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Creating Laboratories to Aid Student Modeling Ability in Calculus I

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Abstract

In this paper we will report on the development and deployment of a laboratory sequence for Calculus 1 students. To aid student understanding of calculus concepts, a multidisciplinary team from Rose-Hulman Institute of Technology created four laboratories to accompany Calculus I instruction. The team worked together developing learning objectives, appropriate concepts, and physical implementations. Laboratory 1 explored differentiation and a Resistor-Inductor circuit. Students learned to build a simple circuit on a breadboard and used an oscilloscope to measure the response of the circuit to a static and varying signal. Laboratory 2 investigated time-varying weight. Students measured the weight of water in a bucket as the bucket filled and then released water. Laboratory 3 modeled the infusion of medicine into a patient using a saline solution. Students examined both a continuous infusion and an infusion pulse and determined time constants for the process. Laboratory 4 required students to build a Resistor-Capacitor circuit and measure voltages. Students also determined gain and phase shift as a function of frequency. Initial observations indicate that students enjoy the laboratory setting. In order to assess the impact of the addition of laboratories, final exam scores from the laboratory section will be compared with final exam scores from the traditional section. In addition, faculty in the introductory instrumentation course will compare preparation of students in the new math class to students from the traditional math classes.

Introduction

Rose-Hulman is in the process of developing a proposed new engineering major, tentatively named Engineering Design, which will embrace an integrated, studio-oriented approach. Among other goals, the program will provide students with a hands-on and design-oriented experience from the very beginning of their time on campus. As part of this initiative, a new first-year calculus sequence was developed that would better support this design focused program. During the 2017-2018 academic year, the first-year program was adopted for incoming Biomedical Engineering majors.

Students at Rose-Hulman, which is on the quarter system, are expected to complete Calculus I, II, and III during their first year. These 5-credit courses meet 5 days per week, for a total of 50 meetings during the 10 week quarter (with a final exam the following week). The new sequence, entitled Introduction to Engineering Mathematics I and II (replacing Calculus I and II) and Applied Multivariate Calculus (replacing Calculus III), consists of 4-credit courses that met for 3 days per week in standard format, along with a double period meeting 1 day per week. This scheduling preserved the 5 contact hours of the standard sequence. However, due to the double period day being used for laboratory activities 4 times per quarter and projects/recitation 6 times per quarter, that day only counted as 1 credit hour toward the total.

The laboratory activities discussed in this paper were developed to enhance the Calculus I variant. The labs were developed by one group of faculty and tested by another group during the preceding summer. The 4 labs were run by the calculus instructor (from the Department of Mathematics, but holding a B.S.E.E. degree) and a graduate teaching assistant from the Biomedical Engineering program, and were held on Wednesdays; the background mathematical material was discussed in class the preceding Friday or Monday. The lab (and, when applicable, the pre-lab) were electronically sent out on Monday afternoons. Write-ups of each lab were submitted the following week, graded, and returned to students, at which time the instructor again discussed the lab and its relation to course material.

The goals of the course were several fold, and included: connecting the theoretical development of the calculus material to physical scenarios via hands-on examples, demonstrating at an early stage the usefulness of the calculus in applications, allowing students to visualize functions as waveforms that might appear on laboratory equipment displays, and developing laboratory skills that students would find useful in upcoming engineering courses in the (semi-) integrated first-year sequence.

First Lab: The RL Circuit

The first lab focused on differentiation and the RL circuit. The lab consisted of two related parts to help students build upon small bits of knowledge. In the first part of the lab, students used a provided current source to apply a selected current to a series circuit with only a resistor. This was done by connecting the in-house created current box to the oscilloscope, and that to the breadboard. Students gained basic breadboard usage skills and learned about the use of the two channels of the oscilloscope, as well as the concept of the output of a circuit and the importance of grounding a circuit. Initially, a DC waveform was applied. A main objective of the lab was to introduce students to electrical equipment, such as oscilloscopes and breadboards, as well as to help them become familiar with being in the lab. Another objective was also to set a tone for the quarter and year of seeing laboratory results in two ways: mathematical models can accurately predict physical systems, and physical systems can produce outputs that clearly resemble mathematical functions and operations.

After building the breadboard with a single resistor and utilizing the current generator box paired with the function generator of the oscilloscope to create a current source to apply to the resistor, the overall setup was as seen in Figures 1 and 2.

