

Critical Life-Cycle Decision Making for Projects under Uncertainty

Dr. K. Jo Min, Iowa State University

K. Jo Min is Associate Professor and Associate Chair, Director of Undergraduate Education in Industrial and Manufacturing Systems Engineering Department at Iowa State University. He teaches courses on production systems, closed-loop supply chains, and engineering valuation. His education research interests include outcome assessment and visualization aids, and his engineering research focuses on application of stochastic optimal control on engineering decision making. He has co-authored numerous papers in The Engineering Economist, IEEE Transactions on Engineering Management, International Journal of Production Research, International Journal of Engineering Education, and other peer-reviewed journals. He has been serving as an ABET program evaluator for EAC and ETAC and as a reviewer for various NSF engineering education panels.

Dr. John Jackman, Iowa State University

John Jackman is an associate professor of industrial and manufacturing systems engineering at Iowa State University. His research interests include engineering problem solving, computer simulation, web-based immersive learning environments, and data acquisition and control.

Michelle Zugg, Iowa State University

Michelle Zugg is a Masters of Science candidate in Industrial Engineering at Iowa State University. Her research interests include supply chain management with specific interest in quality and efficiency.

Critical Life-Cycle Decision Making for Projects under Uncertainty

Abstract

In this paper, we consider how critical life-cycle decisions are made for projects facing significant uncertainties. The key differentiating aspect of our approach from the traditional net present value approach is regarding the timing of such decisions. For example, our emphasis is on the effective dates for the commencement and expiration (i.e., a window of opportunity) for possible actions regarding a project, which is clearly above and beyond a single shot decision of investment or no investment. Our approach is based on elementary stochastic optimal control methods, which often afford closed-form solutions on critical timing information such as the expected remaining life of a project under significant uncertainties. These analytic solutions provide managerial insights and economic implications that are simply absent in numerical results under particular sets of parameter values. We next describe how we present such concepts in an introductory engineering economy course utilizing a short, self-contained module of a few lectures. The context of the lectures focuses on the decisions by wind energy farms to exit and/or enter. For this module, we administer pre- and post- tests as well as self-efficacy surveys, and the results from the assessment of outcomes and the self-efficacy surveys are analyzed for insights. Finally, subsequent steps towards improved teaching and learning in life-cycle decision making for projects under uncertainty are outlined.

Keywords: Optimal Timing of Economic Decisions, Stochastic Optimal Control, Learning Outcomes

Introduction and Research Objective

For engineers, there are many incidents and cases where critical economic decisions are made for important phases of projects throughout their project lives under various uncertainties. For example, for a wind energy farm, a decision maker must decide when to enter into the market, then to expand or contract, and then to repower or decommission and exit from the market.

In this paper, we consider how critical life-cycle decisions are made for projects facing significant uncertainties. The key differentiating aspect of our approach from the traditional net present value approach is regarding the timing of such decisions. That is, the conventional decision-support frameworks typically found in introductory engineering economy textbooks (i.e., the Net Present Value, NPV, approach) may work well with simple engineering projects that are fairly deterministic where it is essentially a single shot framework.

Specifically, for a project, estimates are made for both dollar amount and timing of future cash flows, which lead to a discounted dollar amount at a base time point such as the present time.¹

This necessarily ignores the possibility of various real options as the aforementioned uncertainties such as the prices of input/output unfold with respect to time. For example, if the fossil fuel prices increase significantly, in the context of power plants, the real options representing the corresponding strategic flexibility may be to delay construction, to contract the scale of operations, to mothball, or to decommission - just to name a few.

More recently, in view of the observations stated above, a simple, discrete version of a real options approach has been introduced based on the Black-Scholes formula² found in the finance literature. This is followed by an extension to a multi-period binomial lattice mode.³ This approach, however, has yet to overcome the following critical shortcomings.

