Consistency in Assessment of Pre-Engineering Skills

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A Study of the Consistency in Assessment of Pre-Engineering Skills

Abstract

Assessment tools are often used in a predictive way to gauge the overall skills of first-year engineering students as they begin their engineering education. They are also useful in setting interventions in terms of tutorials, as well as providing self-improvement motivation for the students who achieve scores that are not consistent with earlier high school performance. Previous research[1] has demonstrated that the academic averages obtained in high school, may not necessarily reflect the skill level (competency) of the students entering first-year, especially in mathematics. In addition, a longitudinal study over more than ten years has also indicated that the averages from the math advisory and engineering assessment (Force Concept Inventory) exams did not show a statistically significant decline during that time period. In this study, both the math and engineering assessment results were further analyzed on a per question basis to determine whether or not there were any observable trends in the student responses.

The results for math assessment exams, taken over thirteen years, indicated that the average performance on each question every year is statistically very consistent. The questions that the majority of the students got right each year, and those that the majority got wrong each year showed very little variation in the standard deviation (typically < 5%), which was used as the measure in variability of the mean. The results were further analyzed by categorizing the questions according to three classifications: algebra, trigonometry and geometry. Typically, the questions with the best overall performance were simple algebra questions, and the questions with the worst overall performance involved trigonometric concepts. Moreover, as the complexity of the algebra questions increased, the success rate on those questions diminished as expected. Both assessment exams were time limited and students were not allowed to use calculators. In the high school curriculum in our region, students use calculators regularly in their high school math courses. As a result, their inherent competency in trigonometric functions is lacking, as the average scores (typically less than 30%) on these questions would indicate.

Engineering assessment (Force Concept Inventory) exam results collected over a slightly shorter duration (six years) were also analyzed. The same trends in student responses were observed, but in this case the results were somewhat less striking than the results obtained from the math assessment. It is clear, however, that there is a consistency in the success rate for individual exam questions that test both math and engineering concepts. These results support the anecdotal contention that students collectively have competency in certain areas (algebra) but lack competency in others (trigonometry). It further demonstrates that students often come into first-year engineering with common misconceptions and common math deficiencies.

The results from this study are useful from several perspectives. They can provide a focus for interventions that might address both competency and misconceptions. Secondly, the consistency and repeatability of this data may provide an impetus to work with K-12 educators to address these issues before the students reach university. The consistency of this data also implies that pre-engineering skills are somewhat predictable from year to year.

Introduction

This work is a direct follow-up to previous research that was conducted to examine trends in pre-engineering assessment exams (academic indicators) using results from a longitudinal study of first-year engineering students that spanned just over a decade.[1] In the previous study, mathematical skills for the period 2000-2011 were tested using a 20-question, multiple-choice, pre-calculus Math advisory exam, administered to first-year engineering students (without calculators) prior to entering first-year. Engineering skills were also evaluated during the period 2007- 2011, using the Force Concept Inventory (FCI) Exam, which is a 30-question, multiple-choice exam[2]. Earlier work had indicated that there was little variation in these assessment scores during the time period considered. Furthermore, the results of the previous study did not support anecdotal observations that students' pre-engineering skills were declining with time. It is this anomaly that led to this study, which provides a more detailed analysis of the data that was collected earlier as well as analysis of the additional data that has been collected since the publication of the earlier results. This longitudinal study is ongoing.

The previous work provided a very basic comparative analysis of data, in a rather general sense, using overall averages of exam scores for each group of students collected each year from two different assessment exams. However, earlier work did not consider a detailed analysis of the content of these assessment exams or an analysis of frequency of correct and incorrect responses. This study was initiated to determine whether or not there was also consistency in the student responses for individual questions, to further support the consistency that was observed in the temporal variation in the average scores presented in the aforementioned research.

In this study, math advisory exam data for the period 2000-2013 was examined using data collected from the 20-question, multiple-choice, pre-calculus Math advisory exam administered annually to first-year engineering students (without calculators) prior to entering first-year. Engineering assessment exam data were also evaluated for the period 2007- 2013 using the results obtained from the Force Concept Inventory (FCI) Exam: a 30-question, multiple-choice exam[2]. This research was undertaken to examine the frequency of correct/incorrect students' responses to individual questions for both of these assessment exams, and to describe any trends that might be observed in the data.

