

## Computer-Based Learning Modules For Rigid Body Mechanics

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### Abstract

The use of computer-based modules in teaching and learning some of the difficult to comprehend concepts in mechanics can be very beneficial for both students and instructors. The Vector module discussed in this paper was developed and used by the authors to introduce the concepts associated with the mathematical operations and use of vectors to solve engineering problems including evaluation of vector components, addition, dot and cross products, projection, and unit vectors. The impact of using computer-based modules in a required sophomore-level engineering course, Rigid Body Mechanics, was assessed by special student evaluations. Students' learning capability is generally positively impacted by computer-based modules, and the level of impact varies greatly among students.

### I. Introduction

An exponential growth in the computer software industry in recent years and availability of numerous user-friendly software packages have created a unique opportunity for engineering educators to develop and use computer-based learning tools. Buret et al.<sup>3</sup> developed a computer-aided education (CAE) package to visualize the magnetostatic field in and around the magnetic circuit of a double U-shaped contactor. The modeling of this device was performed by solving in real time a nonlinear finite element problem. From a pedagogical point of view, the package was found to be efficient when used in parallel with a classical lecture. The UMass Amherst tutor team developed a multimedia "injection molding tutor" capable of training a user how to produce cost effective injection molded part designs<sup>8</sup>. The tutor provides visually stimulating animations that clearly illustrate the relationship between part geometry and the tooling required to produce the part. Hansen et al.<sup>5</sup> developed a PC-based diesel engine simulator to supplement existing undergraduate laboratory instruction, so that students could establish a thorough understanding of the response of the engine. Feedback from undergraduates and instructors who evaluated the simulator was very positive. It was regarded as a valuable interactive educational tool. An interactive graphics-based system was employed by Stevenson<sup>11</sup> to rapidly solve statics problems and enhance students' analytic capabilities. Ability to visualize and intuitively evaluate problems was found to complement and sharpen students' analytical prowess.

Gramoll and Abbanat<sup>4</sup> reported the development, implementation, and functionality of an interactive multimedia program designed to assist students in learning basic concepts in engineering dynamics. In this example-based learning process, each example consisted of four parts: Introduction, Theory, Solution, and Simulation. The first three parts introduce a problem to the user, while the fourth part allows them to experiment with a computer-generated simulation of the problem. Computer animation as a tool for learning "Engineering Design/ Graphics Concepts" was discussed by Bradley and Sidler<sup>2</sup>. The following elements were

included: multiview drawings, vectors, and tolerancing. Use of the computer package and animation as a learning tool has been described by others<sup>1, 9, 10, 11</sup>.

Students in freshman and sophomore-level courses, in which some of the difficult to comprehend concepts in mechanics are introduced, can greatly benefit from computer-based self-learning modules as explained in an earlier paper by the authors<sup>12</sup>. Carefully designed modules with visual animation and sound can be effectively used by students as a supplemental learning tool. To this end, four computer-based modules were developed and used by the authors as an aid in teaching a sophomore-level required engineering course, Rigid Body Mechanics. The following topics were covered: (a) Vector Notation and Forces; (b) Truss Analysis; (c) Centroid; and (d) Moment of Inertia. A detailed discussion on the first module is presented in this paper along with an assessment of its impact on the students' learning experience.

## II. Rigid Body Mechanics

The Rigid Body Mechanics (ENGR 2113) course taught at the University of Oklahoma (OU) consists of two major parts. The first part, Rigid Body Statics (RBS), covers vector representation and manipulation of forces for two- and three-dimensional problems with emphasis on the concept of equilibrium, free-body-diagrams, and analysis of structures such as trusses, beams, frames and machines<sup>6, 7</sup>. Friction problems and special topics, including centroid, center of gravity, and moment of inertia, are also covered in this part. The second part, Rigid Body Dynamics (RBD), covers principles of motions for particles and rigid bodies; applications of Newton's second law, energy methods; and special topics including mass moment of inertia, and vibration of spring-mass systems are covered. More than 80% of the time is spent on RBS topics and the remaining time is spent on RBD topics. In many universities a course of this type is taught as two separate courses, RBS and RBD.

## III. Vector Modules Developed

This module introduces the important concepts associated with the mathematical operations and use of vectors to solve engineering problems. Specifically, effort is made: (i) to show how to add forces and resolve them into components using the Parallelogram Law; (ii) to express force and position in Cartesian vector form and explain how to determine the magnitude and direction of a vector; and (iii) application of the dot product operation between two vectors (force vectors) to determine the angle between them, and the cross product operation between two vectors (position and force vectors) to determine moment of a force about a point<sup>6</sup>.