- 1. The Black-Scholes Formula is based on one discrete up or down movement of an underlying asset in a European call option without dividends (i.e., it can be exercised only at the maturity, implying a single period). This is clearly not the case for numerous engineering projects as there are many decision points before the "maturity" when decisions can be made or real options unfold (e.g., if the electric power price becomes too low, the power plant's option to contract its operations becomes viable).
- 2. To mimic the evolution of the underlying asset value, a multi-period binomial lattice model is often employed without a closed-form analytical solution. Even though this approach is necessary in some cases to solve a problem (e.g., for a compound option), it is computationally intensive. And the resulting solutions are numerical in nature and generalizable managerial insights and economic implications are rather limited.

Therefore, such traditional approaches may be less than sufficient, in our view, in addressing critical decision making in major engineering projects as shown in the following question.

"At the current point in time, what is the expected start date of the project?"

This question, which is essential because the resources (such as money, time, and talent) are almost never readily available for such projects at a moment's notice, requires an optimal (or nearly optimal) timing decision making, which is rarely the goal and purpose of the aforementioned traditional engineering economy approaches. In fact, even though this question is central to many engineering projects, the timing decision is somehow decoupled from the resource commitment decision as if they are to be made separately. On the other hand, logically, such decisions on the timing and resource commitment influence each other, and in general cannot be made independently.

In part to answer the question posed above, our approach is based on stochastic optimal control frameworks such as impulse and continuous controls^{4, 5} applied to engineering economy problems for projects. From a stochastic optimal control perspective, the Black-Scholes formula can be considered as a particular application of an impulse control. From the stochastic optimal control approach, for relatively simple classes of aforementioned options, there exist closed-form solutions for the threshold values (e.g., if the electricity price is at this level, we will invest,

mothball, or decommission) as well as the expected time to reach the threshold values. Such values in analytic forms will provide numerous managerial insights enabling students to develop a deeper level of understanding of economic decisions on engineering projects. These threshold values will also help students build practical intuition so as to become better decision makers when working on engineering projects.

Under these circumstances, for such projects, it is essential that engineering students have:

- A. active decision making capabilities exploiting the aforementioned strategic flexibility as the uncertainties such as electric power prices or fossil fuel costs unfold over time.
- B. a useful framework for critical decision making that adds managerial insights and facilitates development of intuition behind decision making under uncertainties. For example, why does volatility increase the value of flexibility (when the flexibility is viewed as an option, its holders do not lose from increased uncertainties if things turn out wrong, but gain if they turn out right because the real options are choices for possible future actions, but not requirements or obligations in a contract).
- C. the rigor in mathematical modeling that facilitates strategic thinking and the ability to focus on just a few key uncertainties to distill sometimes chaotic economic fluctuations observed in engineering projects into a few strategic decisions of importance (e.g., an electric power price threshold to construct a new power plant). This rigor in modeling and the ability to focus will lead to insights and intuition that can be cumulatively applicable to even more engineering projects.

To address these needs, it is highly desirable to introduce the basic concepts of critical decision making in such projects to engineering students and to further show how these concepts are implemented from start to finish.

As a small first step towards this objective, we developed teaching materials and assessments for a short, self-contained module in an introductory engineering economy course with heavy emphasis on concepts (cf. mathematical mastery involving stochastic optimal control itself). The purpose of such construction and teaching is to encourage engineering students to be more attuned to the insights and intuition behind economic decision making on an engineering project during its life-cycle.

The rest of the paper is organized as follows. We first explain the module contents and structure. This is followed by the methodology consisting of the procedure and the participants. We next present the results of this study. This is followed by concluding remarks and comments on future research.

Module Contents and Structure

For the module contents, we utilized Min⁶ as the primary reference paper. This paper is chosen because of relatively straightforward conceptual findings as well as relatively simple mathematical formulation and analysis. For example, this paper formulates and analyzes the

optimal threshold level of the operations and maintenance (O&M) cost above which an aging wind farm needs to be decommissioned and exit from the market where the O&M cost follows a geometric Brownian motion (GBM).⁷ This paper also investigates the expected remaining life of the wind farm evaluated at the said threshold in the O&M cost. We believe that our choice of the primary reference paper is suitable since the aforementioned knowledge and skill attributes for engineering students are shown in the paper.