Background

There has been a substantial amount of literature devoted to the evaluation of mathematical and other pre-engineering skills of students entering first-year engineering, particularly as a predictor of success and retention in engineering. A literature review conducted as a part of the previous research[1] provided the background for the current study, by demonstrating the need for more research in this area. However, the literature review did not uncover any studies that were devoted to the detailed analysis of assessment exams in terms of frequency of correct and incorrect responses.

There were several themes that emerged from the literature referred to in the previous work. One of the more pertinent themes is the deficiency in mathematical problem solving skills that has been observed in engineering programs in North America and the United Kingdom. It has been suggested that addressing these deficiencies might improve the success rate of students in engineering programs. It is these deficiencies that are being explored through results of assessment exams. By examining the frequency of correct/incorrect responses in light of the question type, one may get a sense of common deficiencies in pre-engineering skills. The rationale for this investigation comes from one of the principal themes of research in engineering education which seeks to improve "best practices" in teaching engineering through the analysis of data.

Data

The data for this study was collected through the engineering transfer program at Grant MacEwan University in Edmonton, Alberta, CA. Throughout the course of the study, the sample size of students varied from 109 to 204 (see Table 1); increasing as a result of program expansions during that time period. It should be noted that not all of the students that were admitted to the program participated in the study, due to the way in which the assessment exams were administered. There was tremendous change in the program throughout this study as a result of this growth. It was anticipated that increasing enrolment, while drawing from the same admissions pool, might affect the quality of the students entering the program. The admission requirements* were not changed during this time, however, it was expected that the program expansions would result in taking more students with lower high school achievement, and that there would be considerable variation in the data that was collected during this longitudinal study. The results, shown later in this paper, do not confirm this expectation.

Table 1 - Group Sizes for Student Assessment Exams♪

Year	Math Advisory	Engineering Assessment
2013	202	204
2012	181	182
2011	163	162
2010	185	163
2009	194	198
2008	137	140
2007	116	117
2006	115	
2005	116	
2004	109	
2003	115	
2002	109	
2001	110	
2000	112	

♪The program currently has an admission quota of 216.

* Students admitted to engineering must have Math 30-1, Math 31, Physics 30, Chemistry 30, and English 30-1

The math advisory exam used in this study consisted of twenty multiple-choice questions which were developed to test algebra, geometry and trigonometry skills. The exam questions are provided for reference in Appendix 1. The engineering assessment exam (Force Concept inventory[2]) consisted of thirty multiple choice questions. A classification of the question types is given by Hestenes et al.[2] , but has not been provided or used herein, since there appear to be various versions of FCI being used for assessment purposes. The taxonomy of Hestenes et al. merits further consideration in light of the results obtained in this study. Again, it should be noted that both exams were administered to students prior to the beginning of their first-year program. The students were not allowed to use calculators and the time for the exam was limited to forty minutes. All exam results were scored electronically.

The original analysis of the data, presented in an earlier paper[1], focused on the average scores on two assessment exams for each year in the study. These two exams are referred to as math advisory and engineering assessment, respectively. It is the results of these two exams, which are re-examined in greater detail and summarized in this paper.

The average scores, for these exams each year, were between 50-55% consistently throughout the duration of this longitudinal study. It was noted earlier in this paper, that results for the engineering assessment exam were collected over a shorter time period than for the math advisory exam, since the original focus of the longitudinal study was to collect data regarding mathematical competency.

As a direct result of the research conducted previously, further analysis of the data was undertaken: examining both assessment exams on a per question basis. For each question, the overall average of the responses for all students for a given year was calculated using Excel. In addition, the standard deviation of that average for each question was also calculated. The averages were not weighted, and the standard deviations were based on population rather than sample. This was done to provide a simplistic analysis of the data to observe any trends that might exist. It has also been noted that there were different samples sizes for all of these groups. These results are presented for the math advisory exam in Table 2 and the engineering assessment exam in Table 3.