### III.A. Cartesian Components

The module starts with a graphical representation of a vector ( $\mathbf{F}$ ) in two dimension and an animation of how to find its components ( $F_x$  and  $F_y$ ). In the first step, a vector is drawn on the screen, as an instructor would draw it on a chalkboard. In the second step, the components ( $F_x$  and  $F_y$ ) of the vector are identified as the projection on the x and y axes, respectively, as shown in Figure 1. Along with the animation of how to construct the projections, pertinent mathematical expressions are displayed in the same sequence as the projections appear on the monitor screen. A message is displayed on the top indicating when the user can proceed to the

next page. Two icons are provided at the bottom of each screen; the first icon enables the user to proceed to the next page, while the second icon can be used to display the previous page. A third icon is provided to allow the student to quit the application at any time. These icons are shown in Figure 1.

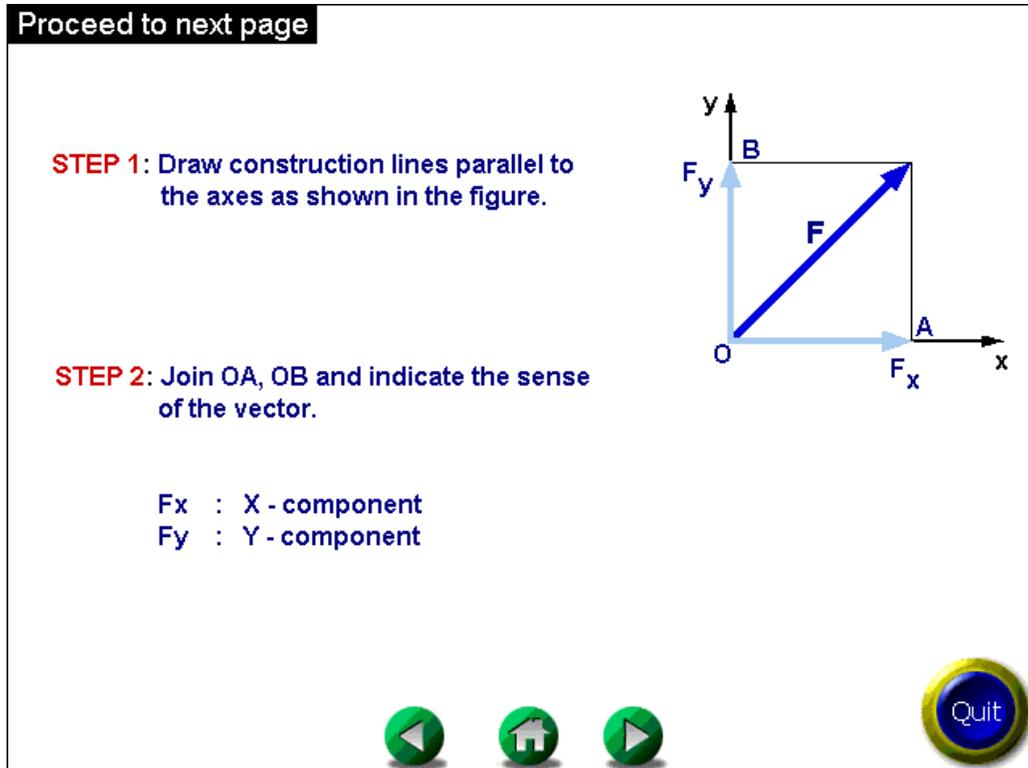


Figure 1 Components of a Vector in Two Dimension

The module then proceeds to find the Cartesian components of a vector in a two-dimensional space, by giving the mathematical expressions and displaying their graphical representation on the same screen. A numerical example is included for illustration, followed by an interactive page that will allow the user to input the desired orientation and magnitude of a force vector, and the module will display the Cartesian component values and animate their representation on a diagram, as shown in Figure 2. In the example shown in Figure 2, the angle and the magnitude of the vector are selected as  $120^\circ$  and 100 N, respectively. This step can be repeated any number of times with different values of orientation and magnitude, giving students a visual perspective of the changes in the Cartesian components of a vector with the corresponding changes in orientation.

