Component Assembly Modeling Using Monte Carlo Simulation: Industry-Based Project

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Abstract

This paper describes an industry-based project in developing processes and tools to conduct Monte Carlo simulation in modeling and analysis of real industrial components. The tool used in the project is the Microsoft Excel spreadsheet. A random selection of part dimensions for 1000 assemblies of automotive torque converter is selected for the analysis. A probability distribution for each dimension is used to represent the expected values. The equations and the spreadsheet were developed by students in one semester time frame. The simulation work was performed in the other semester. This project gives students an integrated understanding of topics in statistics, tolerance analysis and manufacturing applications as well as the opportunity to be involved in an industrial project.

1 Introduction

The ability to assemble components depends on the dimensions of each component. In an ideal assembly process, it would be best if each component were identical and perfect such that they can be interchanged. However, each component has dimensional variation resulted from its preceding manufacturing process. Hence, the manufacture of interchangeable components requires a tolerance for each of the dimensions. The affect is that tolerances reduce the potential for some components to be assembled. The tolerance limits that are applied to component dimensions are usually determined through a stack analysis.

Tolerance stack is the sum of tolerances of two or more dimensional tolerances. The stack may generated by the addition of the overall length tolerance of two or more parts in assembly process [1]. Three types of tolerance stack analysis are limit stacks, probability stacks or Root Sum Square (RSS), and Monte Carlo simulations [2]. The most popular is the limit stack analysis. The limit stack analysis is the summation of all dimensional limits in a stack path. This analysis assumes that all dimensions of the part are at the extreme limits simultaneously. The positive result of this type of analysis is that component tolerances defined using limit stack analysis will always assemble as long as the manufacturing process controls the dimensions to be within the tolerance limits. In fact, some of the assemblies with components that exceed the limits can also be assembled. The negative result of a limit stack analysis is that the dimensional tolerances are more restrictive to the manufacturing processes. The manufacturing processes are driven to more expensive solutions to obtain these smaller tolerance limits. The cost to purchase these processes and maintain the tooling is more expensive than is needed.

The probability stack analysis is referred to as RSS. The stack analysis is determined by taking the square root of the summation of the square of each tolerance. This method requires more calculations and therefore is used less often. Many times this method is used to justify not meeting the tolerances that are required for the limit stack analysis. The RSS method assumes that the tolerance to each dimension is normally distributed around the tolerance mean. While this is closer to the distribution of the tolerance for most processes, it does not allow for other types of statistical distributions or variations in the mean value of the distribution. The variation in the mean value of the distribution is the most serious problem with RSS stacks. The result is that the manufacturing limits are reduced or adjusted to the RSS analysis method results [2].

The third type of stack analysis is the Monte Carlo simulation. This type of analysis is based on the random selection of dimensions for each component. That is, each proposed dimension and the distribution of that tolerance are used to define the actual dimensions of each component. The result is an assembly analysis that mimics the intended manufacturing process. Components that are outside of the limits cannot be assembled. The advantage of the Monte Carlo simulation is that different tolerance variation distributions can be applied to each dimension. The result is a more representative assembly variation than either of the other two types of analyses [3, 4]. The Monte Carlo simulation for tolerance stack analysis has been applied in industry for many years [5, 6].

This project was introduced to students in MIT3600 course, Process Engineering, in the Division of Engineering Technology, in the College of Engineering at Wayne State University. It is an elective and three credit hours course for students' majors in Mechanical Engineering Technology or Manufacturing/Industrial Engineering Technology. The course has been offered in every semester and usually has more than fifteen students. The project was initialized from author's industrial consulting assignment in a major automotive manufacturer. The equations and Microsoft Excel spreadsheet were developed by students in one semester time frame. The simulation work was performed in the other semester. This project gives students an integrated understanding of topics in statistics, tolerance analysis and manufacturing applications as well as the opportunity to be involved in an industrial project.

