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Curricular and Strategic Changes in mathematics to Enhance Institutional STEM Education

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Curricular and Strategic Changes in Mathematics to Enhance Institutional STEM Education

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Abstract

We implemented a project to create transformative changes in the STEM Education at a Hispanic-serving institution by revamping mathematics curriculum and building a system of support in mathematics learning. The project accomplished three main areas of curriculum development: (1) Restructuring the first-year mathematics courses at the college algebra level using a corequisite model; (2) designing and implementing active learning and problem-solving oriented curriculum in Calculus; (3) building a student support system of peer-tutoring and mentoring through peer-led team learning. High impact strategies were used for increasing and sustaining the participation of women and underrepresented minority students in computing. Strategies included hands-on active and collaborative learning pedagogy, experiential learning with real world relevance and applications, and curriculum that incorporates data analysis, data visualization and computational thinking. The support targeted introductory courses to provide early exposure and engagement. This paper describes the curriculum restructuring and development, which led to the institutional implementation and transformation of mathematics curriculum. The paper also reports on the success of the redesigned courses.

Introduction

New York City College of Technology (City Tech), located in downtown Brooklyn, is an open access, non-residential, Hispanic-Serving institution. City Tech, one of the twenty-five colleges in the City University of New York system, provides a diverse multicultural learning environment, reflecting the demographic diversity of the New York metropolitan area. According to the Fall 2020 institutional data, twenty-nine percent of the students self-identified as Black non-Hispanic, thirty-four percent as Hispanic, twenty percent as Asian, ten percent as White, two percent as Other and five percent as Non-Resident. The student body reported 151 different countries of origin. Sixty-one percent reported a household income of less than $30,000. Eighty percent of the first-year students and sixty-seven percent of the returning students received need-based financial aid. Fifty-five percent reported working twenty or more hours per week. Sixty-two percent self-reported that they are first generation college students [19, 17, 18].

City Tech, designated as a college of technology, has a strong science, technology, engineering, and mathematics (STEM) focus. It offers both associate and baccalaureate degree programs in a flexible comprehensive "two plus two" curriculum. This option provides access to college education even if a student has not completed the college preparatory course work; the student may begin in an associate degree program while doing developmental or preparatory coursework and seamlessly continue into baccalaureate degree program without transfer. This flexible comprehensive model has allowed the institution to better support the education of its student population in STEM and lead in the number of associate degrees awarded to minority students. The National Science Foundation (NSF) ranks City Tech 8th in the number of associate degrees awarded to men, 7th in degrees to black students, and 9th to Asian students [4].

A computer science program self-study conducted in 2012-2014 revealed poor graduation and retention rates among students in computer science. Since the computer science program is housed in the mathematics department, the self-study identified three critical areas to address, all pertaining to mathematics: (1) A lengthy non-contributory algebra sequence; (2) low pass rates in calculus courses; (3) need to support students in STEM. In 2015, a team of faculty submitted a grant application to the US Department of Education Minority Science Engineering Improvement Program (US DOE MSEIP) proposing transformative curriculum and strategic changes in mathematics to address the critical needs identified in the self-study. The project was awarded from 2015 – 2018 with the work carried out from 2015 – 2019 (including an extension year). Through curricular and institutional changes, the project impacted thousands of students, beyond the original goal of computer science students.

The project had a strong focus on supporting women and underrepresented minority students in STEM. The curricular and the support strategies were built on high impact education practices that had shown to enhance the learning and the engagement of women and underrepresented minority students in STEM. Research shows active-learning pedagogy such as hands-on collaborative student-centered learning enhances the success of underrepresented minority students in STEM [1, 4].

In this paper, we describe the work done to address the three critical areas. In particular:

1. We describe the design and the adoption of the college algebra with a corequisite model to shorten the time on algebra sequence.

2. We describe the curriculum redesign and support in calculus sequence to improve pass rates.

3. We describe the peer-led team learning (PLTL) and the pedagogical strategies to better support students in STEM.

College Algebra with "Corequisite" Support

The restructuring of the first-year mathematics courses occurred in the algebra sequence. Using a corequisite model and providing content support, students can fulfill their mathematics prerequisite and start on their major course work sooner.

At the institution, the credit level algebra sequence began with Intermediate Algebra and Geometry (MAT 1175), followed by College Algebra and Trigonometry (MAT 1275), and Precalculus (MAT 1375), making it a 3-semester, 12-credit long sequence before reaching Calculus. Part or the entire sequence may be non-contributory towards the degree. Especially in STEM majors, the degree requirement usually starts at the level of precalculus or calculus. Students placed into the prerequisite path would need additional time and money in prerequisite course work before starting their major course work. In computer science, this may mean three additional semesters for an associate degree. The path was even longer for students placed into developmental mathematics. Moreover, these courses were consistently ranked among courses with the highest failure rates. A lengthy prerequisite path in courses with high failure rates would not only lower retention, delay graduation, and may deter students from entering STEM fields or even attaining a degree.

Research has shown that using a corequisite model to accelerate students' prerequisite studies helped more students earn a degree sooner. Studies also examined the demographics of the students who benefited from the corequisite model, it was found that minority students and students with financial needs tended to be those starting at developmental or prerequisite courses. Providing the corequisite support to accelerate students' course work helps give underrepresented students a lift to keep pace with other students, [15, 16, 14].