Specifically, in this paper, rather than passively waiting for the physical life of the wind farm to run its natural course of wear and tear, the wind farm decision maker proactively makes a life-cycle decision to exit using such a strategy as a real option (Attribute A). In addition, via sensitivity analysis, for example, the reason that the value of flexibility increases in volatility is elaborated (Attribute B). Finally, the mathematical rigor in modeling and the focus on insights and intuition are maintained throughout the paper (Attribute C). Hence, even though the emphasis on the module is on concepts, any student interested in further studies can return to the paper for additional information - including a list of further references.

As for the structure of the module, which was presented in an introductory undergraduate engineering economy course, consists of six class periods (50 minutes per period). In this way a balance is struck between covering critical topics of a traditional engineering economy in sufficient details (in 39 class periods) and introducing a new perspective from a stochastic optimal control point of view.

With aforementioned emphasis on concepts (cf. mathematical derivations and manipulations), following materials were presented during the six periods.

- Period 1. A pre-test, traditional net present value approach, questions under uncertainty
- Period 2. Using Min⁶ (for Periods 2-4), introduction to GBM and Bellman optimality principle, hysteresis
- Period 3. Optimal threshold to exit, optimal expected remaining life
- Period 4. Sensitivity of the optimal solution, student contests
- Period 5. An introduction to a decision tree model connecting the approaches of this project and the traditional net present value approach
- Period 6. An epilogue, further studies, and a post-test

Methodology

The teaching materials were used in an undergraduate course on Engineering Economy (taught in the industrial engineering program at Iowa State University) to study the effects, if any on student learning and self-efficacy. This course is required for all industrial engineering students and is used as a technical elective by students in other majors. Our study used a single case design⁸ recommended by the Department of Education, which does not require a control group because it focuses on the assessment of student understanding before and after an instructional intervention. The study was reviewed and approved by the Institutional Review Board (IRB).

For the self-efficacy survey, 10 statements were included based on the General Self-Efficacy Scale of Schwarzer and Jerusalem,⁹ using a Likert scale of 1 to 4. The first part of the survey (up to Question A) is given in Part A.

The pre- and post-test questions (Part B) were constructed to address different levels of Bloom's taxonomy. The three multiple choice questions covered the contents of the new teaching module (Figures used in the test are from Dixit and Pindyck,⁴ on Page 111, and a permission request is under review at this time by Princeton University Press). The first question addressed the lowest order thinking skill test as it relates to remembering a key limitation. The second question was designed to assess students' understanding of an economically rational decision under uncertainty (a higher order thinking skill). The last question assessed students' analysis skill (differentiating scenarios once volatility increases; an even higher order thinking skill).

Participants in the study

A total of 74 undergraduate students participated in the study. The demographics of the students are shown in Figure 1 and Table 1. Industrial engineering majors at the junior level were the largest group in the study.

