Data Acquisition Systems in the Fluid Mechanics Laboratory: Draining of a Tank

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Abstract
This paper illustrates one way in which computer data acquisition systems are being utilized in the laboratory. The examples used are from the first course in Fluid Mechanics. We chose experiments related to the draining of a tank to demonstrate the use of data acquisition systems and their impact on assessing the validity of the quasi-steady analysis that is commonly used in the theoretical formulation of this problem. We compare the theoretical predictions based upon the quasi-steady theory with experimental results.

Introduction

Learning from experimentation is an important aspect of engineering education. The time spent in the laboratory can be used to strengthen the connection between the theoretical models presented in lectures and the actual behavior of substances, machines, devices , processes and systems. This connection is accomplished through observation and experimentation with the aid of measuring and recording devices. Naturally, as technology changes, the ways quantities are measured and recorded change as well. Practice in the teaching laboratory must adjust to these changes in order to enhance learning and to keep the subject matter being taught current. We focus on the draining of a tank.

We consider the efflux of a liquid of constant density rho through an orifice of cross sectional area A,, located at the bottom of a cylindrical tank ‘of cross section A,, We wish to compare the predictions of Bernoulli’s equation on how the tank drains to experimental results. First, we review the quasi-steady analysis that is commonly used in the theoretical formulation of this problem. Then, we discuss the experimental work done in our laboratory to evaluate the theory. Thirdly, we compare the theoretical predictions based upon quasi-steady theory with experimental results; Finally, we summarize our results and conclusions and assess the impact of the Data Acquisition system used in this project.

Theory: Bernoulli’s Equation Applied to the draining of a Cylindrical Tank

Draining of a tank appears either as an exercise or as an example somewhere in the text of most introductory textbooks of Fluid Mechanics. [4,2] These books and others are listed in the references shown below to illustrate both the popularity and the importance of this standard problem. Typically, one considers a cylindrical tank of inside cross sectional area At. The tank is oriented such that its axis of symmetry is vertical. The tank contains a fluid of constant mass density which can exit the tank through a circular orifice of cross sectional area A, that is axisymmetrically located at the bottom of the tank. If the initial height of the free surface of the fluid is Ho and the instantaneous height is h, one can write Bernoulli's equation between two points that are assumed to belong to the same streamline. Let point 1 be on the free surface and point 2 at the center of the effluent jet. The resulting Bernoulli's equation is unsteady and is given by

$$\int_1^2 \frac{\partial V}{\partial t} ds + \frac{V_2^2}{2} + \frac{p_2}{\rho} + gh_2 = \frac{V_1^2}{2} + \frac{p_1}{\rho} + gh_1 \qquad (1)$$

While the equation of continuity corresponding to it is

$$A_1 V_1 = A_2 V_2 \qquad (2)$$

Where V is the velocity of the fluid, t is the time, p is the static pressure, g is the acceleration of gravity and s measures distance along the streamline. The subscripts 1 and 2 refer to quantities at points 1 and 2, respectively. Here, A refers to the cross sectional areas at the indicated locations.

The differential equation governing the height of the free surface above the level of the exit is then obtained by combining equations (1) and (2); it is found to be.'

$$h(g + \frac{d^2h}{dt^2}) = \frac{1}{2}(\frac{dh}{dt})^2[(\frac{A_1}{A_{jet}})^2 - 1] \qquad (3)$$

It is conventional to use the quasi-steady model to simplify these equations and obtain approximate results. In this model, one uses the instantaneous values of parameters that are assumed to be varying slowly together with the governing equations that were assumed in a steady state. In order to achieve this in practice, one assumes that the cross sectional area of the tank is much larger than that of the exit orifice. Under these conditions , it is reasonable to conclude that the free surface would move very slowly and, therefore, that its acceleration would be very small compared to that of gravity. Thus, the second derivative of h with respect to time is dropped from the differential equation. Using this approximation in equation (3) gives

$$hg = \frac{1}{2}(\frac{dh}{dt})^2[(\frac{A_t}{A_{jer}})^2 - 1] \qquad (4)$$

Although the differential equation that results is still nonlinear, it is much easier to solve for the height as a function of time. Equation (4) can be rearranged now to give

$$\frac{dh}{dt} = \pm\sqrt{\frac{2hg}{(\frac{A_t}{A_{jet}})^2 - 1}} \qquad (5)$$

First, we choose the minus sign in equation (5) because the height decreases with time during draining; then, we separate variables and integrate the resulting functions to yield an expression for the height of the free surface of the fluid above the orifice. If integration is done from t=to, when h=ho, to some arbitrary time t, when h = h, one gets

$$1-\left(\frac{h}{h_o}\right)^{1/2}=\frac{t-t_o}{\sqrt{\frac{2h_o}{g_m}}} \qquad (6)$$

rearranging and then squaring both sides of the equation leads to an explicit expression of the height as a function of time, which is given in equation (7).

$$\frac{h}{h_0}=\left(\frac{t-t_0}{t_d}\right)^2-2\left(\frac{t-t_0}{t_d}\right)+1 \qquad (7)$$

where

$$t_d=\sqrt{\frac{2h_0}{g_m}}$$ = the theoretical time it takes the free surface to travel a distance ho. (8)

This time is also the theoretical time it takes to drain the tank completely, if the initial level of the free surface was ho above the exit orifice. Here, gm is the modified acceleration of gravity due to the presence of the exit orifice. It is given by

$$g_m=\frac{g}{\left[\left(\frac{A_t}{A_{jet}}\right)^2-1\right]} \qquad (9)$$

The left -hand side of equation (7) represents the ratio of the current height of fluid remaining in the tank to the original height. In the case of a tank of constant cross section, it is also equal to the fraction of the original volume of fluid that remains in the tank. The fraction that has been drained out of the tank is given by

$$1-\frac{h}{h_0}=-\left(\frac{t-t_0}{t_d}\right)^2+2\left(\frac{t-t_0}{t_d}\right) \qquad (10)$$

Treatment of this problem in texts of fluid mechanics does not explain the extent to which the theory used predicts what actually happens during draining of the tank. Of the fifteen introductory textbooks that were surveyed, only one estimates the area ratio for which the approximation can be expected to hold. [4] To fill this void in our course, we designed experiments to check the accuracy of the equations presented above and incorporated them into the set of experiments that are done during the'semester to illustrate principles of fluid flow.