The consistency in the data for both of these exams for the entire duration of the longitudinal study is quite remarkable, considering that these results were for different groups of students each year, and the sample size varied significantly as well. The small standard deviation (STDV) confirms this consistency for almost every question on both exams. In some cases the standard deviation is just under 0.02, which suggests just less than 2% variation in the mean throughout the time period considered. For the math advisory exam the standard deviation for each question is typically less than 0.05. There is only one question (#14), where the standard deviation is substantially larger than the rest of the data. If question #14 is omitted for the math advisory exam data, the overall average of the standard deviations is 0.04. The same trend holds for the engineering assessment exam. In both cases, the standard deviation of the standard deviations is just under 0.015.

Table 2 - MATH ADVISORY Question Response Distribution

Q	2013	2012	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001	2000	AVE	STDV
1	0.57	0.57	0.66	0.58	0.61	0.61	0.59	0.61	0.59	0.64	0.68	0.52	0.64	0.63	0.61	0.04
2	0.81	0.82	0.76	0.83	0.83	0.81	0.82	0.84	0.91	0.84	0.8	0.74	0.75	0.77	0.81	0.04
3	0.94	0.91	0.96	0.95	0.92	0.96	0.96	0.94	0.93	0.95	0.96	0.89	0.93	0.95	0.94	0.02
4	0.3	0.35	0.46	0.41	0.39	0.39	0.39	0.48	0.44	0.44	0.45	0.41	0.47	0.46	0.42	0.05
5	0.51	0.58	0.58	0.56	0.6	0.54	0.59	0.66	0.52	0.56	0.6	0.62	0.6	0.61	0.58	0.04
6	0.41	0.36	0.39	0.38	0.44	0.36	0.36	0.32	0.47	0.35	0.37	0.37	0.41	0.39	0.38	0.04
7	0.72	0.81	0.77	0.81	0.78	0.72	0.75	0.8	0.81	0.76	0.79	0.77	0.78	0.88	0.78	0.04
8	0.57	0.6	0.61	0.64	0.59	0.64	0.59	0.57	0.55	0.55	0.63	0.64	0.75	0.66	0.61	0.05
9	0.26	0.2	0.15	0.18	0.21	0.2	0.33	0.19	0.26	0.25	0.24	0.24	0.33	0.23	0.23	0.05
10	0.63	0.73	0.71	0.69	0.71	0.72	0.65	0.73	0.76	0.69	0.72	0.65	0.73	0.71	0.7	0.04
11	0.43	0.46	0.49	0.47	0.43	0.47	0.47	0.53	0.49	0.49	0.46	0.54	0.58	0.53	0.49	0.04
12	0.45	0.47	0.56	0.51	0.51	0.56	0.49	0.45	0.52	0.5	0.49	0.43	0.45	0.49	0.49	0.04
13	0.36	0.32	0.29	0.3	0.35	0.28	0.28	0.33	0.33	0.3	0.28	0.33	0.38	0.3	0.32	0.03
14	0.36	0.6	0.55	0.58	0.61	0.48	0.53	0.58	0.78	0.59	0.6	0.62	0.71	0.63	0.59	0.10
15	0.4	0.25	0.31	0.35	0.32	0.34	0.28	0.31	0.44	0.34	0.35	0.28	0.36	0.44	0.34	0.06
16	0.93	0.88	0.88	0.89	0.92	0.88	0.87	0.92	0.93	0.88	0.94	0.85	0.83	0.86	0.89	0.03
17	0.46	0.46	0.51	0.46	0.45	0.44	0.45	0.47	0.51	0.56	0.47	0.36	0.48	0.5	0.47	0.05
18	0.26	0.32	0.42	0.31	0.27	0.25	0.24	0.25	0.3	0.32	0.27	0.26	0.31	0.24	0.29	0.05
19	0.27	0.23	0.24	0.3	0.28	0.28	0.22	0.22	0.33	0.25	0.26	0.23	0.3	0.36	0.27	0.04
20	0.26	0.28	0.29	0.23	0.3	0.19	0.15	0.3	0.31	0.21	0.26	0.3	0.19	0.27	0.25	0.05