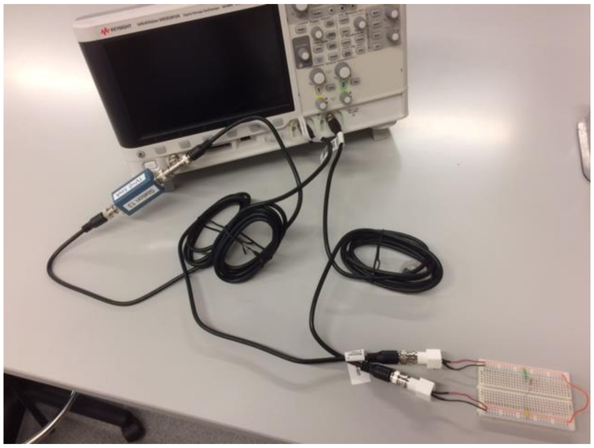


Figure 1: Overall setup of the lab, with the oscilloscope

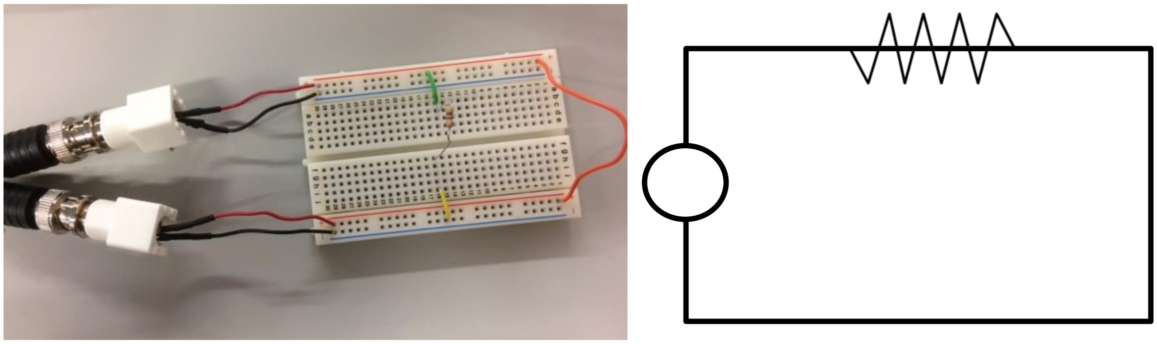


Figure 2: Breadboard setup during Part One of the lab

Students measured the peak-to-peak readings of the output and also the RMS value for various resistors to see the effect of a lower or higher resistance, then added an inductor (which itself had an assumed nontrivial internal resistance), placing the resistor and inductor in series (Figure 3). The students were encouraged to make the connection between the circuit they were building and the simple representation of such a circuit as drawn on a whiteboard in class.

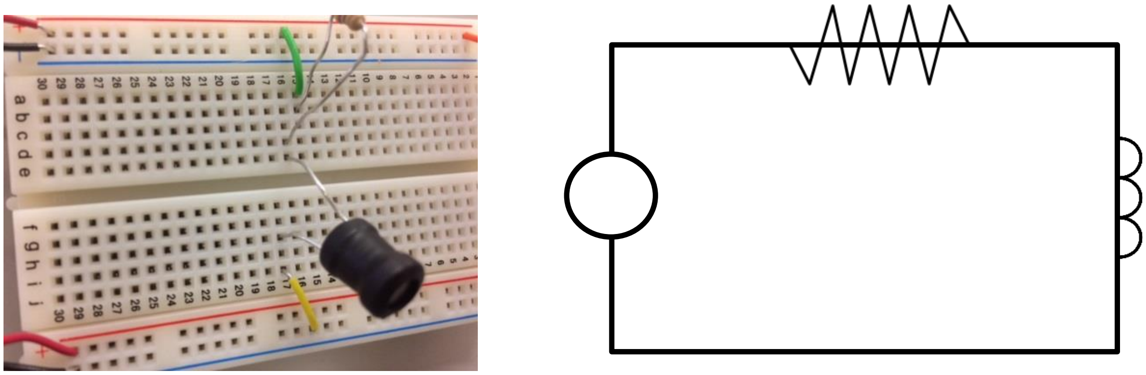


Figure 3: Breadboard setup with the inclusion of an inductor

For the second part of the lab, students measured the peak-to-peak voltage and phase angle (for the inductor-only circuit) with the oscilloscope being used to generate a sine wave. Students recorded their different results in two blank tables in the lab handout.