The College Algebra and Trigonometry with Corequisite (MAT 1275CO) was created following a corequisite model. The model provides two additional hours of the "corequisite component" and strategically offers the "just in time" support in content. For example, before learning a topic, students will review the intermediate algebra topics to prepare for the more in-depth and complex topic in college algebra. This re-organization of topics and how they are presented gives a better content development and continuity.

The College Algebra with Corequisite (MAT 1275CO) was piloted in Fall 2017 and Spring 2018 with 158 and 186 students respectively. The student grade distribution in MAT 1275CO was comparable to that of MAT 1175 and MAT 1275, an indication that students were able to keep up with the intensity and the demand of the course. It was fully implemented in Fall 2018 with more than 1000 students enrolled in MAT 1275CO. The enrollment number in MAT 1175 dwindled until it ceased to be offered in Spring 2020. The algebra courses are no longer a sequence but two parallel tracks of either college algebra or college algebra with corequisite.

Several strategies were incorporated to support the success of the corequisite course. First, WeBWorK, an online open-source homework management system, was adopted by the mathematics department. The WeBWorK homework sets were custom created for the MAT 1275CO students with built-in resources. Second, peer leading support was provided in all MAT 1275CO sections. The peer leaders assisted with both the mathematics content and technology support. Third, workshops were conducted to provide instructors teaching MAT 1275CO with technology training and pedagogy professional development. Active learning pedagogies such as hands-on problem-solving, group work, and peer-led group discussions were recommended; lengthy lectures were dissuaded.

The PLTL support is elaborated in the PLTL section of the paper. The grade distribution of the MAT 1275CO is reported in the results section.

Computing in Calculus

One component of the 2012-2014 computer science program self-study was course assessment in computer science courses. The Fall 2012 Calculus II assessment revealed that students were failing or withdrawing at a very high rate, 44% of men and 49% of women failed or withdrew from the course. See Table 1. The most disappointing discovery was that among the students in Calculus II that semester, only one woman was identified as a computer science major, she withdrew from Calculus II and subsequently withdrew from the college without attaining a degree.

Fall 2012 Calculus II Grade Distributions							
	Total (N)	A	B	C	D	F	W
Men	313	15%	14%	16%	11%	16%	28%
Women	51	21%	12%	10%	8%	18%	31%

Table 1: Fall 2012 Calculus II Grade Distribution.

The discovery prompted the project team to develop an active learning curriculum in Calculus, one that was not just the mechanics of differentiation and integration, but one that integrated concept visualization, real-world problem-solving, and modern computing. We envisioned a calculus curriculum that fostered important 21st century STEM skills, which included modeling, analyzing, computational problem-solving and coding.

Coding in a mathematics class helps with the visualization of the problems, enhances mathematical understanding and builds computational thinking. Studies have shown that intertwining programming and mathematics is beneficial for deeper learning in mathematics as well as for developing coding skills [3, 5, 6, 20]. This helps students without prior coding experience to code in context and build confidence.

From Fall 2016 to Spring 2019, six experimental sections of Calculus I and four experimental sections of Calculus II were implemented. Two instructors in the experimental sections developed visualization lessons and computing modules. The modules on real-world applications in calculus were often challenging and may require coding. Students worked collaboratively under the guidance of the peer leaders in a supportive environment provided by the PLTL workshops.

In addition to Calculus modules, we created extracurricular coding activities with the goal of enriching, enhancing, and engaging students in computing. The activities were offered as winter and summer workshops for students who wanted to enhance their programming skills in an extracurricular setting. The activities included creative art, visualization projects, real-world modeling and problem-solving, using computing tools such as **Python, R, Desmos, GeoGebra, CoCalc, and RStudio Cloud**. Selected visualizations and sample activities are included in the Appendix section of this paper.

In particular, Python and CoCalc were used for a summer coding workshop and some calculus activities, along with Desmos and GeoGebra for the latter; R and RStudio were used for a 3-day summer workshop on implementing visualization projects. One of the authors also used R for a calculus lab, as an experiment, by following the freely available online book *MOSAIC Calculus*, by Daniel Kaplan. This book represents a major attempt at transforming traditional calculus into computational calculus, based on R and the specially designed for this purpose R packages **mosaicCalc** and **mosaic**, developed by **Project MOSAIC**, with the support of the MAA and the NSF, [7]. A collection of projects implemented in R, not just for probability, statistics and data analysis, but also for calculus, can be found in [2, 9].

Many people still think of R as a statistical programming language, but R has evolved, especially around RStudio with its native support for R Markdown technologies, which go beyond notebooks and offer R packages for creating html books, websites and blogs, as well as the ability to create with minimum effort interactive 2D and 3D html visualizations and animations using the state-of-the-art visualization packages of base R and the **Tidyverse** collection of packages developed by RStudio. The package ecosystem built by RStudio is a one-stop solution for many educational needs.

We decided to use R and Python, together with RStudio and CoCalc, respectively, as the high-level programming languages of choice for several reasons. Our choices were based on the computing needs of our students during their college studies and what they need to help them get internships and industry jobs after they graduate. As these needs change over time, we will also change the programming languages that we choose to focus on. For us, it is very important that the programming language of choice has a reasonable, not too steep, learning curve, and that the students can either install and maintain all required software packages on their personal computers, or have free access to the computational environments on the cloud. Last, but not least, one of the most important criteria for a modern computational environment is to have an excellent literate programming implementation, as envisioned by Donald Knuth in the 1980's. We believe that this is the future of scientific computing for creating publication quality, reproducible research and related documentation in different output formats.