2 Overview of Torque Converter Assembly

The efficiency of an automatic transmission torque converter is partially dependent upon the torque converter stator [7]. While the vehicle is coasting, the stator must rotate with the torque converter to minimize drag. During vehicle acceleration the stator re-directs the fluid from the toroid of the turbine to the pump of the torque converter. The force applied to the stator under acceleration must be grounded to re-direct the fluid. Since the torque converter is a rotating sub-system of the transmission, the grounding of the stator is accomplished external to the torque converter. The conditions of rotation in one direction and grounding in the opposite direction are accomplished with a one-way roller clutch that grounds through the inner race and stator shaft [8]. The stator shaft and inner race of the one-way clutch have straight splines to allow ease of manufacture and assembly of the components. The straight splines also eliminate axial thrust and allow variation in the position when the torque converter has internal pressure variation.

The concern with the ability to assemble the components on the assembly line for high volume production is time. A tight fit between the components results in more variation in assembly time. A lose fit reduces the variation in assembly time. This assembly is especially difficult because the mating of the splines is a blind assembly. Variations in assembly time means that there will never be the correct number of people involved in that step of the sequence. Sometimes there will not be enough people to keep up with production and other times there will be too many people. The intuitive solution to reduce the assembly time variation is to increase the clearance between the splines.

3 Application of Monte Carlo Simulation in Torque Converter Assembly

The approach for the Monte Carlo simulation analysis is to virtually build 1000 assemblies and test whether assembly will occur with no assembly aids (additional procedures) other than the direct insertion of the stator inner race on the stator shaft. The percentage of the assemblies that require no additional operations determines the acceptability of the design. All of the dimensions that control the interface between the roller clutch inner race and the stator shaft are used in building the assemblies. The variables are simulated using the Monte Carlo method for the random selection of the dimensions. For most of the variables, the tolerance range is considered to be a normal distribution with 99.7 % of the components between the specification limits. The angular position between the stator shaft spline and inner race spline is considered to have a uniform distribution.

A typical assembly of torque converter is shown in Fig.1. The analysis was based on the interface of the stator shaft to the inner race of the roller clutch. This has been identified by the assembly plant as the area where assembly problems occur. To simplify the analysis several assumptions are needed as:

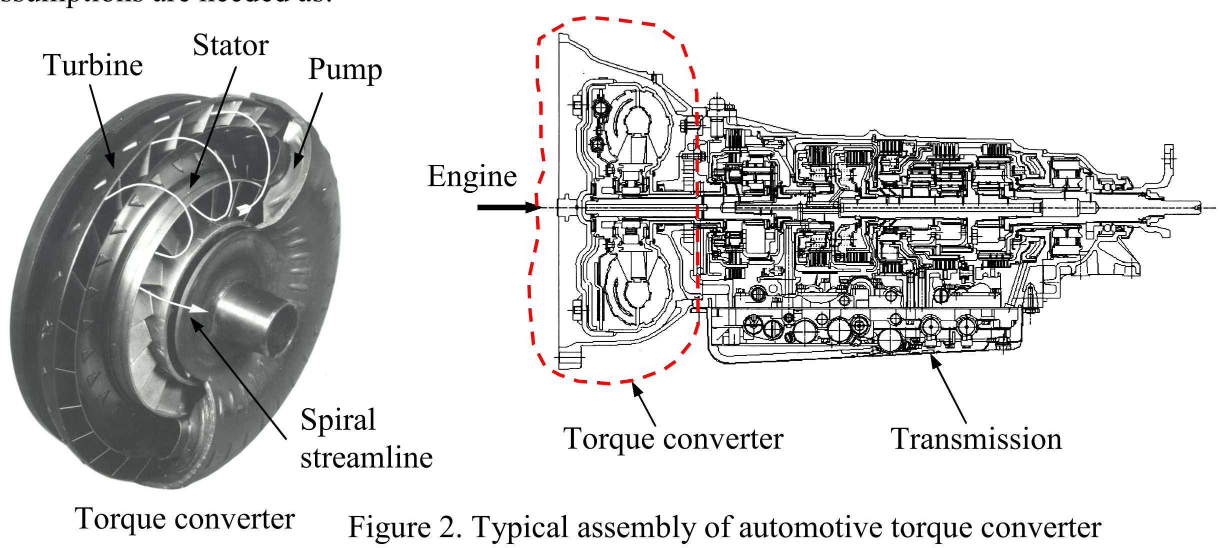

Figure 2. Typical assembly of automotive torque converter

- Both parts are made to specifications illustrated in the part print.
- All splines within a component are made the same to simplify the analysis
- Alignment of the splines during initial assembly is the only consideration. No other considerations, such as rotating the torque converter, are considered.
- The centerlines of the components are coincident.
- The interface between the stator shaft and inner race of the roller clutch is considered frictionless.
- Normally distributed variables are represented by a +/- 3 sigma distribution within the tolerance specification.