These circuits connected differentiation from Calculus I to physical phenomena. Ohm's Law, V$= IR$, indicates that a resistor acts as a multiplier in the context of a known current; multiplication predicts the voltage drop, but from another point of view, a resistor implements the mathematical operation of multiplication. An inductor's behavior is governed by $V = LI'$; differentiation predicts the voltage drop, but the inductor can also be thought of as implementing the mathematical operation of differentiation (i.e., it is acting as a simple analog computer).

Asking students to vary the resistance and inductance in their circuits helped them visually see the application of these voltage drop rules; they also saw that the sine waveform had a sinusoidal derivative while the DC waveform had an essentially zero derivative, matching the material presented in the lectures.

Second Lab: The Big Bucket

In this lab the students analyzed a waterpark-style "big bucket," in which a bucket is filled, then emptied suddenly, then refilled again. The weight of the bucket and its contents were measured as a function of time (see Figure 4 for the setup). The primary goal of this lab was for students to understand the nature of a piecewise continuous function as a model of a physical phenomenon, to compare it to an actual (near-)discontinuity in a physical system, and to see what the derivative of such a discontinuous function might look like experimentally. The bucket itself (red in the figure) was 3D-printed in-house.

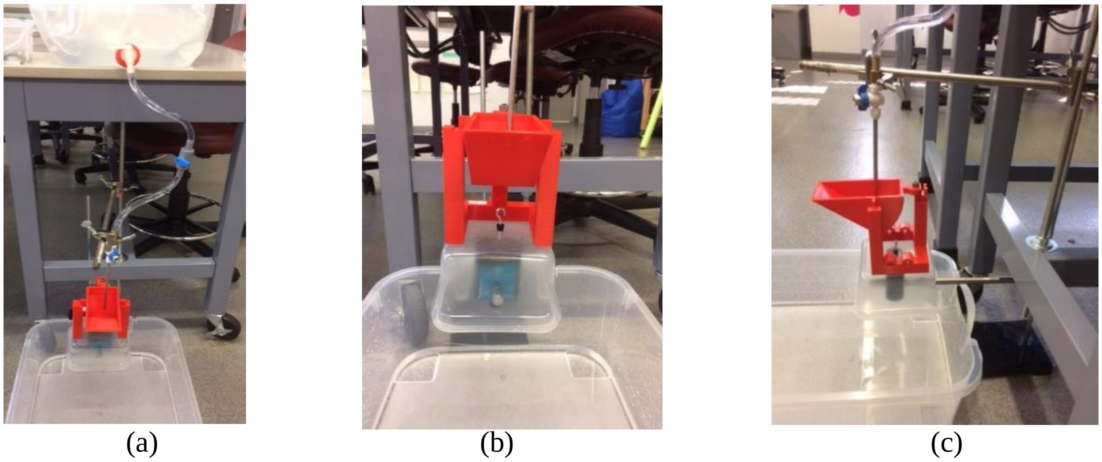

(a) (b) (c)

Figure 4: (a) Lab setup showing the big bucket; (b) a zoomed-in view showing the force gauge below the bucket; (c) side view of setup

The students were first tasked with determining the mass flow rate out of the water supply when the valves were fully opened using a measuring cup and a timer. Next, using a Vernier Go Direct Force and Acceleration gauge, the students determined the weight as a function of time for a 50 second duration using a sampling rate of 50 samples/second. This force gauge reported data via Bluetooth to the students' laptops, and was displayed via the Vernier Logger Pro software in real time. The weight on the support as a function of time is $w(t) = \omega_o + g\mathrm{r}(\mathrm{t})\mathrm{t}$ where ω_o is the weight of the bucket, r(t) is the rate at which water flows into the bucket, t is time, and g is the acceleration due to gravity. Thus, the slope of the line for the data collected, divided by g, is equal to the mass flow rate. The students also used the software to see that the derivative of the line is a constant. In addition, the discontinuities resulted in an undefined derivative, shown as an extremely narrow and noisy zone in the data. (Making this connection between discontinuities as discussed in the Calculus text and discontinues as seen in a physical system was a crucial goal of the lab.) Next, instead of r(t) being constant, the students simulated a sinusoidal wave via rotating a globe valve slowly open and then slowly closed. As with the previous experiment, the students then plotted the force and the derivative. The students' write-ups answered questions such as whether or not the fill times varied and if so, why? In addition, they were tasked with ensuring a constant fill rate for every cycle while rotating the globe valve. Attempting to do this by hand was--as expected and intended--a challenge, reinforcing the idea that replicability requires standardization.