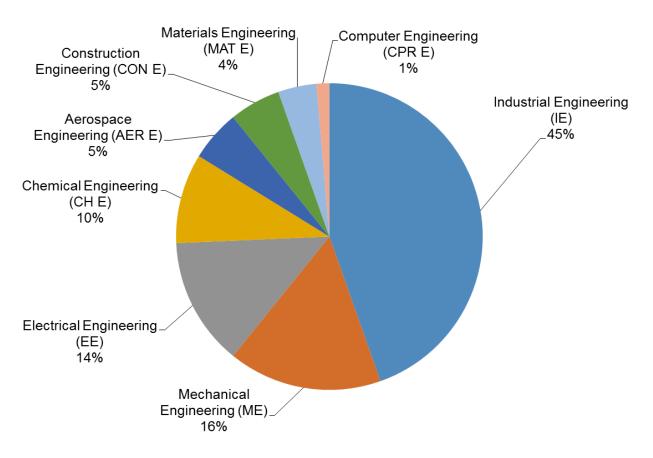


Figure 1 Proportion of students by major

		Majors									
	Total # of										
Year	students	ΙE	ΜE	ΕE	CH E	AER E	CON E	MAT E	CPR E		
2	4	3	0	1	0	0	0	0	0		
3	27	24	1	0	1	0	1	0	0		
4	43	6	11	9	6	4	3	3	1		
Total:	74	33	12	10	7	4	4	3	1		

Table 1 Students by year in the program and major

Procedure

After students learned how to use the traditional Net Present Value approach to decision making, the self-efficacy survey (Part A) was administered followed immediately by the pre-test. The six lectures previously described in the Module Contents and Structure section followed the pre-test. After the last lecture, the self-efficacy survey was administered again followed immediately by the post-test, which is the same as the pre-test. The tests were scored by assigning one point for each correct answer and no points for incorrect answers (i.e., a maximum possible score of 3).

Analysis

Given the single case design, paired t-tests were used in the analysis to determine if the teaching module had a statistically significant effect on student learning and self-efficacy. In addition we calculated descriptive statistics (mean and standard deviation) for the test scores. The null and alternative hypotheses for the paired t-test were H_0 : *there is no difference between pre- and post-test scores* versus H_a : *there is a difference*. We expected that there would be an increase in student scores from the pre- to post-test, which would be indicated by a positive difference. A two-sided t-test was used at a significance level of 0.05. The size of the effects was quantified using Cohen's d statistic where values of 0.2, 0.5, and 0.8 are considered to be small, medium, large effects, respectively.¹⁰

Results

As can be seen in Figure 2, on average, the overall test scores and individual question scores increased from the pre- to the post-test, indicating that the instructional methods had a positive impact on student learning. The average scores on the questions are consistent with how we designed the questions. The scores decreased from question 1 to question 3 due to the increasing difficulty of the questions.

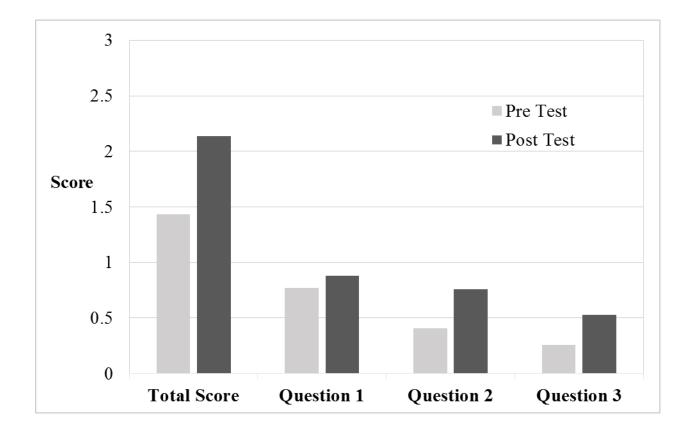


Figure 2: Summary of average score for pre and post test

The paired t-test results in Table 2 indicate that there was a statistically significant difference between the pre- and post-tests for the overall test scores. Cohen's *d* statistic shows that the instructional methods had a large effect on the outcome. While questions 2 and 3 also had significant increases, the students did not perform as well as on question 1. Therefore, changes in instructional methods are warranted.

