Creating Environments for Fostering Effective Critical Thinking in Mathematics Education (Math-EFFECTs)

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Abstract

The objective of this work is to present a strategy for the development and implementation of the Environments for Fostering Effective Critical Thinking (EFFECTs) pedagogical framework in a mathematics classroom, called Math-EFFECTS. A primary goal of Math-EFFECTs is to enhance the timely integration of mathematical solution techniques with engineering, technology and applied science applications.

EFFECTs was developed by a team of researchers at the University of South Carolina under funding from the National Science Foundation. It has been disseminated via the web and has over a dozen practitioners who have applied the framework to concepts such as geotechnical engineering, thermodynamics, mechanics, numerical methods, and scientific visualization, working with students at all stages of their engineering education [1]. The central learning goals of engineering EFFECTs are to (i) improve the understanding and retention of a specific set of concepts that provide core knowledge and (ii) encourage students to recognize and develop critical thinking skills that lead to earlier growth in engineering judgment. The primary application of EFFECTs has focused on enhancing the understanding of underlying engineering concepts and developing critical thinking. However, the framework can also be used to more effectively present and teach core mathematical concepts, encourage the critical mathematical thinking associated with real-world problem solving, and effectively link formal mathematics concepts to students' applications of mathematics at an earlier stage in their education.

Introduction

There are numerous teaching methods described in the literature; their effectiveness has been studied extensively. These often depend on the purpose of a particular class, but include standard lectures, interactive (think-pair-share) lectures, inquiry based, constructivist, mastery-based and flipped classrooms [2].

Correspondingly, there are also curriculum design strategies that are based on learning approaches. In particular, there is a long, and on-going, discussion in mathematics, documented in *Math Wars* [3], concerning the relative merits of teaching with a largely constructivism approach, i.e., students construct their own knowledge, versus a largely formalism approach, based on a set of rules. Various state curricula have been designed to mandate one approach or the other at the high school level. While there is significant evidence of merit in both approaches, time constraints make it difficult to fully incorporate both philosophies in secondary schools.

At the university level, in particular those schools with the goal of a comprehensive education, student learning outcomes often include criteria that students will demonstrate analytical reasoning and problem-solving skills. These outcomes are often satisfied by coursework in Mathematics. These requirements apply to a large population of people who do not see themselves as mathematicians, and often see no need for what they think of as mathematics. Unfortunately, this mindset is often present in science and engineering students who, although they recognize that there is math in their chosen field, see little connection between the math classes they take and their own academic interests.

The EFFECTs methodology provides a structural framework that emphasizes real-world application, encourages self-discovery and analysis, and teaches fundamental concepts, tools, and skills. It was originally designed for geotechnical engineering students. The designers understand that their discipline was as much art as science due to the many variable and unknown conditions [4] and no existence of one `right` answer. The theme of this paper is that this framework can also be used effectively to teach mathematics in a way that provides motivation and context in conjunction with the rote practice of skills; offering a blend of constructivism and formalism.

Background on EFFECTs

The goal of the EFFECTs framework is to formalize the theory-practice-revise problem solving method in a way that promotes and encourages reflective and analytical thinking. The idea is to engage students in a context-rich problem, through the use of a driving question, to guide them through active learning modules exploring core concepts, and to lead them to a solution methodology. The production of a final report serves as a mechanism that allows them to revise their original solution based on a synthesis of the knowledge and understanding gained through the learning modules.

The developmental framework for instructors using EFFECTs begins with the identification of the concepts to be studied; in general these are difficult concepts. Next, these concepts are associated with active learning activities; each concept could be associated with a single activity or multiple concepts can be linked or combined in progressively building activities. The third step in the process is to identify an application or context that requires at its base a solid understanding of the core concepts. Finally, the general application is placed in a realistic context, putting the application in a setting that resonates with students and gets them invested in creating a final solution. From this context an overall driving question is developed and guiding questions constructed. The guiding questions connect back to the concepts that had been identified and serve as motivation for the study of these concepts.

Each of these steps could take a single class, or several classes, and the authors encourage an iterative-growth approach to developing and implementing EFFECTs. In other words, the first time that an EFFECT component is taught, the full process might only consist of one class period that encompasses the context, driving question and an active learning module with reflective writing. Subsequent times teaching the EFFECT might result in additional time spent on the driving question, multiple activities and more reflection. It is also not necessary to teach all concepts using the EFFECTs process; some topics, especially in mathematics, will lend themselves to this approach while others will not. Time constraints could also play a role in how extensively the EFFECTs approach is included in the course.