Third Lab: The Salt Tank

In the third laboratory activity, students used a simple instantiation of a salt tank in order to study the modeling of mixtures. The write-up placed this in the context of the "salt" being a medicine being infused intravenously into a patient. The mathematical objectives included comparing the simple model of a salt tank to reality; again observing the decaying exponential function in action; and, gaining further experience with curve-fitting. The laboratory-focused objectives included gaining further experience with the force meter and the associated Vernier software; practicing good lab safety in a wet environment; and, identifying sources of error that contaminated the results.

Instructors prepared, in advance, two five-gallon plastic "tanks" (in fact, they were inexpensive campground showers): One with fresh water, and one with salt water at approximately 20% solution. These were placed side-by-side at the edge of a table in the laboratory, and connected through a T-valve to a tube that emptied into a plastic cup, supported via the force sensor. (See Figures 5 and 6 below.) This force gauge once again reported data via Bluetooth to the students' laptops, and was displayed via software in real time.

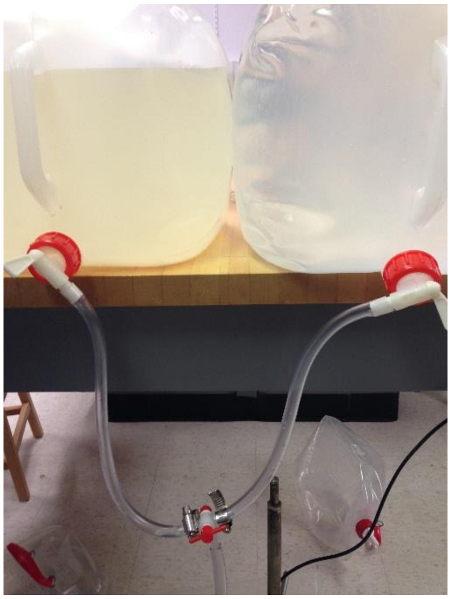
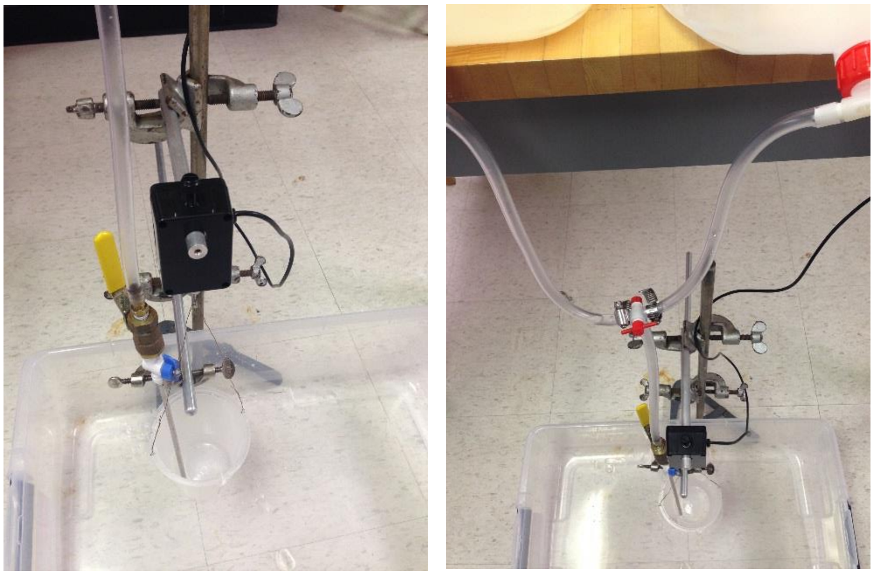


Figure 5: Container connections for the infusion lab



(a) (b)

Figure 6: (a) Measuring cup attached to force sensor and (b) entire setup for the lab