The Cartesian components of a vector in a three-dimensional (x,y,z) coordinate space are introduced next, as shown in Figure 3. As in the case of two-dimensional space, both the mathematical expressions of the three-dimensional (3-D) vector components and their graphical representation are displayed on the monitor screen simultaneously for a visual understanding of the meaning of vectors in a 3-D space. An interactive example is included to give students an opportunity to draw their desired vectors in 3-D and to study how the components of a vector change due to change in orientation.

Enter the angle and the magnitude of the vector

ANGLE = 120 °

MAGNITUDE = 100 N

$F_x = -50\text{N}$        $F_y = 86.6\text{N}$

Continue

Navigation icons: back, home, forward, and a large blue 'Quit' button.

Figure 2 Cartesian Vector Components in 2-D: Interactive Example

Proceed to next page

**STEP 1:**  
Draw the construction lines parallel to these axes as shown in the figure.

**STEP 2:**  
Join OA , OB, OC and indicate the sense of the vector.

Navigation icons: back, home, forward, and a large blue 'Quit' button.

Figure 3 Cartesian Vector Components in 3-D

### IIIB. Parallelogram Law

The parallelogram law module is then introduced to illustrate the method to add two or more vectors. The major steps involved in adding two vectors are displayed and the application of the parallelogram law is shown through animation involving moving the vectors one-by-one to draw the desired sides of the parallelogram, and identifying the diagonal that represents the resultant of the two vectors. Figures 4 and 5 capture the essence of this module illustrating the operations involved in adding two and three vectors.

**Proceed to next page**

Consider two vectors  $P$  and  $Q$  as shown in the figure.

**STEP 1 :**  
Sum of the two vectors is obtained by attaching the two vectors to the same point  $A$ .

**STEP 2 :**  
Construct a parallelogram using  $P$  and  $Q$  as two adjacent sides of the parallelogram.

**STEP 3 :**  
The diagonal that passes through  $A$  represents the sum of the vectors  $P$  and  $Q$ , and the resultant is represented as  $(P + Q)$ .

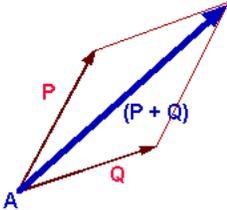




Figure 4 Parallelogram Law: Two Vectors

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To add the vectors  $P$ ,  $Q$ ,  $R$  the following steps are followed.

**STEP 1 :**  
Apply the parallelogram law on any two vectors of three. Consider  $P$  and  $Q$  and obtain resultant  $(P+Q)$  from parallelogram law.

**STEP 2 :**  
Add the resultant  $(P+Q)$  with the vector  $R$  and from the parallelogram law obtain the resultant  $(P+Q+R)$ .

**STEP 3 :**  
Follow the step 2 till all the vectors are added.

**Note :** Order in which the vectors need to be added is not important since the vectors addition obeys associative law.

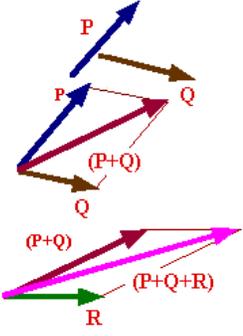




Figure 5 Parallelogram Law: Three Vectors

### IIIC. Unit Vector, Projection of Vectors, and Moment of Forces

The module then introduces the concept of unit vector and projection of a vector on an axis. In each case, both the relevant mathematical expressions and the steps involved are illustrated through animation to give students a visual feel of the associated processes and their physical meaning. To illustrate the method of calculating the moment of a force, the force is resolved into the Cartesian components and the moment of each force is first evaluated and then added to compute the total. The method of superposition, in case of problems involving more than one force, is illustrated through animation involving resolving forces into Cartesian components, then computing the moments of each with correct sign, and finally adding them algebraically. Thus, this module illustrates the application of Varignon's Theorem. It is experienced that beginning students have a difficulty in visualizing the sense of a moment of a force about a point, particularly when there are several forces involved. In this module when taking the moment of a force component, all other forces, except the one being considered, can be made to vanish. The students can then clearly see what is the sense of a moment produced by this force component about the desired point (which acts as the pivot point for the fulcrum considered). A number of interactive features are provided in this module to make the module a self-learning tool. For instance, given the angles and the magnitude of a force vector, the user is asked to compute Cartesian components ( $F_x$ ,  $F_y$ , and  $F_z$ ) of a vector that are then used to evaluate the unit vector. If the user keys in a wrong value, the module indicates it so and gives the user additional chances to correct this error. After five tries, the correct answer is displayed. An example of this interactive feature is illustrated in Figure 6. The procedure for evaluating moments of multiple forces acting on a structure is illustrated in Figure 7.