Currently, the best implementations of the literate programming paradigm are **R Markdown** notebooks in RStudio and **Jupyter** notebooks. R Markdown and its latest iteration **Quarto** notebooks allow one to combine R, Python and other code cells in the same markdown document, infused with plain text narrative and LaTeX expressions, having fine-tuning capabilities to produce a publication quality project reports, presentations and articles as pdf, html, word, and ppt formats, using Pandoc. Both RStudio Cloud and CoCalc offer instructor's accounts to create virtual classrooms and share projects with students and monitor their progress. In particular, RStudio Cloud offers both R Markdown and Jupyter notebooks, and soon the ability to collaborate on a project in real-time, similar to working in a Google Doc.

Peer-Led Team Learning Support

Quantitative data have demonstrated an improvement in student performance where peer-led team learning (PLTL) was an integral component of a course [11, 12, 13]. This was demonstrated in a fundamental mathematics course, as well as in calculus. The PLTL workshops provide an opportunity for collaborative, hands-on active learning. Implementing PLTL increases a sense of community among students when they have consistent interaction with the same peer leader and participants [21]. The PLTL workshops have shown to help cultivate women's confidence by learning through a "community of practice" [10]. Furthermore, peer leaders serve as role models and this is especially compelling when women and underrepresented minorities are role models to other women and underrepresented minorities in STEM. The positive effect of PLTL is not only on the participants in the PLTL program, but also on the peer leaders. Peer leaders who facilitate these workshops feel empowered through training and peer mentoring, which ultimately gives them important leadership and communication skills [8, 22].

We used two different PLTL models to support student learning. The original PLTL uses a workshop model, which takes place outside of the classroom in a setting where the work and the conversations are facilitated by peer leaders. The instructor provides the study materials and guidance, but does not participate in the workshop. In this model, students naturally work in a group setting, and they determine the format and the style of learning without intervention from the instructor [12].

The modified PLTL model resembles the supplemental instruction model where the peer leader assists the instructor in the classroom. In this model, the instructor determines and monitors the work of students with the help from the peer leader. Instructor's design of classroom activities may have a great impact on student engagement.

In the experimental sections of Calculus I and II, we implemented the original workshop model, in which an extra hour per week was scheduled for the workshop. The instructors of the experimental sections provided activities and assignments to support the course work. Often, they were designed to be enriching with a computing component to visualize calculus concepts. While there was no requirement for technology to be used in calculus, the experimental sections regularly incorporated technology or computational problem-solving in the course work and the PLTL workshops. Students were provided with iPads or laptops and worked collaboratively in small groups to complete the assignments.

The modified PLTL model was used in the college algebra with corequisite classes. This model was adopted because of the large number of sections and students needing the support, and the limited availability of classroom spaces. The proposed role of the peer leaders was to assist instructors in problem-solving activities. Depending on the experience and the practice of the instructors, the level of engagement in problem-solving and PLTL varies greatly. Some instructors incorporated group work or technology and had the peer leaders actively engaged in student support. Other instructors lectured the entire time and had little or no utilization for the peer leaders. Given this observation, instructor training in active-learning strategies and in working with peer leaders is a crucial component in a successful PLTL model.

Results

- *The shift to the corequisite model.*

The shift to the corequisite model happened over several years. We started with the pilot sections, then adjusted the prerequisites. The enrollments for the three courses varied greatly from semester to semester depending on the adjustments. See Table 2.

- *Was MAT 1275CO considered successful?*

The success of the college algebra with corequisite support is best considered by its impact on student time to graduation, tuition saved, and course completion. Table 3 and Figure 1

Semester by Semester Enrollment and Pass Rate of MAT 1175, MAT 1275, and MAT1275CO			
	MAT 1175 Enrollment (Pass Rate)	MAT 1275 Enrollment (Pass Rate)	MAT 1275CO Enrollment (Pass Rate)
Fall 2015	834 (62%)	1454 (62%)	
Spring 2016	623 (58%)	1019 (64%)	
Fall 2016	853 (59%)	1274 (66%)	
Spring 2017	729 (56%)	900 (63%)	
Fall 2017	1618 (57%)	900 (66%)	158 (62%)
Spring 2018	886 (53%)	1081 (70%)	186 (65%)
Fall 2018	305 (50%)	1008 (63%)	1103 (57%)
Spring 2019	49 (41%)	481 (61%)	1125 (51%)
Fall 2019	32 (82%)	654 (62%)	2192 (45%)
Spring 2020		464 (52%)	1001 (57%)
Fall 2020		1267 (58%)	1259 (54%)
Spring 2021		419 (56%)	668 (56%)
Fall 2021		1487 (55%)	980 (47%)

Table 2: Semester by Semester Enrollment and Pass Rate of MAT 1175, MAT 1275, and MAT1275CO.

below show a comparison of the three courses MAT 1175, MAT 1275, and MAT 1275CO with cumulative grade distributions over 13 semesters from Fall 2015 to Fall 2021. The MAT 1275CO sections show lower pass rates than MAT 1175, but still advance students at the rate of 51.8% to precalculus, compared with the 2-semester MAT 1175/1275 sequence. This is game changing for the students because it opens options in their access to their major course work and degree program. While the benefits of the corequisite model can be quantified by reduced time to graduation and tuition, there are other potential educational benefits such as increased motivation, confidence, and interests in STEM that are not readily quantifiable.