The procedure, in outline, was as follows: First, students measured the weight of fresh vs. salt water by manipulating the T-valve, and calculated the salt concentration from it. (This also served to allow them to iron out any difficulties in sending the data to their laptop and locating it in the software.) Next, they mimicked a continuous infusion by filling the cup to the very top with fresh water (representing the patient's total blood volume), then switching to salt water. The water level did not change, and hence the added weight represented the amount of medicine in the bloodstream as it came in via the tube and left the "body" via overspill. They later fit a curve to this data and compared it to the expected solution of $x'(t)=r_ic_i-r_ox/V$, with V the known, constant volume of the cup, then discussed the limiting behavior of the concentration by comparing their data to the formal limit of the provided analytical solution of the differential equation. (Students had been taught to create such simple ordinary differential equation models, but not yet to solve them.) Finally, they mimicked the effects of an injection or fast-acting pill by filling the cup with fresh water again, letting fresh water flow into it for 60 seconds (while collecting data), then allowing salt water to flow for 30 seconds, after which they returned to supplying fresh water only to the cup. Analyzing this data allowed them to see the decaying exponential curve expected after the salt water flow was turned off, when the governing equation was simply $x'(t)=-r_o x/V$.

The lab appeared to be successful in allowing students to see the exponential function in action, including the significance of the limit as time tends to infinity of the function $\exp(-a\ t)$. They also had to connect the a in $\exp(-a\ t)$ to the displays on their screens, giving them experience in interpreting the rate of growth inherent in the simple separable ordinary differential equation $y'=ay$, the solution of which they *had* studied. The switch from salt water to fresh in the last part also demonstrated a clear cusp—a continuous, but nondifferentiable, function. (The second lab had demonstrated the case of a discontinuous, nondifferentiable, function, which appears earlier in the context of the course lectures.) After the lab, the calculus instructor was able to refer to such discontinuous and continuous-but-nondifferentiable functions during lectures and remind students that these occurred naturally in physical contexts—as they should well know because they had previously “made” them happen! Students’ comments on possible sources of error showed that they were capable of identifying physical difficulties that violated the assumptions of the underlying model and/or affected the data collection process, including most notably the lag and the jiggle from switching the T valve’s position. Spirits were high during the lab, and many students stated that they had found both water labs enjoyable and interesting experiences.

Fourth Lab: The RC Circuit

In the final lab of the first term, students—who by this point were beginning to see antidifferentiation and the basics of definite integration in their coursework—created an RC circuit. (See Figures 7 and 8 below.) They initially confirmed the expected zero-current result for a DC voltage source, then used an AC source to generate a sinusoidal voltage waveform. The solution of the corresponding differential equation model was provided to the students and they were asked to verify it by differentiation and find the steady-state solution by taking a limit. Within the lab period, they used the oscilloscope to verify predictions about phase shifts and about the locations of maxima, minima, and zero-crossings.

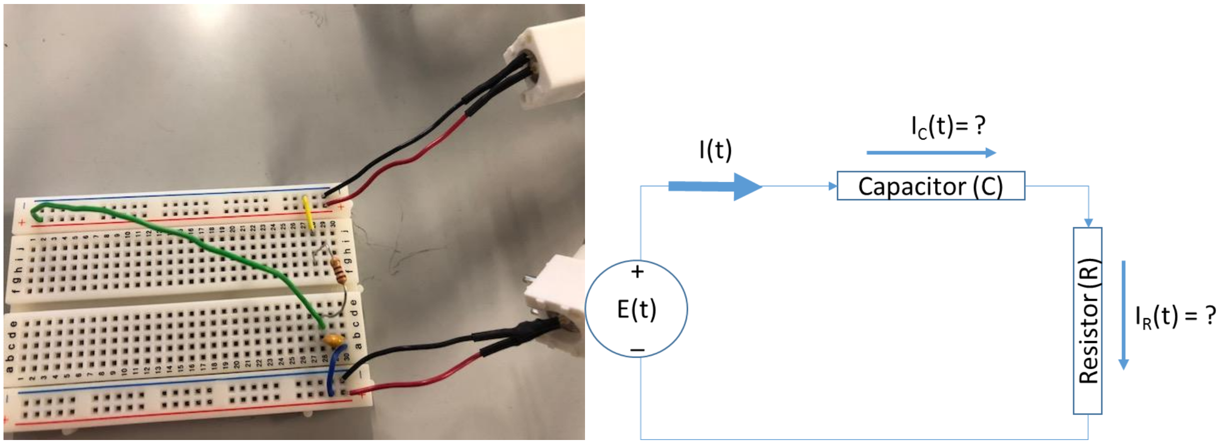


Figure 7: Circuit on the breadboard showing how to connect a capacitor and a resistor to the function generator and the oscilloscope