The inner race of the roller clutch is designed with a 50 degree lead-in chamfer on the splines. The stator shaft is designed with a 30 degree lead-in chamfer on the splines. The result is that first contact will occur near the inner diameter (minor diameter) of the splines. This is shown in the sketch of Fig. 2.

The concentric centers are justified by the variation in the chamfer angles. As long as the splines are chamfered as shown in Fig. 2, the first contact will be at the base of the splines on the stator shaft even if the centers are not concentric. If the centerlines were not concentric, the probability of assembly would change. The off-center position is not analyzed in the study. The actual contact point is also dependant upon several dimensions: the minor diameter of the inner race (internal splines), the corner radius of the splines and the angle of the spline face. The vertical distance from the centerline of the corner radius to the tangent point of the radius to the spline face is subtracted from the sum of the minor radius and corner radius. This dimension defines the contact point radius for the two splines and is shown in Fig. 3.

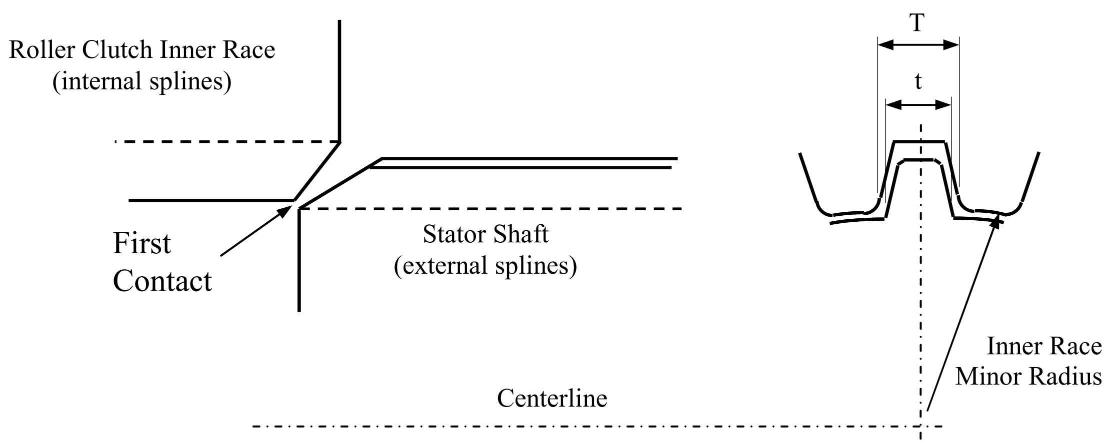

Figure 2. Relationship of stator shaft splines to the roller clutch inner race splines

The angle of the spline face (pressure angle) does not have a specified tolerance. Therefore, the angle is considered to be a constant. Equation (1) defines the radius to the contact point as:

$$R_{CP} = R_r + R_c - R_c\,(\sin 30^\circ) \quad (1)$$

where R_{CP} is radius of contact point (CP), R_r is minor radius of the inner race, and R_c is a radius of the spline corner.