Table 3 - ENGINEERING ASSESSMENT Question Response Distribution

Q	2013	2012	2011	2010	2009	2008	2007	AVE	STDV
1	0.61	0.65	0.63	0.73	0.66	0.73	0.66	0.67	0.05
2	0.30	0.32	0.30	0.27	0.28	0.27	0.24	0.28	0.03
3	0.61	0.63	0.71	0.55	0.57	0.62	0.63	0.62	0.05
4	0.31	0.29	0.33	0.25	0.22	0.21	0.17	0.26	0.06
5	0.16	0.19	0.23	0.23	0.16	0.14	0.17	0.18	0.04
6	0.71	0.75	0.76	0.72	0.72	0.79	0.76	0.74	0.03
7	0.71	0.68	0.72	0.64	0.66	0.68	0.64	0.67	0.03
8	0.68	0.73	0.72	0.63	0.73	0.70	0.68	0.70	0.04
9	0.53	0.54	0.54	0.51	0.60	0.62	0.69	0.58	0.06
10	0.74	0.71	0.66	0.67	0.69	0.75	0.71	0.70	0.03
11	0.26	0.26	0.35	0.38	0.23	0.29	0.31	0.30	0.05
12	0.82	0.82	0.80	0.80	0.73	0.84	0.82	0.80	0.04
13	0.28	0.21	0.35	0.28	0.21	0.30	0.22	0.26	0.05
14	0.56	0.56	0.70	0.64	0.60	0.67	0.60	0.62	0.05
15	0.22	0.24	0.29	0.22	0.16	0.16	0.12	0.20	0.06
16	0.62	0.70	0.66	0.65	0.53	0.71	0.62	0.64	0.06
17	0.22	0.24	0.24	0.25	0.21	0.29	0.23	0.24	0.03
18	0.23	0.22	0.27	0.20	0.24	0.22	0.27	0.24	0.03
19	0.64	0.60	0.69	0.65	0.70	0.66	0.66	0.66	0.03
20	0.51	0.55	0.66	0.51	0.58	0.62	0.61	0.58	0.06
21	0.51	0.45	0.57	0.57	0.41	0.56	0.54	0.52	0.06
22	0.49	0.48	0.57	0.54	0.47	0.53	0.42	0.50	0.05
23	0.52	0.54	0.61	0.52	0.51	0.59	0.56	0.55	0.04
24	0.81	0.73	0.80	0.71	0.78	0.79	0.74	0.77	0.04
25	0.27	0.26	0.25	0.26	0.23	0.34	0.28	0.27	0.04
26	0.18	0.14	0.17	0.20	0.18	0.19	0.18	0.18	0.02
27	0.62	0.58	0.70	0.67	0.62	0.73	0.71	0.66	0.05
28	0.38	0.42	0.51	0.37	0.36	0.31	0.27	0.37	0.08
29	0.59	0.63	0.60	0.45	0.55	0.55	0.55	0.56	0.06
30	0.27	0.30	0.32	0.38	0.22	0.34	0.28	0.30	0.05