**EXAMPLE**

Given  $F = 100 \text{ lb}$ ,  $\theta_x = 60^\circ$ ,  $\theta_y = 45^\circ$ ,  $\theta_z = 120^\circ$ . Compute the components and unit vector along  $F$ .

**Step 1: Components of the force,  $F$ .** Given the angles defining the direction & the magnitude of the force, we can compute the components of  $F$ .

$F_x = F \cos \theta_x =$

**Wrong, Try again!**

$F_y = F \cos \theta_y =$

$F_z = F \cos \theta_z =$

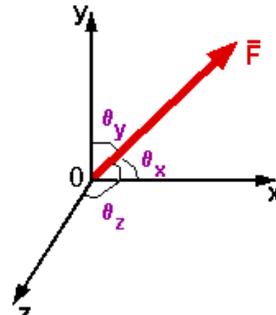
**HINT**

$$\bar{F} = F\bar{\lambda}$$

$$\bar{\lambda} = \lambda_x\bar{i} + \lambda_y\bar{j} + \lambda_z\bar{k}$$

$$\bar{\lambda} = \cos\theta_x\bar{i} + \cos\theta_y\bar{j} + \cos\theta_z\bar{k}$$

$$\bar{F} = F(\cos\theta_x\bar{i} + \cos\theta_y\bar{j} + \cos\theta_z\bar{k})$$



⏪
🏠
⏩
Quit

Figure 6 Example of Interactive Feature: 3-D Vector Components

**Procedure to calculate moment of a 2-D force system about a point**

**Step 1: Choose x, y, coordinate system**

**Step 2: Resolve each force into x and y component**

**Step 3: Compute the moment about desired point due to each component**

**Step 4: Add all moments to calculate total moment**

**Sign Convention**  
 Counterclockwise : A +ve  $M_A$   
 Clockwise : A -ve  $M_A$

$$M = \bar{M}_{1x} + \bar{M}_{1y} + \bar{M}_{2x} + \bar{M}_{2y} = (F_{1x})(d_{1x}) + (F_{1y})(d_{1y}) + (F_{2x})(d_{2x}) + (F_{2y})(d_{2y})$$

Figure 7 Moments of Two Forces about a Point

The concept that when a force is translated to another point, the equivalent force system will include a force (acting in the same direction as the given force) and a moment (equal to the moment of the force about the desired point) is used. Thus, the concepts introduced in the previous module are further expanded upon. This gives an appreciation to the students why it is essential to master these skills before going into their applications.

### IIID. Equivalent Forces

Another very important feature of the previous module developed is its application to find evaluation of equivalent forces. This module is developed to find the simplest equivalent force system for a structure in which several forces are acting. For illustration, consider the rigid structure in Figure 8 subjected to a 90 kip concentrated load acting vertically downward and a 20 kip horizontal load as shown. The goal is to let the students walk through the important steps involved in representing these two forces by a single force and a couple acting at point O. If the student clicks on the 20 kip force, two equal and opposite forces (20 kip each) appear at point O. In the next step, the two equal and opposite 20 kip forces are transformed into a couple (100 kip-ft) acting at O and the 20 kip force is moved to point O. In the next step, the module asks the user to move the 90 kip force to the desired position to account for its effect in calculating the equivalent force at O. If the user places the force at a wrong location, the force retracts back to the original position. After five tries, the module moves the force to the correct location. Steps

such as these continue until the assigned task is completed. A pictorial representation of the steps (except the last step) is shown in Figure 8.