	Total Score		Ques	Question 1		Question 2		tion 3
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
	Test	Test	Test	Test	Test	Test	Test	Test
Mean	1.432	2.135	0.770	0.878	0.405	0.757	0.257	0.527
Standard Deviation	0.742	0.833	0.424	0.329	0.494	0.432	0.440	0.503
df	73		73		73		73	
Calculated t-value								
for paired t-test	6.813		2.192		4.982		4.005	
t-value threshold to								
reject H _o	1.993		1.993		1.993		1.993	
p-value	< 0.001		0.0315		< 0.001		< 0.001	
Comparison to H_0 Reject H_0		$\operatorname{ct} H_0$	Cannot reject H_0		Reject H_0		Reject H_0	
Cohen's d Value	0.89		0.29		0.76		0.57	

Table 2: Summary of test score results for all students

We found differences in performance between industrial engineering (IE) majors and other majors as shown in Tables 3 and 4. Based on Cohen's *d* statistic, a larger effect was observed for IE majors that the other majors. The IE majors had a larger improvement on question 2 than the other majors and there was not a significant improvement on question 3 for IE majors. The other majors had a larger improvement on question 3 and it was statistically significant. Further investigation is needed to determine what is causing these differences.

	Total Score		Question 1		Question 2		Question 3	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
	Test	Test	Test	Test	Test	Test	Test	Test
Mean	1.333	2.121	0.727	0.879	0.364	0.818	0.242	0.424
Standard Deviation	0.736	0.781	0.452	0.331	0.489	0.392	0.435	0.502
df	32		32		32		32	
Calculated t-value	5.796		1.971		5.164		2.248	
t-value threshold to reject H_0	2.021		2.021		2.021		2.021	
p-value	< 0.001		0.057		< 0.001		0.032	
			Cannot Reject				Cannot Reject	
Comparison to H_0	Reject H_0		H_0		Reject H_0		H_0	
Cohen's d Value	1.04		0.38		1.03		0.39	

Table 3: Summary of test scores for Industrial Engineering students

Table 4: Summary of test scores for all other Engineering students

	Total Score		Question 1		Question 2		Question 3	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
	Test	Test	Test	Test	Test	Test	Test	Test
Mean	1.512	2.146	0.805	0.878	0.439	0.707	0.268	0.610
Standard Deviation	0.746	0.882	0.402	0.333	0.503	0.461	0.447	0.498
df	40		40		40		40	
Calculated t-value	4.193		1.138		2.557		3.332	
t-value threshold to								
reject Ho	2.021		2.021		2.021		2.021	
p-value	o-value < 0.001		0.2612		0.014		0.002	
			Cannot Reject					
Comparison to H_0	Reject H_0		H_0		Reject H_0		Reject H_0	
Cohen's d Value	0.	78	0.16		0.63		0.58	

The results based on the students' level (junior or senior) are shown in Tables 5 and 6. Surprisingly, the juniors exhibited a larger effect (based on Cohen's *d*) on questions 2, while seniors exhibited a larger effect on question 3. It should be noted that the majority of juniors were industrial engineering majors, so there could be interaction effects.

	Total Score		Question 1		Question 2		Question 3	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
	Test	Test	Test	Test	Test	Test	Test	Test
Mean	1.296	2.074	0.704	0.889	0.407	0.815	0.185	0.370
Standard								
Deviation	0.775	0.781	0.465	6	0.501	0.396	0.396	0.492
df	26		26		26		26	
Calculated								
t-value	5.381		1.333		3.376		1.333	
t-value threshold								
to reject Ho	2.056		2.056		2.056		2.056	
p-value	< 0.001		0.1942		0.0023		0.1942	
Comparison to H_0	Reject H_0		Cannot Reject H ₀		Reject H_0		Cannot Reject H_0	
Cohen's d Value	1.00		0.46		0.90		0.41	

Table 5: Summary of test scores for juniors (or year 3)

Table 6: Summary of test scores for seniors (or year 4)

	Total Score		Questi	Question 1		Question 2		on 3
		Post		Post		Post		Post
	Pre Test	Test	Pre Test	Test	Pre Test	Test	Pre Test	Test
Mean	1.512	2.163	0.814	0.860	0.395	0.721	0.302	0.628
Standard								
Deviation	0.736	0.898	0.394	0.351	0.495	0.454	0.465	0.489
df	42		42		42		42	
Calculated t-value	4.388		0.703		3.313		3.313	
t-value threshold								
to reject H_0	2.018		2.018		2.018		2.018	
p-value	< 0.001		0.0175		< 0.001		< 0.001	
			Cannot Reject					
Comparison to H_0	Reject H_0		H_0		Reject H_0		Reject H_0	
Cohen's d Value	0.79		0.12		0.69		0.68	

