper published by Boyle & Liu reported student feedback on pseudographic kinematic analysis for fourbar linkages [12]. The position analysis proposed in this paper is suitable for implementing in the TK Solver software. Boyle and Lui also state that the traditional mathematical steps for solving the fourbar are somewhat tortuous. They recognize that the graphical methods are more intuitive but are limited to solving one driving link position at a time. The projection method offers the advantage of being comparable to graphical approaches while being easy to implement in computational tools such as MATLAB.

Research Approach

This section describes the study procedure and population recruited for the study. The next subsection explains the data analysis procedure and the rubrics used for grading student performance.

Study procedure

Data collection was performed in two rounds, each nearly two hours long. Both rounds were moderated and taught by the same researchers. Table 1 details the structure of the data collection sessions. Participants first received a fill-in note packet and attended a short lecture on the **half-angle method**. The participants then solved a problem, where Group A received Problem 1 and Group B received Problem 2. The time required to complete the problem was recorded. Participants repeated this process for the **projection method**, except problem distribution was reversed, where Group A now received Problem 2 and Group B received Problem 1. Both Problems 1 and 2 were very similar, only containing different linkage lengths, linkage orientations, and application images.

One of the faculty authors is the originator of the projection method and teaches classes on design of mechanisms. Another faculty author teaches design of mechanisms and favors the projection method for teaching fourbar linkages in her classes. Two undergraduate researchers who are co-authors of this study took class on design of mechanisms. To avoid introducing any bias from the mechanisms course instructors, we invited a third faculty author, who does not teach design of mechanisms or related courses, for this study. He was an unbiased observer and lecturer for both methods. Both problems were chosen and set up by undergraduate researchers again to avoid any potential bias. Both authors who teach classes on design of mechanisms have biases toward the projection method and prefer teaching only the projection method in the classroom. They are of the opinion that projection method allows students to check their answers at intermediate steps along the way and that is how the method is taught in the classroom. Hence, we did not recruit students from our classes but instead opened the study recruitment to all campus and compensated participants for their time.

Study population

The participants recruited for this study were undergraduate engineering students enrolled at Rose-Hulman Institute of Technology. The participants received a gift card to the bookstore on campus for participation in the study. It was essential that the participants attended both lectures for this study so that they could complete the self-perception of the difficulty level of the material covered. Participants were randomly assigned to the 2 groups. Group A consisted of 13 students and Group B consisted of 14 students. The self-reported GPAs of the undergraduate students in both groups are similar as shown in Figure 2.

Table 1. Overview of the data collection method

Activity	Group A	Group B	Time (mins)	Description
Introduction			10	Consent form signing Overview of evening session
Lecture on Method 1			20	Half Angle lecture notes distributed Fourbar linkage animations shown Note packet filled in and derivation explained
Problem Solving Session 1	Problem **1**	Problem **2**	25	Participants given time to solve given problem
Lecture on Method 2			20	Projection lecture notes distributed Fourbar linkage animations shown Note packed filled in and derivation explained
Problem Solving Session 2	Problem **2**	Problem **1**	25	Participants given to solve given problem
Wrap-up			10	Participants given Feedback and Demographic forms, then allowed to leave

Self-Reported GPA range	Not reported	0-2.5	2.5-3.0	3.0-3.5	3.5-4.0	**Total**
Group A	2 (7.4%)	0 (0.0%)	1 (3.7%)	3 (11.1%)	7 (25.9%)	**13** (48.1%)
Group B	2 (7.4%)	1 (3.7%)	0 (0.0%)	4 (14.8%)	7 (25.9%)	**14** (51.8%)
Total	**4** (14.8%)	**1** (3.7%)	**1** (3.7%)	**7** (25.9%)	**14** (51.8%)	**27**

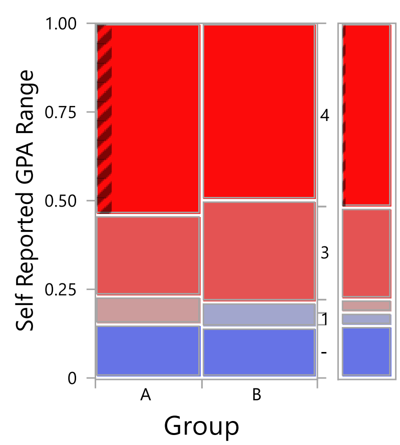


Figure 2. Self-reported GPA range for both groups of students

Data Analysis and Grading Rubric

Round one: Original grading rubric

Two researchers independently graded all problems using the same rubric. This rubric was made with the intention of identifying individual mistakes or misunderstandings rather than penalizing mistakes made earlier more heavily than later mistakes. For more details on the methods itself, kindly refer to the prior published work [10]. The following ground rules as well as Table 2 and 3 layout the grading scheme:

1. If the final answer is correct, full points are awarded.
2. For each step, if the answer is wrong but the formula is correct (calculator error), half points are awarded.
3. If values are off due to an error on an earlier step, but the process and calculation with that early mistake is correct, full points are awarded.

Table 2. Grading rubric for the half-angle method

Solution step	Point value (total 30)
$K_1 = \frac{d}{a}$	3
$K_2 = \frac{d}{c}$	3
$K_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}$	3
$A = K_3 - K_1 - (K_2 - 1) \cdot cos(\theta_2)$	4
$B = -2 \cdot sin(\theta_2)$	3
$C = K_3 + K_1 - (K_2 + 1) \cdot \cos(\theta_2)$	4
$\theta_4 = 2 \cdot \arctan\left(\frac{-B + \sqrt{B^2 - 4AC}}{2A}\right)$	10

Table 3. Grading rubric for the projection method

Solution Step	Point value (total 30)
$r = d - a \cdot \cos(\theta_2)$	2
$s = a \cdot \sin(\theta_2)$	2
$f^2 = r^2 + s^2$ (or $f = \sqrt{r^2 + s^2}$)	4
$\delta = \arccos\left(\frac{b^2 + c^2 - f^2}{2bc}\right)$	6
$g = b - c \cdot \cos(\delta)$	3
$h = c \cdot \sin(\delta)$	3
$\theta_3 = \arctan\left(\frac{hr - gs}{gr + hs}\right)$	5
$\theta_4 = \theta_3 + \delta$	5

Round two: Revised grading rubric

As will be later discussed in more detail in the **Results and Discussion** section, the first round of grading using this rubric resulted in an undesirable interrater reliability metric. All completed problems were regraded using a revised rubric. This rubric distinguished between calculation errors and concept errors. We deemed this weighting to be better suited to testing the learning of the method itself rather than algebra. The revised ground rules are as follows:

1. If the final answer is correct, full points are awarded.
2. For each step, if the answer is wrong but the formula correct, full points are awarded for that question, but a math error is deducted.
3. Math errors are treated as –2 points for a minor error, e.g., solving a single step in an equation wrong, or –4 points for a major or repeated error, e.g., mistakenly keeping a calculator in radians rather than degrees.
4. If the formula is wrong with no correct numerical answer, no credit is awarded.

Round three: Final grading rubric

To enable computing interrater reliability metrics, the final round involved converting the scores from the revised rubric (round two) into percentage form, dividing those values by 10, and assigning the resulting scores a single value depending on the range of values the scores fell in between. For instance, all scores between 24 to 26.99 out of 30 from the revised rubric were converted to 80.0% to 89.9% and assigned a score of 8. This allowed us to put all scores in 10 bins for comparison between datasets.

Results and Discussion

This section presents the results of the study. The first sub-section reports the interrater reliability of the final scores obtained by the participants. The second sub-section compares the time required for solving and the total scores obtained by participants with both methods. The final sub-section discusses participant feedback received and the perceived difficulty level of each method.

Interrater reliability

The weighted Cohen's Kappa values for the interrater reliability of the scores received on the graded problems are reported in Table 4. Two raters graded the complete dataset as mentioned earlier. With each rating iteration, the grading rubric was revised to reduce any discrepancies. The first iteration of kappa value was compared based on the score out of 30 points. For the second iteration, the rubrics were revised to achieve better consistency, but were still based on a 30-point scale. In the final iteration, the scores were converted into a 10-point ordinal scale based on percentages of scores. This adjustment was done to simulate the actual percentage or letter grades that student would receive on an exam. The conversion to 10 bins allowed for better statistical analysis of the total scores.