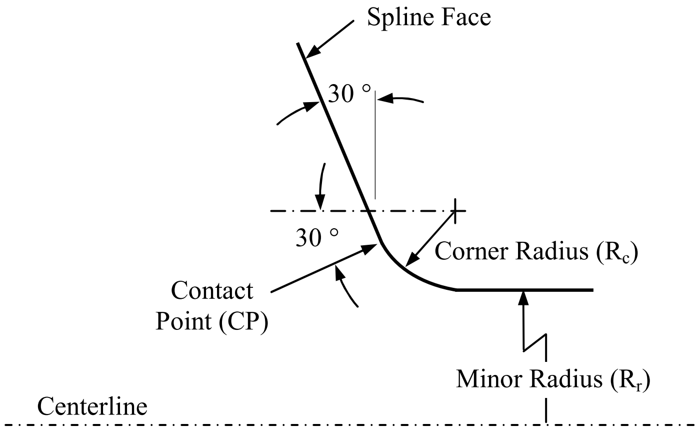

Figure 3. Relationship of contact point to dimensions on the inner race

The width of the gap (T) to the adjacent spline is dependent upon the variation in the gap between adjacent splines of the inner race at the contact point. The width of the gap is defined on the print at the pitch diameter. This gap must be translated to the contact point radius using the linear relationship of the spline face angle to the radial line of the inner race. Since each face is treated separately, the change in gap must be twice the calculated value. This is shown as:

$$T = G + 2\,(R_P - R_{CP})(\tan 30°) \qquad (2)$$

where G is gap at the pitch diameter and R_P is pitch radius.

The width of the stator shaft splines (t) is determined in a similar manner. The contact radius defined for the inner race and the width at the pitch radius is used to calculate the width of the tooth. As previously stated, the equation is similar to equation (2) and is shown as:

$$t = W + 2\,(R_p - R_{CP})\,(\tan 30°) \qquad (3)$$

where t is spline width at contact point, W is width of stator splines, and R_p is pitch radius of stator shaft.

The next step in the analysis is to determine the angular position of the stator shaft relative to the inner race. The range of this variable can be limited to the angle from the centerline of one spline to the centerline of the adjacent spline. This simplifies the analysis and is valid because the splines are assumed consistent. These components have 27 splines. Therefore, the angle is represented by equation (4).

$$A = 360° / 27 \text{ splines} = 13.33° / \text{spline} \qquad (4)$$

where A is an assembly angle.

The most significant difference in the variable is the type of distribution of angle (A). There is an equal likelihood of selecting any of the angles. Therefore, a uniform distribution was selected for the analysis. The ability to assemble the components depends upon placing the stator spline in the gap of the inner race. To define whether this has occurred, the angle formed by the thickness of the spline at the contact point is added to the assembly angle (A) and the sum is subtracted from the angle of the gap. That calculation is shown as

$$Ø = a_T - (a_t + A) \tag{5}$$

where Ø is a difference in angle between components, a_T is gap angle at the contact radius, and a_t is spline width angle at the contact radius.

The inner race gap and stator shaft spline angles are calculated from the spline variables. The difference in the angles is used to determine whether the components will assemble. If the angle (Ø) is equal to or greater than zero the components will assemble. However, if the angle is negative the components will have interference and not assemble.

The first analysis is referred to as the baseline. That combination includes all of the component dimensions except waviness or scallops. It also includes the variation in assembly angle. The manufacturing process selected for the forming of the stator shaft splines was hobbing. The hobbing forms the scallops in the surface of the splines. A sketch representing the surface waviness is shown in Fig. 4. The waviness of the stator shaft spline face can affect the ability to assemble the components. Waviness is a variable that subtracts from the spline width on each side of the spline. Therefore, the effective spline is defined by equation (6).

$$\text{Effect width} = t - 2*w \tag{6}$$

where w is spline waviness.