Analysis of the self-efficacy survey indicated an increase in scores for statements A and B that were statistically significant with p-values < 0.003. The other statements did not have a significant difference. Scores for all the survey statements were at the high end of the Likert scale as shown in Figure 3. The effects of the module contents on self-efficacy indicate a positive impact on students' self-efficacy, but further investigation is warranted to explore the effects.



Figure 3: Average Likert score for self-efficacy

Concluding Remarks and Future Works

In this paper, we considered how critical life-cycle decisions are made for projects facing significant uncertainties via elementary stochastic optimal control methods, and described how a brief teaching module was developed emphasizing managerial insights and economic implications. We then how such a module was presented in an introductory engineering economy course. For this module, we administered pre- and post- tests as well as self-efficacy surveys, and the results from the assessment of outcomes and the self-efficacy surveys were statistically analyzed for insights. For example, such a statistical analysis showed that, on average, the overall test scores and individual question scores increased from the pre- to the post-test, indicating that the instructional methods had a positive impact on student learning.

At the time of this writing, we are continuing our efforts for effective and efficient teaching and learning of how critical life-cycle decisions are made for projects under uncertainties. For example, we are teaching an experimental course aimed at undergraduate senior and graduate level engineering majors titled, Advanced Engineering Economy for Complex Engineering Projects. Concurrently, we are in the process of converting journal publication contents into teaching materials^{11, 12, 13} with their corresponding visual aids.¹⁴

As we deepen our understanding of the teaching and learning effectiveness of this important topic, we plan to increase our dissemination efforts as well, and we hope to positively contribute

to the education of engineering majors who will be making critical life-cycle decisions for projects in the near future.

Acknowledgment and Disclaimer

This material is based upon work supported by the National Science Foundation under Grant No. 1504912. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. We would like to thank the regular instructor and the teaching assistant, Dr. Mike Helwig and Mr. Fikri Kucuksayacigil, respectively, for their cooperation and contributions such as accommodation, advice, and a decision-tree based guest lecture. We also would like to thank the Department of Industrial and Manufacturing Systems Engineering at Iowa State University for generous support in the form of teaching assistantship. Finally, we would like to thank the constructive and prompt reviews and decisions with professionalism by the two referees and the Engineering Economy Division Program Chair.

References

¹ Sullivan, W., E. Wicks, and J. Luxhoj, 2003, *Engineering Economy*, 12th Edition, Prentice Hall, New Jersey. ² Black, F. and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *The Journal of Political Economy*, Vol. 81, pp. 637-654.

³ Park, C., 2007, *Contemporary Engineering Economics*, 4th Edition, Pearson Prentice Hall, New Jersey.

⁴ Dixit, A. and R. Pindyck, 1994, *Investment under Uncertainty*, Princeton University Press, New Jersey.

⁵ Stokey, N., 2009, *The Economics of Inaction: Stochastic Control Models with Fixed Costs*, Princeton University Press, New Jersey.

⁶Min, K. J., C. Lou, and C. Wang, 2012, "An Exit and Entry Study of Renewable Power Producers: A Real Option Approach," *The Engineering Economist*, Vol. 57, pp. 55-75.

⁷ Ye, M., 1990, "Optimal Replacement Policy with Stochastic Maintenance and Operation Costs," *European Journal of Operational Research*, Vol. 44, pp. 84–94.

⁸ Kratochwill, T., Hitchcock, J., Horner, R., Levin, J., Odom, S., Rindskopf, D., Shadish, W., 2010, *Single-Case Designs Technical Documentation*. Retrieved from What Works Clearinghouse website: http://ies.ed.gov/ncee/wwc/pdf/wwc_scd.pdf .