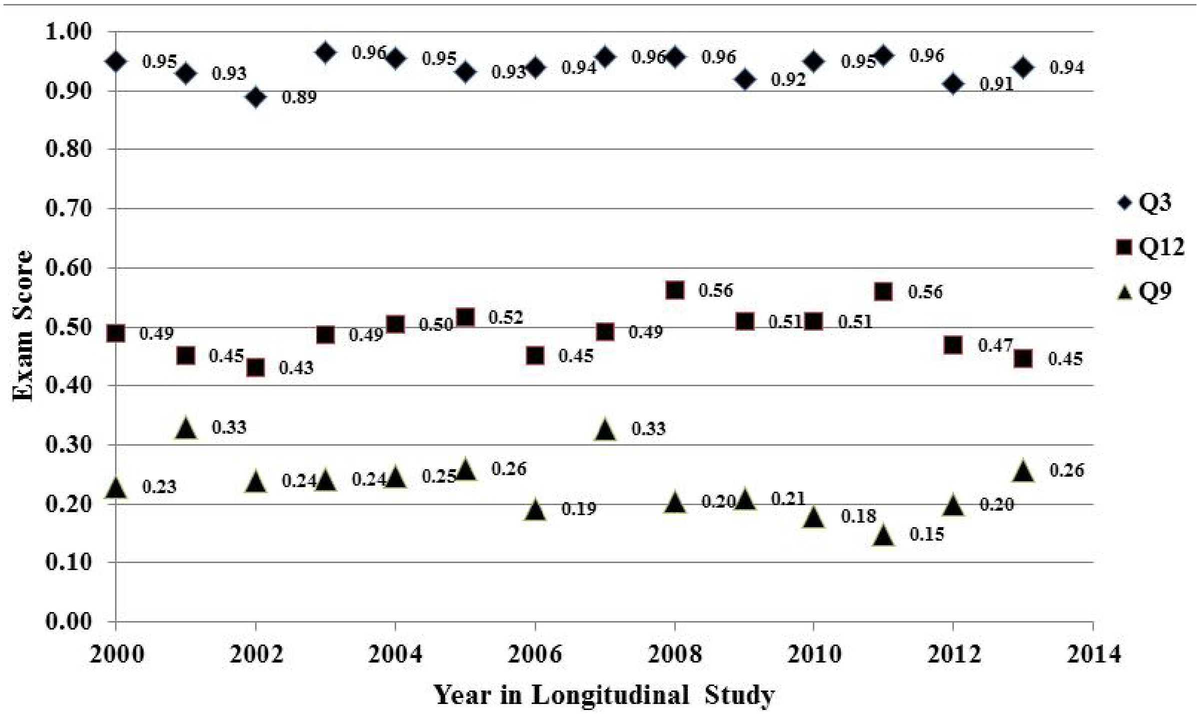

Figure 1: Temporal variation in Math Advisory exam scores for three typical questions.

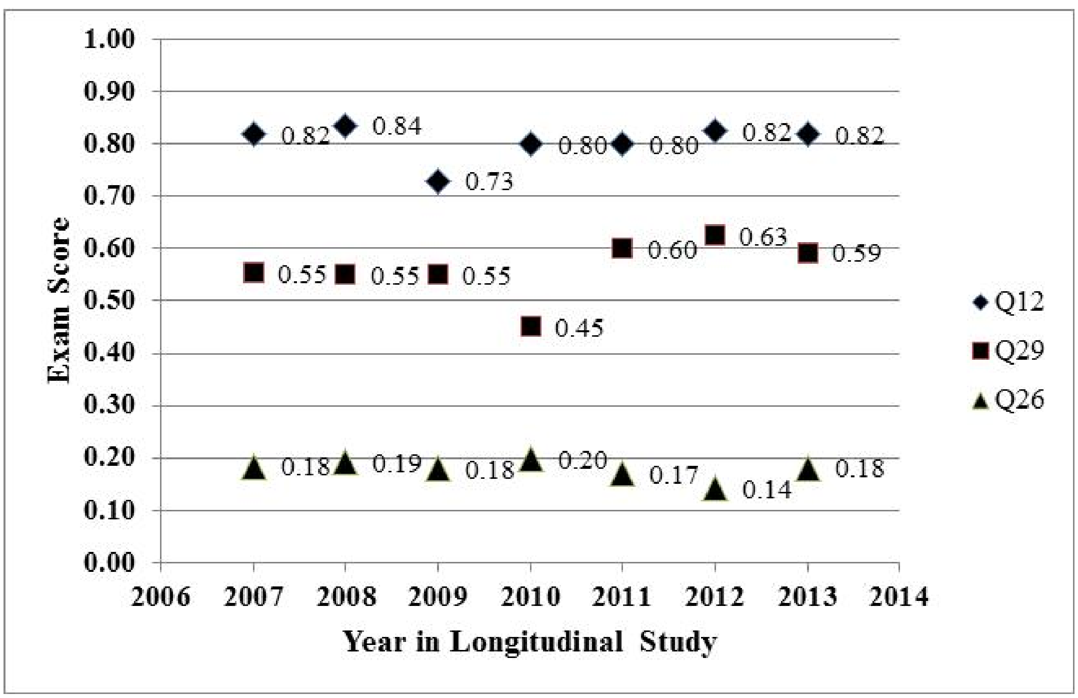

Figure 2: Temporal variation in Engineering Assessment exam scores for three typical questions.

To visually demonstrate this consistency in temporal profiles, data is presented in Figure 1 for the math advisory exam, and Figure 2 for the engineering assessment exam. Figure 1 shows temporal profiles of the average student responses for three of the twenty questions from the math advisory exam: one at the top end of the score spectrum, one at the bottom end of the spectrum and one in the middle. Likewise, in Figure 2, similar results are shown for the engineering assessment exam from the thirty-question set. In both figures, the question response averages have little variation over the time period considered. In a few cases, as discussed previously, there are data points that deviate more than others. It might be interesting to explore this further in future research.