	Total (N)	ABC Grades	ABCD Grades	F	W
MAT 1175	5,929	42.3%	57.3%	24.6%	18.1%
MAT 1275	12,408	49.1%	61.8%	19.3%	18.6%
MAT 1275CO	8,672	40.8%	51.8%	23.2%	24.3%

Table 3: Grade Distribution of MAT 1175, MAT 1275, and MAT 1275CO from Fall 2015 - Fall 2021.

- *Were students performing better in the experimental sections of Calculus I and II?*

The experimental sections of calculus I and II were carried out in semesters Fall 2016 – Spring 2019. In the experimental sections, the instructors designed exploratory calculus activities with a strong technology and computing component, providing students with

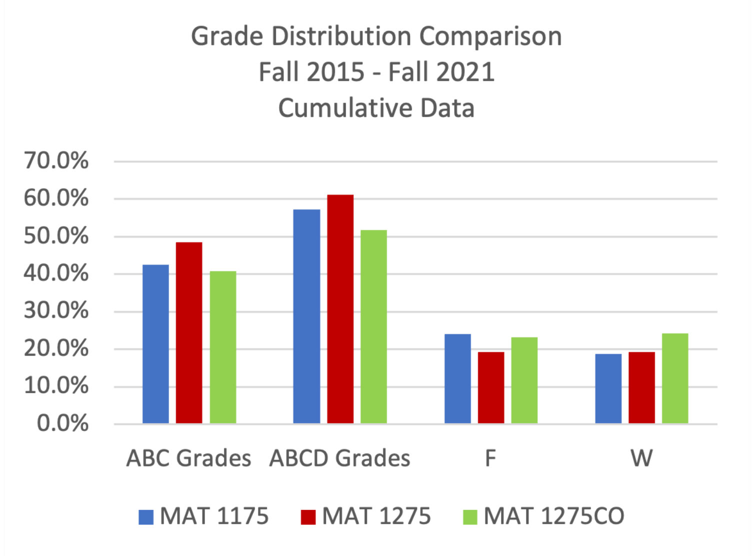
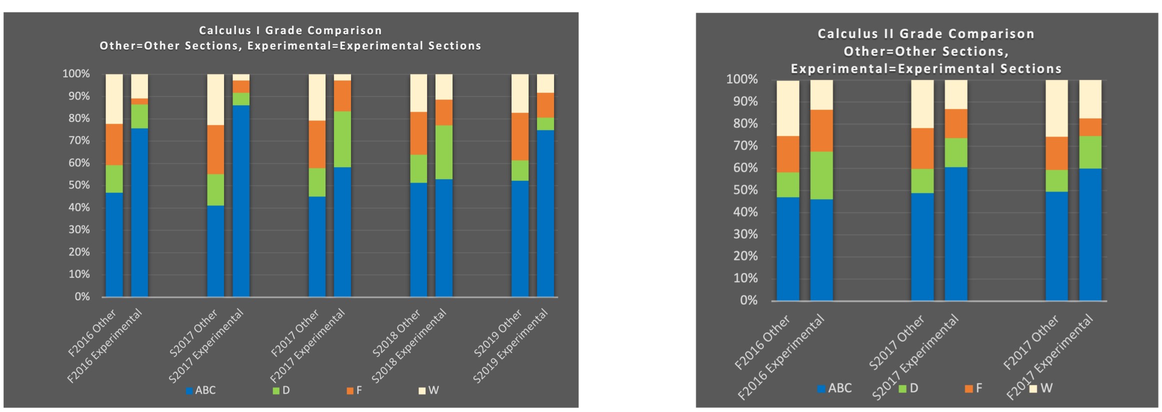


Figure 1: Grade Distribution Comparison Between MAT 1175, MAT 1275, and MAT 1275CO.

hands-on learning and problem-solving usually with the help of peer leaders. There was no special selection process, and no special designations for the experimental sections. They were listed in the system as Calculus I and II. Instructors were usually not assigned in the system until close to the start of the semester to reduce student bias against or tendency towards a particular section. A comparison of the pass rates shows that the experimental sections have higher pass rates and lower withdrawal rates than the other sections of Calculus I and Calculus II. Figure 2 shows the semester by semester comparison.



(a) Calculus I Grade Comparison

(b) Calculus II Grade Comparison

Figure 2: Calculus I and II Grade Comparisons.

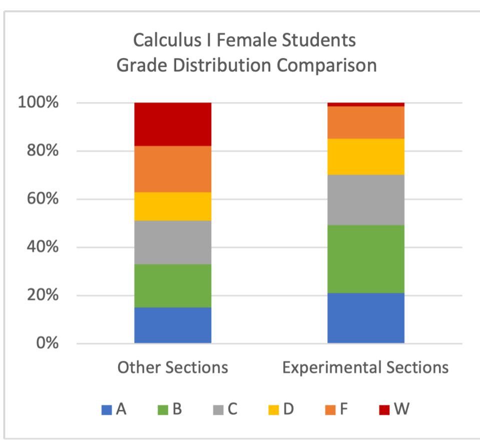
Furthermore, data revealed women were positively impacted in the experimental sections. Not only were the pass rates of women earning A, B, C, D grades higher in the experimental sections than those in the other sections, 85.1% versus 62.8% in Calculus I, and 75.8% versus 59.9% in Calculus II, the most telling result was the completion rates. Course completion means students stay until the end of the semester earning a letter grade A, B, C, D, or F in the course. The overall completion rate for women in the Calculus I experimental sections was 98.5% (66 out of 67), and 90.9% (30 out of 33) in the Calculus II experimental sections, they are significantly higher than the completion rates of 82.2% and 71% respectively for women in the other Calculus I and II sections. See Table 4 and Table 5. The higher completion rates suggest women responded positively to the curriculum, the pedagogy, and the support received in the experimental sections and felt encouraged to stay and complete the course. See Figure 3.