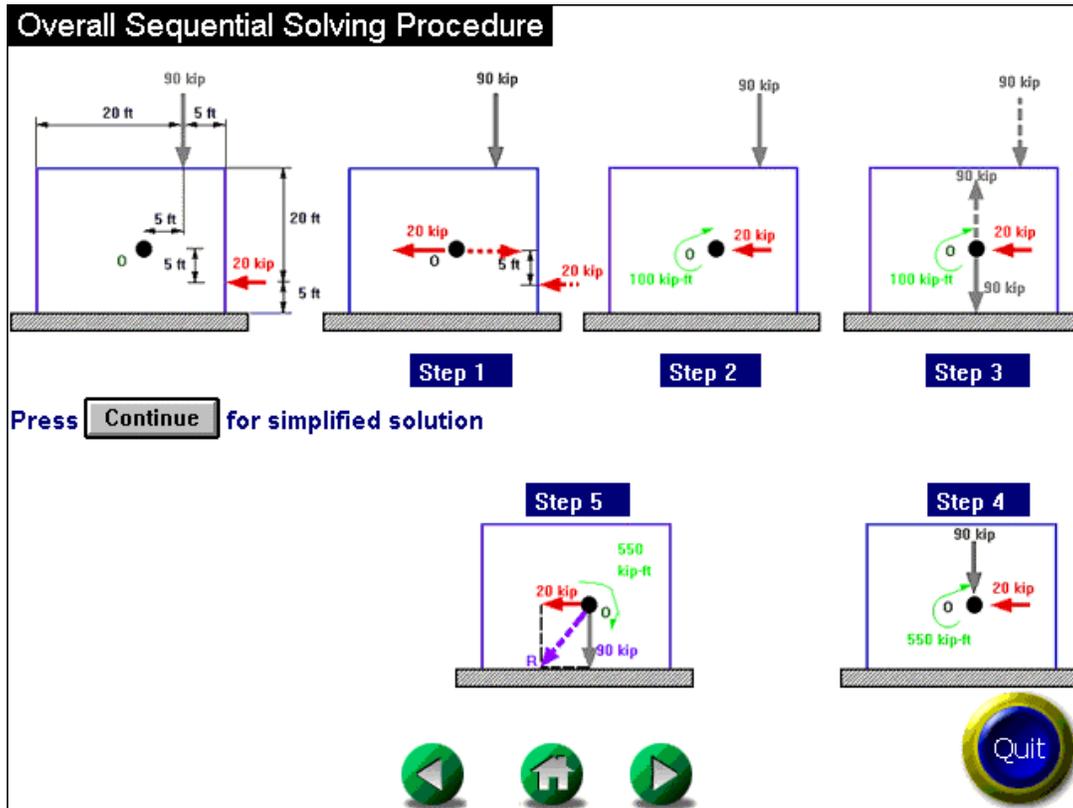


Figure 8 Equivalent Force Example

#### IV. Assessment of Impact

To assess the impact of the vector module presented in the preceding section as well as the other three modules used in the course, a questionnaire was designed to seek students' feedback. Three questions were specifically related to the computer-based learning (or teaching) module addressed in this paper: Q47: The idea of using teaching modules is very appropriate for this type of course; Q48: The teaching modules covered appropriate topics; and Q49: Having teaching modules in the course Home Page is very helpful. Also, student feedback was obtained through the regular student evaluations at the end of the semester using the standard evaluation form provided by the College of Engineering. Both the custom-designed and standard evaluations indicate that the students' learning was positively impacted by the computer-based modules. The level of impact, however, appeared to vary greatly among the students. In answering Q47, 73% of the respondents agreed with the statement of which 18% agreed strongly. The remaining 27% respondents indicated that they were unsure about the level of impact. In answering Q48, about 90% of students believed that the teaching modules covered appropriate topics, but their relative ranking (i.e., "agree strongly" versus "agree") varied somewhat. The response to Q49 had the following distribution: Strongly Agree--18%, Agree--42%, Probably--24%, Not at all--9% and No Answer--7%. Overall, most students found these modules to be an effective supplemental tool, not the primary learning tool, indicating that such

modules can enhance learning when used with other materials such as lecture notes, classroom lectures, and textbook. From the written students' comments it was evident that although some students found it difficult to use these modules on their own, a majority of them found that the animation features enhanced the understanding of the underlying concepts. Also, using the computer-based modules in a group environment was more effective than using them on an individual basis.

## V. Concluding Remarks

The use of computer-based modules in teaching and learning some of the difficult to comprehend concepts in mechanics was found to be beneficial for both students and instructors. The level of benefit may depend on various factors including topics which require a visual representation of the mathematical concepts introduced, and the user friendliness of the module. Based on the limited experience gained by the authors, computer-based modules can serve as a supplemental learning tool rather than a sole learning tool. It is time consuming and costly to develop such modules, but benefits were found to outweigh the effort, time, and cost, particularly considering projected long term benefits.

## Acknowledgements

The authors are thankful to Mr. Suresh Nedunuri and Mr. Nelson Er for their assistance in developing the computer-based learning modules. Also, thanks are extended to their colleague, Dr. K. Muraleetharan, for his useful comments.

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