The results for both exams were further analyzed by considering the correctness of the average responses in the context of the question type, to establish some sort of trend in the results. The data were sorted for each exam from the most correct average responses to the least correct average responses and tabulated with question type. These results are presented in Table 4 for the math advisory exam and Table 5 for the engineering assessment exam. Question descriptions were also provided for the math advisory exam.

Table 4 - MATH ADVISORY Question Type & Description

Q	AVE	STDV	Type	Description
3	0.94	0.02	A	add polynomials together - 3rd order highest
16	0.89	0.03	A	evaluate a function of a function
2	0.81	0.04	A	simplify quotient with numerator addition
7	0.78	0.04	A	simplify exponents and evaluate
10	0.70	0.04	A	solve a simple algebraic linear expression
8	0.61	0.05	A	factor a cubic polynomial
1	0.61	0.04	A	simplify quotient with denominator addition
14	0.59	0.10	G & T	identify a graphical equation - circles, ellipses, parabolas, hyperbolas
5	0.58	0.04	A	rationalize a denominator with square root
12	0.49	0.04	G	find slope of a line perpendicular to another line
11	0.49	0.04	A	solve verbal question two equations two unknowns
17	0.47	0.05	G	verbal problem with geometry
4	0.42	0.05	A	sqrt function of addition of squares
6	0.38	0.04	A	equivalent expressions with square roots
15	0.34	0.06	G	find vertex of a parabola
13	0.32	0.03	G	find points on a line with given slope
18	0.29	0.05	T	tan of an angle - without a calculator - knowledge of unit circle
19	0.27	0.04	T	simplify trigonometric expressions
20	0.25	0.05	T	solve a trigonometric function
9	0.23	0.05	A	solve a quadratic with an unknown coefficient

Table 5 - Engineering Assessment Question Type			
Q	**AVE**	**STDV**	**Type**
12	0.80	0.04	Forces - constant - acceleration - 1D
24	0.77	0.04	Forces & Kinematics 2D
6	0.74	0.03	Forces - curvilinear - kinematics
10	0.70	0.03	Forces- kinematics - 2D
8	0.70	0.04	Forces- kinematics - 2D
7	0.67	0.03	Forces - curvilinear - kinematics
1	0.67	0.05	Energy
27	0.66	0.05	Forces - constant 1D
19	0.66	0.03	Kinematics - 1D
16	0.64	0.06	Forces - N 3rd law
14	0.62	0.05	Forces - N 3rd law
3	0.62	0.05	Energy
20	0.58	0.06	Kinematics - 1D
9	0.58	0.06	Forces- kinematics - 2D
29	0.56	0.06	Forces - FBD
23	0.55	0.04	Forces & Kinematics 2D
21	0.52	0.06	Forces & Kinematics 2D
22	0.50	0.05	Forces & Kinematics 2D
28	0.37	0.08	Forces - N 3rd law
30	0.30	0.05	Forces - FBD
11	0.30	0.05	Forces- kinematics - 2D
2	0.28	0.03	Energy
25	0.27	0.04	Forces - constant 1D
13	0.26	0.05	Projectile motion
4	0.26	0.06	Forces - 3rd law
17	0.24	0.03	Forces - F = ma
18	0.24	0.03	Forces - FBD
15	0.20	0.06	Forces - N 3rd law
5	0.18	0.04	Forces - curvilinear - FBD
26	0.18	0.02	Forces - constant - 1D

In the case of the math advisory exam, three categories were used for question typing: i) A for algebra, ii) G for geometry and iii) T for trigonometry. In addition, verbal descriptions were also provided in Table 4 to give a sense of the nature of the question being considered (see Appendix 1). The question types and their descriptions were used to make conclusions regarding trends in the data. The majority of the questions used in the math assessment exam were algebraic in nature, since the assessment exam used at our institution is pre-calculus assessment.