The Grade Distribution of Women in Calculus I (Fall 2016, Spring 2017, Fall 2017, Spring 2018, Spring 2019)								
	Total Enrollment	Total No. of Women	A	B	C	D	F	W
Other Sections	3033	718	14.9%	18.1%	18.1%	11.7%	19.4%	17.8%
Experimental Sections	215	67	20.9%	28.4%	20.9%	14.9%	13.4%	1.5%

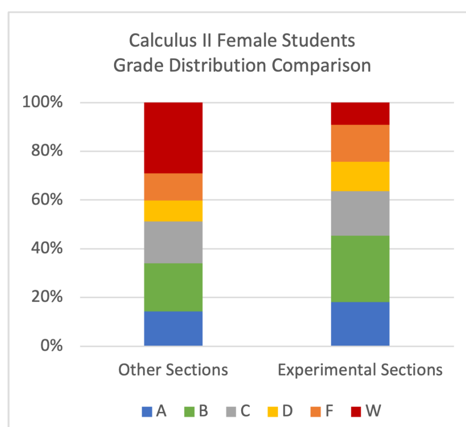
Table 4: The Grade Distribution of Women in Calculus I.

The Grade Distribution of Women in Calculus II (Fall 2016, Spring 2017, Fall 2017)								
	Total Enrollment	Total No. of Women	A	B	C	D	F	W
Other Sections	1229	217	14.3%	19.8%	17.1%	8.8%	11.1%	29.0%
Experimental Sections	148	33	18.2%	27.3%	18.2%	12.1%	15.2%	9.1%

Table 5: The Grade Distribution of Women in Calculus II.



(a) Calculus I Grade Comparison for Women.



(b) Calculus II Grade Comparison for Women

Figure 3: Calculus I and II Grade Comparisons for Women.

Conclusion

The mathematics curriculum changes have transformed the STEM education at a college of technology impacting thousands of students many of whom were identified as minority, first generation students with financial needs and aspired to become scientists, technologists, engineers, and mathematicians.

The reform in the first-year algebra courses using the corequisite model led to faster completion of the prerequisite and start of major course work by the students. This has an impact on reducing time to graduation which may impact student motivation, retention, and eventual graduation.

The redesign of Calculus to incorporate problem-solving and computing curriculum led to better understanding and better performance by the students. The peer leaders facilitated the hands-on collaborative learning in the PLTL workshops. Data shows women were retained in the course (completing the course) at a significantly higher rate in the experimental sections than in the regular sections.

The project incorporated strategies that promoted best practices in the education and the engagement of women and underrepresented minority students in STEM. Integrating real-world computational problem-solving and coding provided students with important 21st century skills for STEM success. Recruiting and training women and underrepresented minorities to be peer leaders and role models further empower and enhance the engagement of more women and underrepresented minorities in STEM to form a community of support.

The project lasted four years from Fall 2015 to Fall 2019. In Spring 2020, no one could foresee, we were faced with COVID-19 and the sudden transition to remote learning. The curriculum reform, the adoption of online open resources, the integration of technology in problem-solving, and the faculty training workshops fell into place and prepared us to continue our mission as educators during the pandemic. It is extraordinary to see our work has helped lead a transformation in STEM education at our institution.

Dedication

We dedicate this paper to our team member, colleague, and friend, Professor Janet Liou-Mark, whose kindness, empathy, and devotion to helping students and colleagues will always be remembered.

Acknowledgment

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References

[1] Cissy J. Ballen, Carl Wieman, Shima Salehi, Jeremy B. Searle, and Kelly R. Zamudio. Enhancing Diversity in Undergraduate Science: Self-Efficacy Drives Performance Gains with Active Learning. *CBE life sciences education*, 16(4):ar56, 2017.

[2] Nadia Benakli, Boyan Kostadinov, Ashwin Satyanarayana, and Satyanand Singh. Introducing computational thinking through hands-on projects using R with applications to calculus, probability and data analysis. *International Journal of Mathematical Education in Science and Technology*, 48(3):393–427, April 2017.

[3] Chantal Buteau, Eric Muller, and Bill Ralph. Integration of Programming in the Undergraduate Mathematics Program at Brock University. In *Online Proceedings of the Math + Coding Symposium*, London, Canada, 2015.

[4] Kelsey Hood Cattaneo. Telling Active Learning Pedagogies Apart: From theory to practice. *Journal of New Approaches in Educational Research*, 6(2):144–152, July 2017.

[5] K. Cline, J. Fasteen, A. Francis, E. Sullivan, and T. Wendt. Integrating Programming Across the Undergraduate Mathematics Curriculum. *PRIMUS*, 30(7):735–749, 2020.

[6] Leslie B. Jones and Britney J. Hopkins. Teaching a Course in Mathematical Programming. *PRIMUS*, 30(5):571–600, May 2020.

[7] Daniel Kaplan. MOSAIC Calculus. http://www.mosaic-web.org/MOSAIC-Calculus/index.html. Accessed: 04/09/2022.