Using this backward design process from core concepts to driving question is very helpful in identifying the most important learning objectives in a course. This approach helps the instructor focus more on building conceptual understanding and less on the quantity of material covered. The backward design process is discussed in more detail in *Understanding by Design* [5].

The instructional framework for students operates in reverse. The student begins with the real world context, often presented in a media rich visual format. Students are then presented with a decision worksheet, directly linked to the context, which poses a driving question. Driving questions are deliberately offered with minimal information so that students are encouraged to perform estimations and must begin to evaluate what they need to know in the context of the application. In response to the driving question, students are prompted to start asking conceptually-based questions that motivate subsequent active learning modules. Thus the goal of the EFFECTs framework is to create an integrative, rather than additive, module based approach.

The most non-traditional component of this teaching (student centered learning) model is a reflective writing assignment that usually occurs after each module in the form of a journal entry and which is featured as a significant component of the final report. These reflections, in combination with the decision worksheet, are a critical part of the process. With the decision worksheet, students begin to take responsibility for the problem and its solution. Within the modules, a more traditional learning environment exists, where information is provided by instructors, discovered by peer groups, or obtained through experiment. Through the journal entries students are asked to summarize what they learned, and discuss why it is important or relevant to the problem at each stage. By requiring students to reflect on what they learn and explain its relevance, ownership of the problem and its solution methodology are returned to the student, emphasizing their individual responsibility for learning and understanding, and documenting their changing perspective on the topic.

Relevance to Mathematics

An obvious goal of educators is to teach in a way that encourages and deepens student learning and comprehension. In particular we try to promote the higher order thinking skills discussed in Bloom's Taxonomy [6]; those that demonstrate true comprehension and not merely memorized repetition of facts and simple concepts.

The goal in engineering courses is to examine the applications of the proper math tools to solve particular physical problems. The tendency in mathematics classes, however, often is to teach as many different solution methods as possible so that students have a full suite of tools to use when they get to the applications. The challenge in teaching students who do not describe themselves as mathematicians is to provide a context in which they can see the advantage of the suite of tools. Not only does an application strengthen their skill set, but it holds the potential to enhance understanding, deepen their comprehension of the topic and build mental pathways to help them retain it longer. EFFECTs embodies a paradigm that formalizes this process.

Adaptation to Math Concepts

EFFECTs have been applied to a number of engineering fields [7, 8]. In each of these "an EFFECT" has been defined as the complete package of 1) a context-rich driving question; 2) a decision worksheet with an estimated solution; 3) several active learning modules; 4) a daily reflective writing component; and 5) a final report with revised solution using methods and justifications from the learning modules, and reflective analysis.

Core Concepts: The specific set of concepts and core knowledge for which students need to gain a deep understanding through the EFFECT

Context: The real-world scenario and problem descriptions

Decision Worksheet - Driving Questions:

The goal of the driving question is to engage students through the importance and relevance of the problem, and draw them in to the process for uncovering a path towards the solution note discover is a loaded word in education circles The Decision Worksheet can be used as a way to gauge a prior knowledge of the topic, and identify potential biases regarding physical constraints, solution methods, or estimation procedures. At the conclusion of the decision worksheet students are asked to propose a solution to the Driving Question.

Active Learning Modules:

Next, students explore core concepts through 2-5 active learning modules during the next few class sessions. These lessons are designed to provide depth in each core concept needed to formulate a sophisticated solution to the driving question. Because of student engagement with the problem content, they are often more interested/engaged in participating in the learning modules [4]. Active learning modules can be in the form of traditional lectures with think-pair-share responses. For math these modules should include sample problems and individual practice exercises. For engineers and students in applied fields, these activities can include taking measurements, making a sketch, or building a model.

Reflective Writing

Students are required to keep a journal in which they enter comments at the end of each class session. In many cases the instructor can prompt these journal entries with specific questions, e.g. What did you learn today? Was there anything surprising in what you observed? This activity, even done in minimal quantity, can have a significant effect on student learning perspectives. The grading rubric should include two components: understanding of core knowledge, including proper use of mathematical terminology, and critical thinking as evidenced by the analytical nature of the reflective statements.

Final Report

At the conclusion of an EFFECT, students create a comprehensive final report that demonstrates their understanding of the importance of the driving question, presents a well-supported solution to the problem using information derived from the active learning modules, and concludes with an analysis of what they learned and how their understanding progressed throughout the course of the EFFECT. The reflective portion of the assignment is really the key that proves the synthesis of content into a more sophisticated (and hopefully correct) solution. Students can use their journal entries as a guide to this analysis.