⁹ Schwarzer, R. and M. Jerusalem, 1995, *Generalized Self-Efficacy Scale*. In J. Weinman, S. Wright, and M. Johnston, Measures in health psychology: A user's portfolio. Causal and control beliefs (pp. 35-37). Windsor, UK: NFER-NELSON.

¹⁰Cohen J., 1988, *Statistical Power Analysis for the Behavioral Sciences* (2nd ed.), Lawrence Earlbaum Associates, New Jersey.

¹¹ Shi, W. and K. J. Min, 2014, "Product Remanufacturing: A Real Options Approach," *IEEE Transactions on Engineering Management*, Vol. 61, pp. 237-250, 2014.

¹² Shi, W. and K. J. Min, 2014, "Product Remanufacturing and Replacement Decisions Under Operations and Maintenance Cost Uncertainties," *The Engineering Economist – Special Issue on Engineering Economics in Reliability, Replacement and Maintenance*, Part 1, Vol. 59, pp. 154-174.

¹³ Wang, C. and K. J. Min, 2013, "Electric Power Plant Valuation Based on Day-Ahead Spark Spreads," *The Engineering Economist*, Vol. 58, pp. 157-178.

¹⁴ Min, K.J., J. Jackman, and J. Chan, 2014, "Visual Models for Abstract Concepts towards Better Learning Outcomes and Self-Efficacy," Educational Research and Methods Division, *ASEE Proceedings*, Indianapolis, Indiana.

Appendix Part A: Self Efficacy Part B: Quiz (Mandatory)

Please circle your answers for the following 3 problems.

#1. For an investment decision problem on a project, in a traditional net present value approach, the sum of present values of incoming and outgoing cash flow is computed. Next, the investment rule is that, if the sum of present values is positive, then the decision maker invests. If negative, then the decision maker does not invest. Which of the following decisions are not supported by the traditional net present value approach?

a) the optimal starting time of the project.

b) the optimal termination time of the project.

c) the optimal length of the period during which this investment decision rule is valid.

d) all of the above.

#2. A commercial popcorn-making machine for a movie theatre business has a fixed physical life of 10 years. Let us assume that the level of profit at a time *t*, x(t), evolves according to Brownian motion with drift (Bmwd).

Bmwd implies that, for any time interval, the profit decreases on average proportional to the size of the time interval while the variance of the profit increases proportional to the size of the time interval. This is because as the machine ages, the operations and maintenance (O&M) cost increases on average, but its volatility also increases (e.g., the range of the repair cost for your car 10 years down the road will be far greater than the cost 1 year down the road).

Under the circumstances described above, the graph below shows the threshold function, $x^*(t)$ versus *t* (in years). If the profit falls below this curve at time *t*, then the decision maker retires this machine.

See Figure 4.1.a in Reference 4 (Dixit and Pindyck 1994; Permission to use is still pending at Princeton University Press).

As you can observe, the threshold profit level to retire is quite negative. Why does the decision maker not retire the machine at the first time point at which the profit turns negative? That is, why wait?

- a) Most of loss is sunk cost. That is, the machine is already paid for.
- b) The new generation of popcorn-making machine is not yet available.
- c) If there is some time left until year 10, then profit may become positive due to variance.
- d) The government might provide a movie ticket subsidy in the near future.

#3. Suppose volatility (a measure of uncertainty; proportional to variance) has increased. What would the threshold graph now look like (depicted in the dashed curves below)?

- (a) Figure 4.1.a in Reference 4 with a dashed line below.
- (b) Figure 4.1.a in Reference 4 with a dashed line crossing from above.
- (c) Figure 4.1.a in Reference 4 with a dashed line crossing from below.
- (d) Figure 4.1.a in Reference 4 with a dashed line above.
- (Dixit and Pindyck 1994; Permission to use is still pending at Princeton University Press).