[8] S Khan. Mentorship of College Women in Science. *The Journal on Excellence in College Teaching*, 19(1):81–101, 2008.

[9] Boyan Kostadinov, Johann Thiel, and Satyanand Singh. Creating Dynamic Documents with R and Python as a Computational and Visualization Tool for Teaching Differential Equations. *PRIMUS*, 29(6):584–605, July 2019.

[10] Jean Lave and Etienne Wenger. *Situated Learning: Legitimate Peripheral Participation*. Learning in Doing: Social, Cognitive and Computational Perspectives. Cambridge University Press, 1991.

[11] J. Liou-Mark, S. Han, M. Ahmed, F. Anglade, and J. A. Young. Strengthening Foundational Courses through the Implementation of Peer-Led Workshops. In *Proceedings of the 2013 Peer-Led Team Learning International Society*, University of Houston-downtown, Houston, Texas, 2013.

[12] Janet Liou-Mark, A. E. Dreyfess, Sandie Han, Laura Yuen-Lau, and Karmen Yu. Aim for success: Peer-led team learning supports first-year transition to college-level mathematics. *Journal of Learning Development in Higher Education*, December 2015.

[13] Janet Liou-Mark, Urmi Ghosh-Dastidar, Diana Samaroo, and Melanie Villatoro. The peer-led team learning leadership program for first year minority science, technology, engineering, and mathematics students. *Journal of Peer Learning*, 11(1):65–75, 2018.

[14] A. W. Logue, Daniel Douglas, and Mari Watanabe-Rose. Corequisite Mathematics Remediation: Results Over Time and in Different Contexts. *Educational Evaluation and Policy Analysis*, 41(3):294–315, September 2019.

[15] A. W. Logue, Mari Watanabe-Rose, and Daniel Douglas. Should Students Assessed as Needing Remedial Mathematics Take College-Level Quantitative Courses Instead? A Randomized Controlled Trial. *Educational Evaluation and Policy Analysis*, 38(3):578–598, September 2016.

[16] Alexandra W. Logue, Mari Watanabe-Rose, and Douglas Daniel. Reforming Remediation: College students mainstreamed into statistics are more likely to succeed. *Education Next*, 17(2):78–84, 2017.

[17] NYCCT Institution Fact Sheet 2020-2022. `https://citytech.cuny.edu/about-us/docs/facts.pdf`. Accessed: 04/09/2022.

[18] NYCCT Institutional Data Dashboard, Institutional Research and Effectiveness (AIRE). `http://air.citytech.cuny.edu/data-dashboard/`. Accessed: 04/09/2022.

[19] NYCCT Institutional Description. `https://facultycommons.citytech.cuny.edu/sponsored-programs/institutional-description/`. Accessed: 04/09/2022.

[20] Christopher J. Sangwin and Claire O'Toole. Computer programming in the UK undergraduate mathematics curriculum. *International Journal of Mathematical Education in Science and Technology*, 48(8):1133–1152, November 2017.

[21] Carolee Stewart-Gardiner. Using peer led team learning to assist in retention in computer science classes. *Journal of Computing Sciences in Colleges*, 25(3):164–171, 2010.

[22] Sarah Beth Wilson and Pratibha Varma-Nelson. Small Groups, Significant Impact: A Review of Peer-Led Team Learning Research with Implications for STEM Education Researchers and Faculty. *Journal of Chemical Education*, 93(10):1686–1702, 2016.

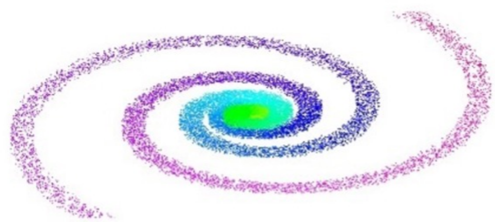
Appendix

We showcase some of the activities that we developed.

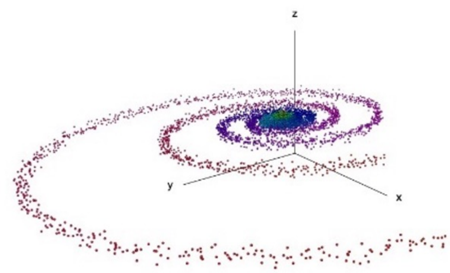
Mathematics as Creative Art Using R

We developed a number of scaffolded visualization activities for our **Code in R** 3-day summer program. Neither calculus nor coding experience was a prerequisite for the workshops, nevertheless, students learned the basics of coding in R in a couple of days, and showed their creativity in visualizing complex functions using R. For students with little or no prior coding experience, we chose R for this activity due to its easy setup and not very steep learning curve, as well as the ability to create beautiful graphics and publication quality reports using R Markdown in RStudio.