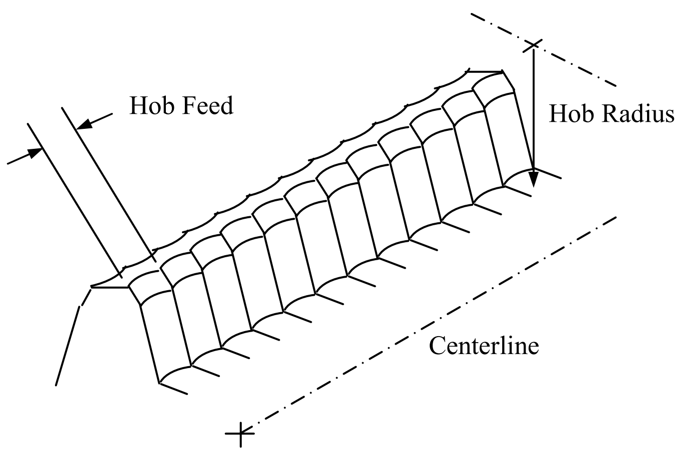

Figure 4. Representation of the waviness (scallops) that occur with hobbing

The second analysis added the effect of the waviness (scallops). The dimensions specified for the splines are to be checked over pins. That means the dimension to the peak of the scallop is measured but not the valleys. The scallops occur like a spiral along the axis of the shaft. During assembly anything from the peak to the valley can occur. The likelihood of getting a peak verses a valley is not a normal distribution. The valley is more likely than the peak. However, for simplicity of the analysis, the distribution is assumed to be normal across the depth of the scallops.

4 Analysis Results

The application of a Monte Carlo simulation is to randomly select a specific dimension from the range of possible dimensions for each variable. A partial example of how this is done is shown in Table 1 for 10 assemblies. The calculations are accomplished with a spreadsheet in Microsoft Excel. Each column represents a variable. The random numbers for the analysis are generated in Data Analysis of the Tools menu bar. All of the information is entered through a data entry sheet that is presented. The remaining columns are simply the application of the equations developed earlier and a calculation of the percent assembled.

Table 1. Analysis calibration to define the expected number of paired components that can be assembled

Effective gap width (T=G+2*X)	Pitch dia. ($2*R_p$)	Y' dim. (R_p-R_{CP})	X' dim. (Y' *tan30)	Spline thick. (W)	Spline wav. (w)	Effec. spline (W+2*X' -2*w)	Gap angle (a_T)	Spline angle (a_t)	Assm. angle (A)	Delta angle (°)	Assm.? 1 = Yes 0 = No	Percent Assembled
2.3566	34.3522	0.3596	0.2076	1.9014	0	2.3167	8.04	7.90	0	0.07	1	77.7%
2.4842	34.3530	0.4787	0.2764	1.9388	0	2.4915	8.53	8.56	0	-0.01	0	(98.2%
2.5289	34.6868	0.5498	0.3174	1.8966	0	2.5314	8.64	8.64	0	0.00	0	Expected
2.4920	34.2101	0.3900	0.2252	1.9528	0	2.4032	8.55	8.24	0	0.15	1	Value)
2.3329	34.2826	0.3864	0.2231	1.8392	0	2.2854	7.98	7.82	0	0.08	1	
2.5565	34.3580	0.3949	0.2280	1.8805	0	2.3365	8.74	7.98	0	0.38	1	
2.4464	34.4827	0.4380	0.2529	1.9239	0	2.4296	8.35	8.29	0	0.03	1	
2.5011	34.5128	0.4797	0.2769	1.9056	0	2.4595	8.55	8.41	0	0.07	1	
2.4483	34.3241	0.3887	0.2244	1.9299	0	2.3787	8.37	8.13	0	0.12	1	
2.4017	34.3058	0.4218	0.2435	1.9042	0	2.3913	8.23	8.20	0	0.02	1	

Only 10 of the 1000 assemblies are shown in Table 1. The first column is the effective gap width of the stator inner race (equation 2). Column seven is the effective spline width of 10 mating stator shafts (equation 3). The result of assembling all 1000 of the inner races and stator shafts is shown in the last column as 77.7 %. Since the relative angle is held constant at zero, 98.2 % of the population should have assembled. The main reason for the lower number of paired components not assembling is the assumption that all dimensions use +/- 3 sigma of the tolerance range. Components are normally manufactured to a tighter tolerance then specified to insure that the dimensions will always stay within the specification range.

The 77.7 % of the paired components that assembled is used as the upper limit for the other analyses. Any modification to the components should not result in a number of assemblies that exceed 77.7 %.

A sample of the table used to define the results of the baseline assemblies is shown in Table 2. With the assembly angle added as a variable, the number of component pairs that can be assembled decreases to 2.7 % of the population. The drop from 77.7 % is specifically due to the potential of the splines to overlap. The expected value dropped slightly to 97.9 % because the number of variables that are being analyzed increased by one.