Classifying question type for the engineering assessment exam (FCI concept inventory) was a bit more difficult than the math advisory exam as the categories for the FCI were less simplistic. The FCI exam is more conceptually based rather that skill-based, hence the question types are more difficult to classify. The descriptions were created in an attempt to establish a way to recognize question complexity. Again, in this case, the motivation for the question typing was to determine possible trends in the data. The classification scheme and trend determination was less successful with the engineering assessment exam. It might be necessary to explore this assessment exam using Hestenes'[2] taxonomy for the FCI exam before any conclusions can be reached.

Discussion

The results from the previous study and the results presented here show that the academic indicators (math advisory and engineering assessment) did not vary significantly throughout this longitudinal study, which has now spanned just over thirteen years. Furthermore, this study has now shown a striking consistency in the average response profiles for each question for both of these assessment exams during the same time period. Previous research showed statistically insignificant temporal variation in the overall averages on these exams. The current research has now confirmed that the correctness/incorrectness of the student responses is also very consistent from year to year. The consistency in these results is quite interesting, since clearly the size of the groups and many other variables have changed throughout the study. Many of these variables are known to influence academic aptitudes and skills, so these results were not anticipated.

It is clear from the data shown in Tables 4 and 5, that there are some questions that the majority of the students give the correct response to: math advisory Q3, Q16 and Q2, and engineering assessment Q12, Q24 and Q6. Likewise, there are some questions that the majority of students give an incorrect response to: (math advisory Q19, Q20 and Q9, engineering assessment Q15, Q5 and Q26). Yet, there is little variation from year to year in the average responses on these questions. This suggests that the "preparedness" of the students in the context of these exams is quite similar. This information can now be used to determine areas of strengths and weaknesses of the students by going back to the original questions to define remedial activities to correct the competency in these areas. Likewise, this data can be provided to K-12 curriculum developers to provide some insight regarding the success of achieving their course outcomes.

A comparison of the results in Tables 4 and 5, show some similarities and some differences. One trend that is observed is that the frequency of correct responses for the students is the greatest simple mathematical algebra questions, and least when the questions are more complex, or in the case of the FCI exam, the concepts are more difficult. The questions with the highest number of incorrect responses (0.18) occurred on the FCI exam, and the question with the highest number of correct responses occurred on the math advisory exam. In general, the performance of students on the math advisory exam (skill based) was better than the FCI (concept based).

Moreover, if the indicators used in this study are also predictors of success in engineering, one might expect that the success rate of the students in this study would be relatively constant over time as well. It would be useful to examine the GPAs of these students in light of the research to date to see if this trend is observed. In addition, one might expect that, in the future, if these indicators change with time, it could be indicative of substantial changes in the K-12 curriculum and/or some other contributing factor(s) that the students experience prior to enrolment in first-year engineering. This longitudinal study may also provide a foundation for assessing the success or failure of initiatives that are being undertaken or those that may be proposed in the future to improve the success and retention of first-year engineering students at our institution.

Summary

Further analysis of the data obtained from just over a decade of pre-engineering assessment scores demonstrates consistency in the data over this time period. Previous research indicated that students' admission averages were very constant throughout the longitudinal study, and the overall averages from year to year on these assessment exams remains constant as well. Examination of these results on a per question basis revealed that the correctness/incorrectness of the responses of students was also consistent from year to year. A comparison of these results to question types elucidated trends that could be used directly to deal with deficiencies and misconceptions that students might have as they enter first-year engineering. More importantly, this information could be useful to K-12 educators in determining their success/failure in satisfying their course outcomes in preparation for first-year engineering programs. An additional important benefit of this research is that the data from this longitudinal study can be used as the "canary in the coal mine" to perhaps predict unwanted changes in pre-engineering skills by monitoring results from these assessment exams.

Bibliography

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2. Hestenes, D. , Wells, M. and Swakhamer. "Force Concept Inventory", The Physics Teacher, Vol. 30, March 1992, 141-158.