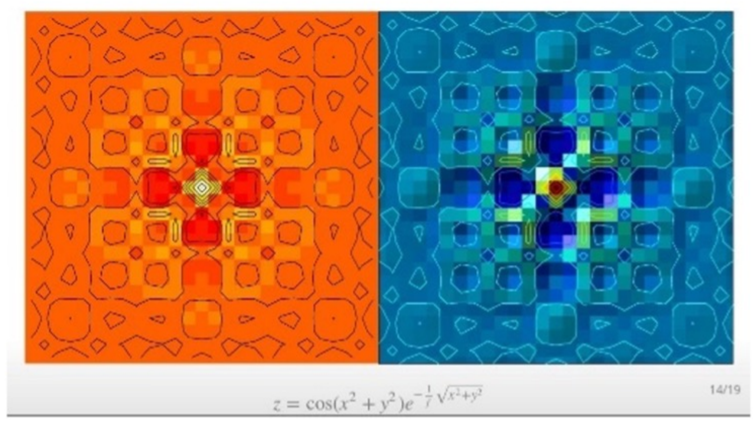
Visualizing Spiral Galaxies in 2D

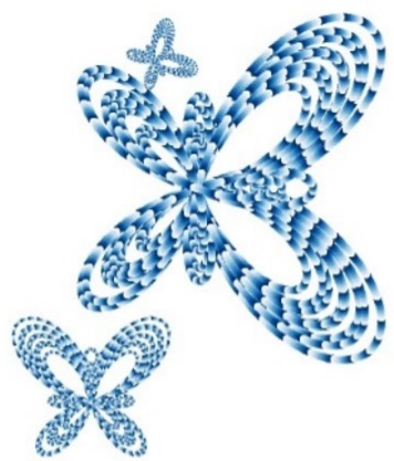


Interactive 3D Plot of a Spiral Galaxy



'Native American' Art as Contour Projections





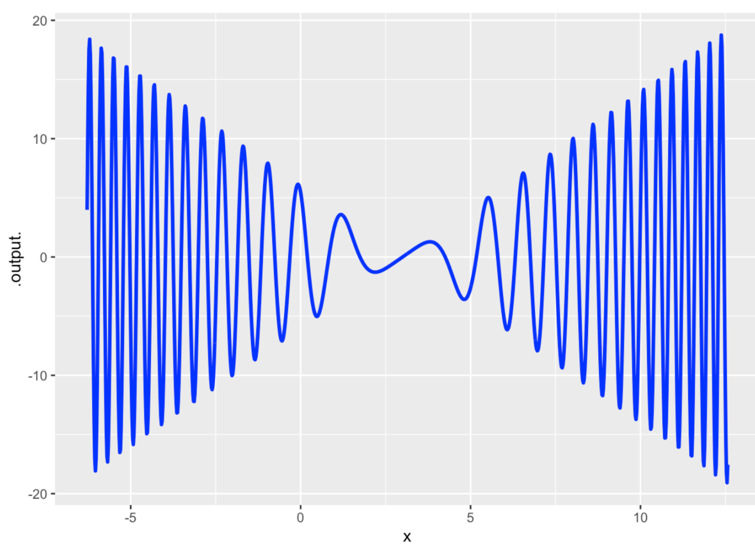
$$r = e^{\cos(\theta)} - 2\cos(4\theta) + \sin^5(\theta/12)$$

$$f(x) = e^{it} + \frac{1}{2}e^{6it} + \frac{i}{3}e^{-14it},\ t \in (0, 2\pi)$$

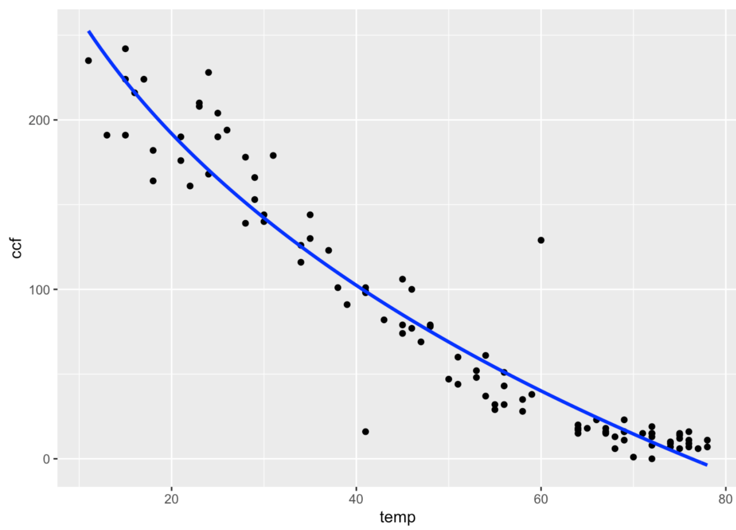
Calculus Lab Using R

We used R together with the **mosaic** and **mosaicCalc** R packages developed by Project Mosaic for an experimental section of a calculus lab, based on the *MOSAIC Calculus* book, being developed by Daniel Kaplan, and supported by Project Mosaic [7].

```
h <- D(sin((x - 3)^2) ~ x) # derivative function
slice_plot(h(x) ~ x, domain(x=range(-2*pi,4*pi)), col="blue", size=1.1, npts = 1000)
```



```r
g <- fitModel(ccf ~ A + B*temp + C*sqrt(temp), data = mydata)
gf_point(ccf ~ temp, data = mydata) %>%
  slice_plot(g(temp) ~ temp, col="blue", size=1.1)
```



Modeling Logistic Growth Using Desmos

In this activity students derived the logistic growth model using the method of separation of variables. They fitted a logistic growth model curve to the given data using Desmos.