Table 2. Baseline assembly analysis including the assembly angle

Effective gap width (T=G+2*X)	Pitch dia. ($2*R_p$)	Y' dim. (R_p-R_{CP})	X' dim. (Y' *tan30)	Spline thick. (W)	Spline wav. (w)	Effec. spline (W+2*X'-2*w)	Gap angle (a_T)	Spline angle (a_t)	Assm. angle (A)	Delta angle (°)	Assm.? 1 = Yes 0 = No	Percent assembled
2.3566	34.3522	0.3596	0.2076	1.9014	0	2.3167	8.04	7.90	-6.6447	-6.51	0	2.7%
2.4842	34.3530	0.4787	0.2764	1.9388	0	2.4915	8.53	8.56	5.008904	-5.03	0	(97.9%
2.5289	34.6868	0.5498	0.3174	1.8966	0	2.5314	8.64	8.64	-1.67994	-1.69	0	Expected
2.4920	34.2101	0.3900	0.2252	1.9528	0	2.4032	8.55	8.24	3.281962	-2.98	0	Value)
2.3329	34.2826	0.3864	0.2231	1.8392	0	2.2854	7.98	7.82	4.588562	-4.43	0	
2.5565	34.3580	0.3949	0.2280	1.8805	0	2.3365	8.74	7.98	6.122626	-5.37	0	
2.4464	34.4827	0.4380	0.2529	1.9239	0	2.4296	8.35	8.29	3.069553	-3.01	0	
2.5011	34.5128	0.4797	0.2769	1.9056	0	2.4595	8.55	8.41	0.514949	-0.37	0	
2.4483	34.3241	0.3887	0.2244	1.9299	0	2.3787	8.37	8.13	4.512062	-4.27	0	
2.4017	34.3058	0.4218	0.2435	1.9042	0	2.3913	8.23	8.20	6.614992	-6.58	0	

A sample of the table used to analyze the effect of waviness is shown in Table 3. The effect of waviness (scallops) is to increase the probability of assembly by 0.1 %. Even with the waviness in the stator shaft splines, the number of pairs that can be assembled without aid is unacceptable. The total percentage that can be assembled is shown in the last column of Table 3. The expected value for the number of pairs that can be assembled is slightly lower because of the number of variables increased from seven to eight.

The optimum internal spline design was only applied to the inner race of the roller clutch. A sample of the table that was used for the analysis of this condition is shown in Table 4. The difference in this table is due to column one. The gap at the lead-in to the spline for the inner race increased significantly. With the increase in the gap, the number of component pairs that can be assembled increased to 23.5 %. This is a significant increase in the number of completed assemblies. Additional operations are not required on this percentage of the production population.

Based on the assembly validation of 77.7 % and the analysis of the optimum design applied to the inner race (23.5 %), approximately 30 % of the production assemblies do not require any type of assembly aid. Seventy percent require additional aids such as removal and re-installation of the torque converter, rotation of the converter during assembly or rocking of the torque converter while it is sitting on the stator shaft.

Table 3. Analysis of the effect of waviness on the number of component pairs that can be assembled.

Effective gap width (T=G+2*X)	Pitch dia. (2*R_p)	Y' dim. (R_p-R_{CP})	X' dim. (Y' *tan30)	Spline thick. (W)	Spline wav. (w)	Effec. spline (W+2*X'-2*w)	Gap angle (a_T)	Spline angle (a_t)	Assm. angle (A)	Delta angle (°)	Assm.? 1 = Yes 0 = No	Percent assembled
2.3566	34.3522	0.3596	0.2076	1.9014	0.000112	2.3165	8.04	7.90	-6.6447	-6.51	0	2.8%
2.4842	34.3530	0.4787	0.2764	1.9388	0.002895	2.4857	8.53	8.54	5.008904	-5.01	0	(97.6%
2.5289	34.6868	0.5498	0.3174	1.8966	0.005866	2.5197	8.64	8.60	-1.67994	-1.65	0	Expected
2.4920	34.2101	0.3900	0.2252	1.9528	0.003935	2.3953	8.55	8.22	3.281962	-2.95	0	Value)
2.3329	34.2826	0.3864	0.2231	1.8392	0.002668	2.2801	7.98	7.80	4.588562	-4.41	0	
2.5565	34.3580	0.3949	0.2280	1.8805	0.002118	2.3323	8.74	7.97	6.122626	-5.36	0	
2.4464	34.4827	0.4380	0.2529	1.9239	0.005247	2.4191	8.35	8.26	3.069553	-2.98	0	
2.5011	34.5128	0.4797	0.2769	1.9056	0.00383	2.4518	8.55	8.38	0.514949	-0.35	0	
2.4483	34.3241	0.3887	0.2244	1.9299	0.003459	2.3718	8.37	8.11	4.512062	-4.25	0	
2.4017	34.3058	0.4218	0.2435	1.9042	0.005163	2.3810	8.23	8.16	6.614992	-6.54	0	