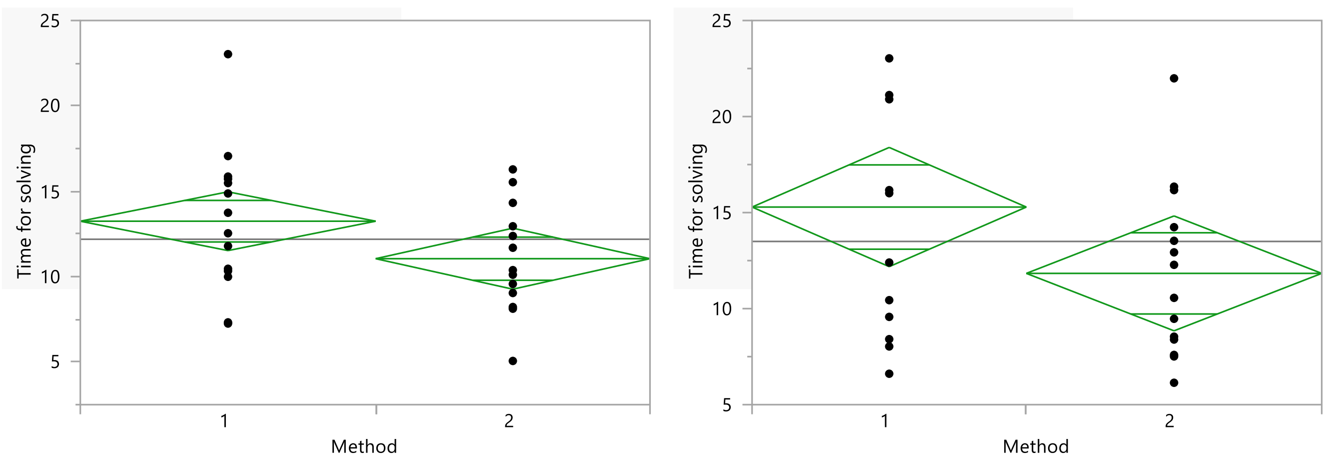
Table 4. Weighted Cohen's Kappa (linear-weighted) for total scores for the complete dataset

	Round one	**Round two**	**Round three**
	30-point continuous scale	*30-point continuous scale*	*10-point ordinal scale*
Problem 1	0.68	0.63	0.76
Problem 2	0.58	0.64	0.69

Comparison Between the Two Methods

Once we established acceptable interrater reliability, we compared the results for the control and experimental conditions along with the total time needed to solve the problems in this sub-section. Since this is an exploratory study, the criterion for statistical significance is set at $\alpha = 0.10$. The control condition is the method that is taught traditionally in kinematics classes: the half-angle method. The experimental method is the projection method that has been recently developed by one of the authors.

Figure 3 compares the total time taken by the participants for both methods *(1 = half-angle method, 2 – projection method)*. The results of the total time for solving indicate that the average amount of time required by the projection method was significantly lower than the half angle method (p-value < 0.10). This is observed for both problems, so we can confidently conclude that the projection method is a faster solution approach compared to the half-angle method.

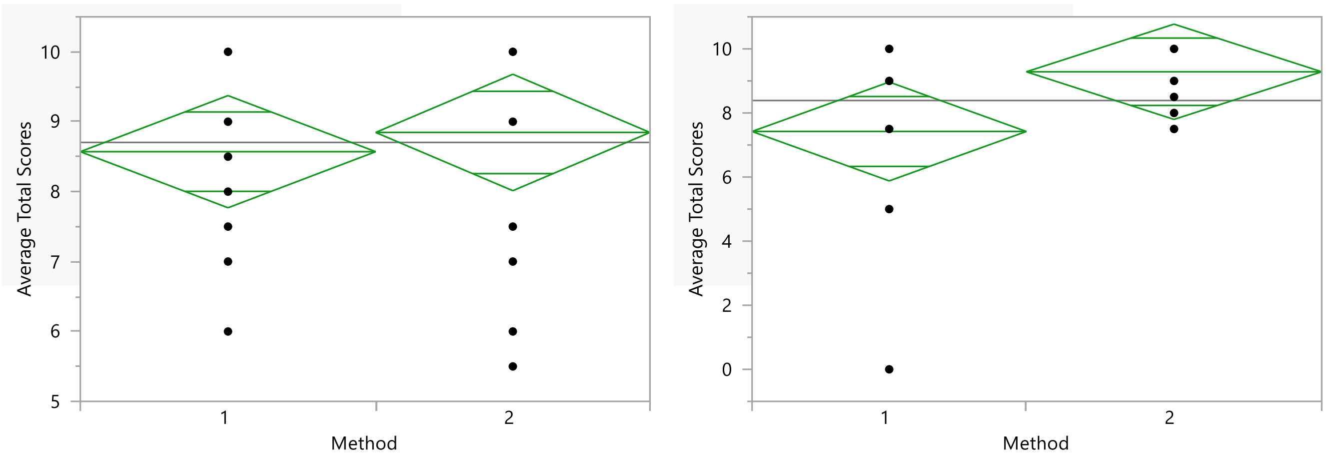


Problem 1 washer (p-value 0.07)