Example Math-EFFECTs

The following are sample components of several Math-EFFECTS.

Math-EFFECT #1

Core Concept: Estimation

Context: Gather information to design an HVAC system for open air courtyard.

Decision Worksheet - Driving Questions: What would be the "cost" of the HVAC system? How does one provide an accurate but relatively quick estimate for a project?

Active Learning Modules:

- Lecture on statistical metrics; upper and lower bounds;
- Have students measure the length of hallways using estimated measures of feet, wing-span and stride; calculate ensemble statistical metrics;
- Guess and discuss the dimensions of common objects; ceiling and floor tiles; square footage in classroom;
- Actual on-site estimations of the size of the open air courtyard.

Reflection:

- How accurate are `back of the envelope estimates"? Can you imagine doing them as an engineer?
- How many trials are needed for a good sample?
- Were the volumes and dimensions bigger or smaller than initial guesses?
- What other information, not provided in the problem statement, would be helpful?

Math-EFFECT #2

Core Concepts: Measurement & Estimation; Geometric formulas

Context: Design the audience space for a concert in a city plaza.

Decision Worksheet - Driving Questions: How many tickets should you sell to a concert?

Active Learning Modules:

- Guess and discuss the dimensions of common objects; ceiling and floor tiles; square footage in classroom;
- Have students measure the length of hallways using estimated measures of feet, wing-span and stride; calculate ensemble statistical metrics for comparison;
- Worksheet to review geometric formulae; calculations of areas and volumes for complex and/or composite shapes;
- Actual on-site estimated measurements of the city plaza and drawing of its layout.

Reflection:

- How often is information required that has no 'right' answer?
- Are `exact' measurements really exact?
- How much space does a person at a concert require?
- What other information, not provided in the problem statement, would be useful?

Math-EFFECT #3

Core Concept: Solutions of systems of linear equations; vectors and matrices.

Context: A supercargo load is to travel across the state. Two possible routes have been suggested. Both include taking the load across truss bridges.

Decision Worksheet - Driving Questions: What is the 'best' route for a super cargo load?

Active Learning Modules:

- Lecture on: Vectors and matrices; matrix multiplication; connections between linear system of equations and matrix equations; solutions to matrix equations, determinants, inverses;
- Actual measurements and calculations for geometry of truss bridge;
- Lecture: Concept of Equilibrium in Vectors; resultant forces equal zero; could be tied into Statics class;
- Calculations of system of equations; conversion to matrix; Training using a technological platform (Matlab, Mathcad, Maple) for inverting matrix.

Reflection:

- What is meant by `best'? Is there always one `best` answer?
- What assumptions are being made in the truss analysis and how realistic are they?
- Why are matrix methods preferred when solving systems of linear equations?

Math-EFFECT #4

Core Concept: Numerical Solution to First Order Differential Equations

Context: The BuzzBeeToys Company makes a small nerf/foam bullet gun with a three bullet clip. It is marketed under the name Air Warriors Tek 3. They are interested in modifying the firing mechanism; potentially increasing the initial velocity of the bullets.

Decision Worksheet - Driving Questions: Is the initial speed of the foam bullet dangerous to small children if it is fired at close range?

Active Learning Modules:

- Euler's method; simple integration, initial and boundary conditions;
- Measurements of distance foam bullet travels; difficulty in accurate measurement; how many data points are sufficient?

Reflection:

- What speed would be too fast? For what age child?
- Were the measured/approximated speeds surprising?
- How could one more directly measure initial velocity? Average velocity?

Assessment

Assessment of the effectiveness of EFFECTs is still in the preliminary stage. So far it consists of applying a rubric, developed at the University of South Carolina, that serves to measure critical thinking. This rubric involves an evaluation of student journal entries. What they observed, based on this rubric, is that critical thinking improves. This preliminary assessment is discussed in [8]. The EFFECTs researchers are in the process of doing an evaluation that employs an independent, external instrument. The format of this external evaluation is a multiple choice test, for which they have initial validation.

Conclusions

Concept based study is engaging to students because it gets them more closely involved in the work they will eventually do as professional engineers and mathematicians. It's a fun way to approach core concepts, and provides current, realistic content with active learning modules to reinforce retention and understanding. By including reflective writing, teachers can promote higher order critical thinking and return responsibility for learning back to the students. While it is true that rote practice is still essential in mathematics, the developers stress that even implementing parts of an EFFECT are an excellent way to engage students and start the development process of creating entire units of study designed around the EFFECT framework.

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