Table 4. Analysis of the optimum design applied to the roller clutch inner race

Effective gap width (T=G+2*X)	Pitch dia. (2*R_p)	Y' dim. (R_p-R_{CP})	X' dim. (Y' *tan30)	Spline thick. (W)	Spline wav. (w)	Effec. spline (W+2*X'-2*w)	Gap angle (a_T)	Spline angle (a_t)	Assm. angle (A)	Delta angle (°)	Assm.? 1 = Yes 0 = No	Percent assembled
3.1304	34.3522	0.3596	0.2076	1.9014	0.000112	2.3165	10.68	7.90	-6.6447	-3.86	0	23.5%
3.2801	34.3530	0.4787	0.2764	1.9388	0.002895	2.4857	11.27	8.54	5.008904	-2.27	0	(97.6%
3.2696	34.6868	0.5498	0.3174	1.8966	0.005866	2.5197	11.17	8.60	-1.67994	0.89	1	Expected
3.2689	34.2101	0.3900	0.2252	1.9528	0.003935	2.3953	11.22	8.22	3.281962	-0.28	0	Value)
3.1627	34.2826	0.3864	0.2231	1.8392	0.002668	2.2801	10.83	7.80	4.588562	-1.56	0	
3.3059	34.3580	0.3949	0.2280	1.8805	0.002118	2.3323	11.30	7.97	6.122626	-2.79	0	
3.2055	34.4827	0.4380	0.2529	1.9239	0.005247	2.4191	10.95	8.26	3.069553	-0.38	0	
3.2949	34.5128	0.4797	0.2769	1.9056	0.00383	2.4518	11.27	8.38	0.514949	2.38	1	
3.2387	34.3241	0.3887	0.2244	1.9299	0.003459	2.3718	11.08	8.11	4.512062	-1.54	0	
3.1440	34.3058	0.4218	0.2435	1.9042	0.005163	2.3810	10.78	8.16	6.614992	-3.99	0	

Conclusions

This paper describes an industry-based project in developing process and tool to conduct Monte Carlo simulation in modeling and analysis of real industrial components. This project gives students an integrated understanding of topics in statistics, tolerance analysis and manufacturing application as well as the opportunity to be involved in an industrial project. The tool used in the project is the Microsoft Excel spreadsheet. The equations and the spreadsheet were developed by students in one semester time frame. The simulation work was performed in the other semester. A random selection of dimensions for 1000 assemblies is selected for the analysis. A probability distribution for each dimension is used to represent the expected values. An automotive torque converter for automatic transmission was used as an application example.

The application example demonstrates that the ability to assemble the torque converter is dependent upon the width of the spline root of the stator shaft and the gap at the tip of the inner race spline. The critical areas are near the minor diameter of the splines on each component.

Allowing the surface waviness of the stator shaft to increase would not affect the ability to assemble the components but has previously been shown to increase the failure rate in customer service.

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Biography

GENE LIAO, currently an assistant professor at the Wayne State University, received his B.S.M.E. from National Central University, Taiwan, M.S.M.E. from the University of Texas, Mechanical Engineer from Columbia University, and the Doctor of Engineering degree from the University of Michigan, Ann Arbor. His research and teaching interests are in the areas of mechanical design, multibody dynamics, and CAE applications in manufacturing. Dr. Liao has 15 years of industrial practice in the automotive sector prior to becoming a faculty member.