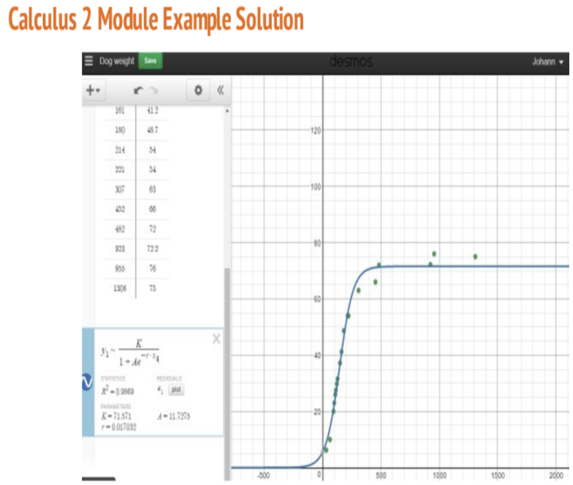
Logistic Growth Model

Objectives: Construct a logistic growth model for the weight of an animal.

1. Let $w := w(t)$ be the weight of an animal over time. A weight growth model, known as the logistic growth model, is given by the differential equation
$$\frac{dw}{dt} = r \cdot w\left(1 - \frac{w}{K}\right)$$
where r and K are constants. The goal is to find a general solution, that is, function w that satisfies the differential equation given above.

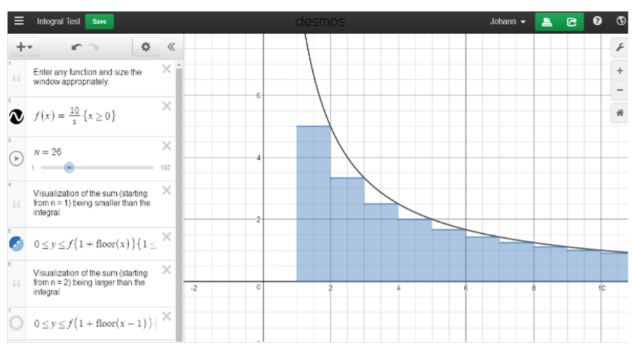
2. The method of separation of variables (a topic in MAT2680 - Differential Equations) says that we can find a general solution by computing both sides of the following equation:
$$\int \frac{1}{w\,(1 - w/K)}\, dw = \int r\, dt.$$

3. Show that
$$\int \frac{1}{w\,(1 - w/K)}\, dw = \ln\left|\frac{w}{K - w}\right| + C.$$
Hint: Use the fact that $\frac{1}{w(1-w/K)} = \frac{K}{w(K-w)}$ and partial fraction decomposition.



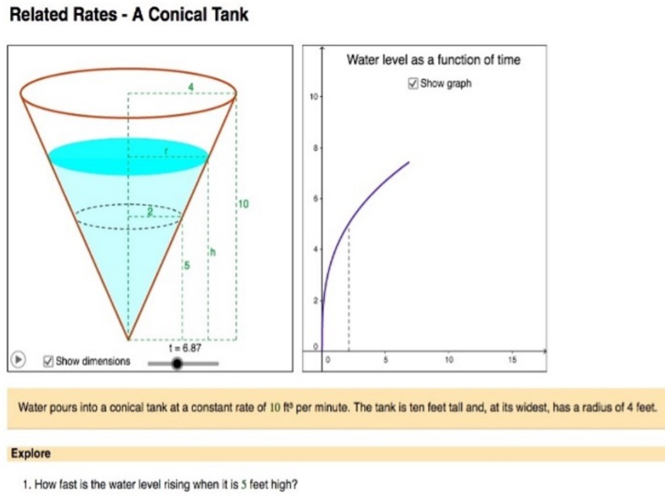
Visualization Tools Using Desmos

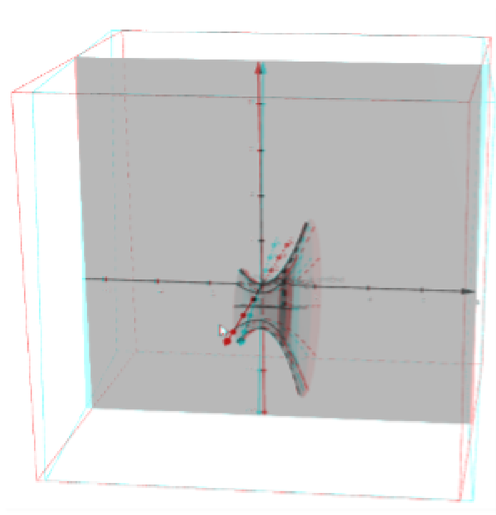
Desmos was used throughout the course to visualize various concepts. In this instance it is shown how an infinite series can be compared to an improper integral to try to determine whether or not the series converges.



3D Visualization Tools Using GeoGebra

In these examples we used GeoGebra's anaglyph generating tool to help students visualize a 3D conical tank and the solids of revolution.





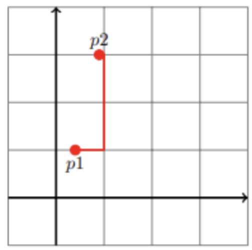
Problem-Solving Using Python

In this activity, students first designed an algorithm to solve a driving distance problem. Their algorithm was then implemented using Python.

Driving Distance

Consider a perfectly square, infinite grid. Define the driving distance between two points on the grid as the shortest path between the points when traveling along the gridlines.

Example: For points `p1 = [0.4,1]` and `p2 = [0.9,3]`, the driving distance is `0.6+2+0.1=2.7`.



Goal:

1. Write a function `ddist(p1,p2)` that, given two points (each point is given as a list of the form `[x,y]`), returns the driving distance (as a float) between the points.

Test Cases:

1. points: `[2.4,1]` and `[5,7.3]`, driving distance: `8.9`
2. points: `[0,0.4]` and `[1,0.6]`, driving distance: `2`