1. $\frac{a}{x+b} =$

a) $\frac{a}{x} + \frac{a}{b}$ b) $\frac{a}{x} + \frac{1}{b}$ c) $\frac{1}{x} + \frac{a}{b}$

d) both b and c e) none of these

2. $\frac{a + bx}{a} =$

a) $1 + bx$ b) $1 + \frac{b}{a}x$ c) $1 + abx$

d) both a and b e) none of these

3. Combine terms to simplify:

$(-2x^2 + 3x - 9) - (4x^2 - x + 2) + (x^3 - 2x^2 + 1)$

a) $x^3 - 8x^2 + 3x - 6$ b) $x^3 - 4x^2 + 2x - 6$ c) $x^3 - 8x^2 + 4x - 10$

d) $x^3 - 2x^2 + 2x - 6$ e) none of these

4. $\sqrt{x^2 + y^2} =$

a) $\sqrt{(x+y)^2}$ b) $x + y$ c) $|x| + |y|$

d) $x^{\sqrt{2}} + y^{\sqrt{2}}$ e) none of these

5. Rationalize the denominator: $\frac{4}{5 - \sqrt{3}}$

a) $\frac{5 + \sqrt{3}}{4}$ b) $\frac{10 + 2\sqrt{3}}{11}$ c) $\frac{20 + \sqrt{3}}{22}$

d) $\frac{20 + \sqrt{3}}{2}$ e) none of these

6. Insert the required factor in the parentheses:

$5\sqrt{x+3} - \frac{5x}{2\sqrt{x+3}} = \frac{1}{2\sqrt{x+3}}(\quad)$

a) $2\sqrt{x+3} - 5x$ b) $5x + 30$ c) 15

d) $5 - 5x$ e) none of these

7. Evaluate: $\dfrac{9(3)^{-1}}{(2)^{-3}(2)}$

a) $\frac{3}{4}$ b) $\frac{9}{8}$ c) $\frac{3}{2}$

d) 12 e) none of these

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8. Factor: $8x^3 - 27$

a) $(2x-3)^3$ b) $(2x-3)(4x^2-12x+9)$ c) $(2x-3)(4x^2+6x+9)$

d) $(2x+3)(4x^2-6x+9)$ e) none of these

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9. Determine values of k for which the trinomial $4x^2 + 3x + k$ can be factored using only integers.

a) 6 b) 0 c) -1

d) both a and b e) both b and c

---

10. Solve for x:

$$3\left[2x - (7x - 1)\right] = 5x + 13$$

a) 4/5 b) -2 c) $\frac{7}{13}$

d) $-\frac{1}{2}$ e) none of these

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11. Find the smaller of two consecutive positive integers such that the one number times twice the other equals 612.

a) -18 b) 12 c) 18

d) 17 e) none of these

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12. What is the slope of the line which is perpendicular to the line $y = 7$?

a) 0 b) undefined c) $\frac{1}{7}$

d) $-\frac{1}{7}$ e) none of these

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13. Determine which points lie on the line that contains the point (2, -3) and has a slope of -7/4

a) $\left(4, -\frac{13}{2}\right)$ b) $(-2, 4)$ c) $\left(0, \frac{1}{2}\right)$

d) all of these points e) none of these points

14. Identify the graph of: $6x^2 + 2y^2 - 12x + 4y - 7 = 0$

a) Circle b) Hyperbola c) Ellipse

d) Parabola e) none of these

15. Find the vertex of the parabola: $x^2 - 4x - 4y + 16 = 0$

a) (2,12) b) (3,2) c) (2,4)

d) (2,3) e) none of these

16. Given: $f(x) = x + 2$

$g(x) = x^2 - 7$

Then $f(g(3)) =$

a) 18 b) 10 c) 7

d) 4 e) none of these

17. Find the height of a tree that casts a 20 foot shadow when the angle of elevation is $60°$.

a) $10\sqrt{3}$ feet b) $20\sqrt{3}$ feet c) 30 feet

d) $20\sqrt{2}$ feet e) none of these

18. Find the exact value of: $\tan \frac{5\pi}{6}$

a) $\frac{\sqrt{3}}{2}$ b) $\sqrt{3}$ c) -1

d) $-\frac{\sqrt{3}}{3}$ e) none of these

19. Perform the subtraction and simplify: $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x}$

a) csc x b) tan x c) cot x

d) $\cos^2 x$ e) none of these

20. Find all solutions of: $\sin x = \frac{1}{4 \sin x}$ in the interval $[0, 2\pi)$

a) $\frac{\pi}{6}, \frac{5\pi}{6}$ b) $\frac{7\pi}{6}, \frac{11\pi}{6}$ c) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

d) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ e) none of these