Problem 2 wheel (p-value 0.06)

Figure 3. Total time taken by the participants for solving for both problems
(1 = Half-angle method, 2 – Projection method)

Figure 4 compares the total scores obtained by the participants for both problems by both methods *(1 = half-angle method, 2 – projection method)*. The total score compared in the analysis is the average of the two scores reported by each rater. Even though the average total score is marginally higher for Problem 1 using the projection method, the difference is not statically significant. However, for Problem 2, the average total scores from the projection method are statistically higher than those associated with the half-angle method. Based on the data collected, we conclude that the performance of students on graded solutions to the problems using the projection method is at a minimum equal to, if not higher than, those using the half-angle method.



Problem 1 washer (p-value 0.31) Problem 2 wheel (p-value 0.04)

Figure 4. Total scores obtained by the participants for both problems
(1 – Half-angle method, 2 – Projection method)

<u>Participant Feedback</u>

Feedback was collected from the participants to understand their perspective on the two methods for solving fourbar linkages. This feedback was collected at the end of the study after all participants attended both lectures and solved problems using both methods. One participant did not complete the feedback form so that datapoint was eliminated from the analysis.

As shown in Table 5, two-thirds of the students had not heard about fourbar mechanisms before participating in the study. All the participants reported that they understood the half-angle method. Ninety six percent said that that projection method lecture was easy to follow. A similar trend was observed in the response to the question pertaining to the ability to explain the methods to somebody else with provided notes. Based on the responses, we conclude that the lectures conducted for each method are clear and easy to follow.

Table 5. Participants' feedback on lectures on the half-angle method and the projection method

Feedback Questions	No	Some-what	Yes
Have you heard about or worked on fourbar mechanism before?	18 (69%)	3 (11%)	5 (20%)
Do you feel comfortable to solve engineering problems using trigonometry?	1 (4%)	11 (42%)	14 (54%)
Were the problem statements clear?	1 (4%)	2 (8%)	23 (88%)
Was the training for **half angle method** easy to follow?	0 (0%)	5 (20%)	21 (80%)
Was the training for **projection method** easy to follow?	1 (4%)	8 (31%)	17 (65%)
Are you able to explain **half angle method** to somebody else with notes provided?	0 (0%)	11 (42%)	15 (58%)
Are you able to explain **projection method** to somebody else with notes provided?	1 (4%)	12 (46%)	13 (50%)

Table 6 show the participant's perception on ease of understanding, effort required, and ease of implementation. Participants were split between the two methods in terms of ease of understanding, maybe slightly leaning toward half-angle method. Participants perceived that the half-angle method took fewer steps to finish even though the time for completion was significantly higher with half-angle method, as discussed in earlier in Figure 3. Finally, when asked about the ease of implementation using computational tools, the participants were equally divided between the two methods. A few participants were unsure, and the same number thought they were almost the same.

Table 6. Participants' perception on ease of understanding, efforts required, ease of implementation

Feedback Questions	Half Angle method	Almost the same	Projection method	Unsure
Which method is **easier to understand**?	11 (42%)	5 (20%)	10 (38%)	0 (0%)
Which method took **fewer steps or less effort** to finish the problem?	15 (58%)	3 (12%)	8 (30%)	0 (0%)
Which one do you think is **easier to implement** in MATLAB or Maple?	8 (30%)	5 (20%)	8 (30%)	5 (20%)

Conclusions

This paper presents results of an exploratory study to compare two teaching methods for analyzing fourbar linkages. Participants were asked to attend lectures pertaining to the two different methods and they then solved problems using those methods. Feedback was collected at the end of the study about participants' perceived ease of understanding, level of effort, and implementation.

The authors acknowledge that a more thorough analysis may be needed particularly through reversing the order in which the methods are introduced to assist in establishing more robust results. However, due to limitations in time and availability of participants, the study was limited to presenting the half-angle method first and the projection method second. We welcome collaborators from larger schools who may be interested in conducting a thorough study with a robust population group.