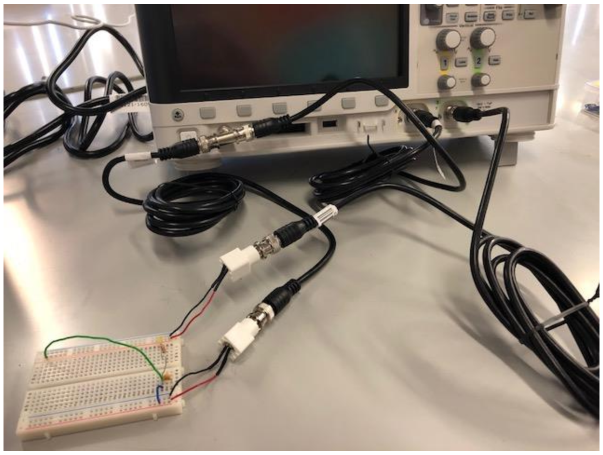


Figure 8: The entire setup of the circuit with the oscilloscope connected

Following this, the students treated the voltage drop across the resistor as the output of the circuit and recorded it for various frequencies. They were asked to use this data to calculate the gain at those frequencies, allowing them to get further understanding of the basic concept of gain, and see how it can be frequency-dependent. Following classroom discussions extended the simple plot of the gain as a function of frequency to the notion of a simple filter.

Discussion

Verbal comments during and after the labs as well as anonymous course evaluations (100% return rate; 16 and 22 students, respectively) allowed for the instructors to study the students' experiences with the laboratory activities in a qualitative way with a goal of understanding how to improve the results of this initial trial of the concept of such "math labs" for follow-on years, both with respect to the planned second year of the trial within the Biomedical Engineering department and as part of the integrated experience within the expected new Engineering Design major, in which the mathematics courses are intended to be even more closely coordinated with material taught in the studio classes. The mathematics department was also interested in whether aspects of this approach might be applied to the standard Calculus sequence.

Student comments on the end-of-term course evaluations were very positive. This student's comment was fully in-line with the course goals: "I really enjoyed the labs because they were hands on and helped me see what happens on the white boards and paper come to life in the real world. It demonstrated graphs and equations from the board in a real-world application." Another student wrote: "I enjoyed the labs honestly, they were a bit stressful occasionally but they helped us apply our calc knowledge and even if you got really confused, you could explain your logic and reasoning for why you did what you did." Another student: "I really enjoyed the Labs, making the connections in real life and in our subject of study." Comments about the benefits of learning to use a breadboard early on were plentiful. Surprisingly, no negative comments on the lab were received. Anecdotal discussions with groups of students or individual students conveyed similar sentiments.

From the lecture instructor's point of view, the labs allowed for strong reinforcement of otherwise somewhat dry classroom material on continuity, differentiability, exponential decay, and modeling in general. The ability to sketch a function on the board and remind students that they have created similar waveforms in the labs was intended as a more convincing way to "sell" students on the utility of the material they were studying, and this allowed for a more physically-based presentation of the subject matter. A much greater-than-usual emphasis had been placed on the creation of differential equation models during the term—indeed, a differential equations textbook was required of the students in addition to the standard calculus textbook—and students had significant opportunity to see how they applied and to see the analog solutions of them on a screen at a time when they could only check them analytically by differentiation of a provided solution.

Conclusion

From the perspective of both the students and of the instructors involved, the lab activities were successful: In isolation, in their connection with the calculus course, and as preparation for further courses, such as the second-quarter electrical circuits material presented by an instructor from Biomedical Engineering within the studio format. The instructional team hopes to refine the labs and extend the planned 12 labs of the full 3-course sequence. (The sequence will run for the entire Academic Year 2017-2018, but the second and third quarters are anticipated to have a reduced number of labs due to time constraints the preceding summer.) The intent is to also create similar lab experiences for the two planned sophomore year differential equations courses.

Lab activities such as these allow students to gain early experience connecting mathematical models to simple electrical and mechanical devices, as well as seeing how such simple devices naturally generate outputs that correspond to standard topics of the calculus sequence: derivatives model inductors, and inductors model derivatives, for example. Students saw how the functions they found in class could be viewed as outputs of these devices. Mathematics, science, and engineering are inter-related, inter-connected fields of study, and the current approach represents a means of bringing this perspective in at an earlier stage of undergraduate engineering education.