The results of the study show that the scores received by the participants on graded solutions to the problems are at a minimum equal, if not higher, with the projection method. The required solution time is (statistically) significantly less for the projection method compared to the half-angle method. The participant perception for both methods is similar. Based on the observations from this study, we recommend that the projection method be considered for adoption by instructors who teach fourbar linkage analysis to undergraduate students.

References

[1] A. Cayley, "On Three-bar Motion," *Proceedings of the London Mathematical Society,* vol. VII, pp. 136-166, 1876.

[2] S. Roberts, "On Three-bar Motion in Plane Space," *Proceedings of the London Mathematical Society,* vol. VII, pp. 14-23, 1875.

[3] C. W. Wampler, "Solving the Kinematics of Planar Mechanisms," *Journal of Mechanical Design,* vol. 121, no. 3, pp. 387-391, 1999.

[4] R. Norton, *Design of Machinery*. New York: McGraw-Hill, 2012.

[5] K. J. Waldron and G. L. Kinzel, *Kinematics, Dynamics and Design of Machinery*. Wiley, 2004.

[6] G. H. Martin, *Kinematics and Dynamics of Machines*. Prospect Heights, IL: Waveland Press, 1982.

[7] D. H. Myszka, *Machines and Mechanisms*. Upper Saddle River, NJ: Prentice Hall, 1999.

[8] S. Dordević and R. Bulatović, "On the optimum synthesis of a four-bar linkage using differential evolution and method of variable controlled deviations," *Mechanism and Machine Theory,* vol. 44, pp. 235-246, 2009.

[9] C. E. Wilson and J. P. Sadler, *Kinematics and Dynamics of Machinery*. Upper Saddle River, NJ: Prentice Hall, 2003.

[10] E. Constans, K. Dyer, and S. Sangelkar, "A New Method for Teaching The Fourbar Linkage and its Application to Other Linkages," in *2019 ASEE Annual Conference & Exposition*, 2019.

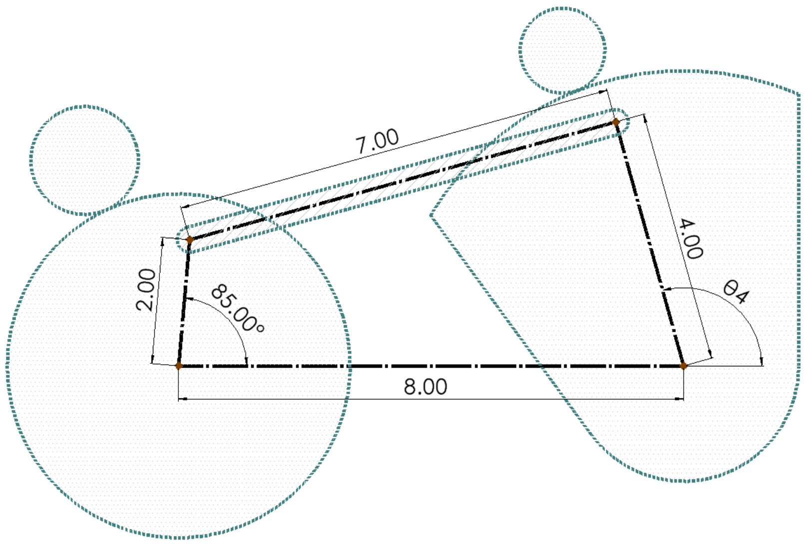
[11] E. Constans, T. R. Chandrupatla, and H. Zhang, "An efficient position solution for the fourbar linkage," *International Journal of Mechanisms and Robotic Systems,* vol. 2, no. 3-4, pp. 365-373, 2015.

[12] W. Boyle and K. Liu, "The fourbar linkage: pseudographic kinematic analysis," *International Journal of Engineering Education,* vol. 13, no. 4, pp. 262-268, 1997.

Appendix A: Problem 1

Problem Washing machine

For the fourbar mechanism shown in the figure, calculate the angle of the rocker (θ_4) when the angle of the crank is 85 deg.



Appendix B: Problem 2

Problem Wheel drive

For the fourbar mechanism show in the figure, calculate the angle of rocker (θ_4) when the angle of crank (θ_2) = 75 deg. Crank is the link that is going in full circle on the wheel.