The added experiments were intended to allow us to specifically examine the following six quantities: 1) the position of the free surface as a function of time and as a function of position;
2)the velocity of the free surface as a function of time and as a function of position;
3)the acceleration of the free surface as a function of time and as a function of position;
4)the volume flow rate of fluid leaving the tank as a function of time and as a function of the position of the free surface.
5)the total time to empty the tank as a function of the ratio of the tank cross section to the that of the exit orifice.
6)the variation of pressure within the tank as a function of time and as a function of position.

Experimental Setup and Data Acquisition

Description of the Tanks:

We designed five different cylindrical tanks to measure the time it takes fluid to drain; each had a constant cross section; four of these tanks had circular cross sections and the fifth one was square. Each tank was open to the atmosphere at one end and closed at the other. The closed end had orifices built into it such that any of them could be opened or shut depending upon the need. The ratios of the areas of the tank cross sections to those of the exit orifice were designed to vary from 169 to about 698. The heights and inside diameters varied as well. The length of the orifices in contact with the water varied from 0.375 to 0.75 inches. The orifices were threaded to allow for the introduction of transducers. In all tests reported here, the orifices led to the open atmosphere of the lab. Table 1 summarizes the data for each tank.

Description of the Data Acquisition

Our data acquisition system consists of WorkBenchMac/LE 4.0: Data Acquisition and Control Software by Strawberry Tree. Its features include a worksheet screen, display and control windows, calculation blocks, a library of math and logic functions, Icons and Icon connections. With appropriate hardware, it is capable of measuring data such as pressure, temperature, displacement, flow, and time and converting them into on-screen displays. The displays can be graphic or digital; also, ranges of analog inputs and sampling rates can be chosen to suit a particular application. Recorded data can be logged onto disk in text format that is compatible with databases, spreadsheets and other programs. By selecting different functions, which are represented by icons, and interconnecting them in specific ways, it was possible to configure WorkBench to do the tasks that were needed. These included data logging, monitoring, controlling, testing, analysis and simulation. Data could be input and output via the serial port or our Macintosh computer, thus it was possible to communicate with other laboratory instruments.

Table 1. Description of Draining Tanks

	Tank 1	Tank 2	Tank 3	Tank 4	Tank 5
cross section	circular	circular	square	circular	circular
Height (in)	33.00	23.75	33.00	23.00	10.80
Inside Diameter (in)	11.60	8.62	10in. X 10in.	5.75	5.69
Orifice Diameter (in)	0.425	0.435	0.695	0.425	0.435
Orifice Length (in)	0.75	0.75	0.375	0.75	0.75
Area Ratio (At/Ao)	698.47	394.3 1	269.38	172.73	169
Draining Time (Sec.)	284.38	133.11	109.68	59.62	39.97
Average Discrepancy with theory (Eq. 7)	10%	7%	30%	7%	2%

Experimental Setup

Each tank was placed vertically such that the open end faced up. This way, it was used as a container into which different amounts of fluid can be added. Also, pressure transducers were inserted in one or more of its orifices a the lower end. The transducers made it possible to measure the pressure at one point or at several points at the bottom of the tank simultaneously. Thus, when the fluid inside the tank was at rest and maintained at a fixed height, one was able to measure hydrostatic pressures. It was also possible to drain the tank continuously and record the local height of the free surface as well as the pressure at the bottom of the tank corresponding to the current location of the falling free surface.

The transducers were calibrated using hydrostatic tests. For this, Tank 1 was used to gather data; it was filled with water to a height of 32 inches. During calibration experiments, fluid was added in two-inch increments until the tank was full. After data collection, calibration followed a two-step process. The collected data were used to determine whether or not the voltages registered were directly proportional to the height of water in the tank at which they were measured; and secondly, to find if the relation between the recorded voltages and the hydrostatic pressure was linear as well. Both relationships were found to be linear experimentally. Therefore, hydrostatic tests yielded calibration curves that were used throughout the rest of the project; the slopes of these curves made it possible to convert voltages from the transducers directly into pressures by entering the conversions obtained from calibration into the Data Acquisition System. To insure that conversion factors were reliable, calibration was done at three points. Accordingly, transducers were placed at three points: the center of the tank (r/R=0) , a point half-way between the center and the wall (r/R=0.5) and the last one near the wall (r/R= 7/8). Here, R denotes the inside radius of the tank. The conversion factors obtained from the three points were practically identical.

Collection of Data

The test themselves were run as follows: the tank being tested was completely filled with water. It was necessary to wait until the water came to rest because filling the tank caused the water in the tank to move. Sloshing and oscillations on the free surface as well as the movement of submerged particles and bubbles were used to determine whether or not the water had come to rest. After the water had come to rest, the orifice was suddenly unplugged by unscrewing the threaded bolt that was used as a plug away from the water. From this point until the tank was completely empty, output voltages from the transducers were recorded automatically into a Data Acquisition system in a continuous manner; they were simultaneously being converted into pressures in a calculation block using calibration data and recorded; and the height of the free surface of water was entered manually into the data file next to the voltages at one-inch increments by clicking on the computer mouse. Voltages, heights and pressures were now available as functions of time. By utilizing standard software, these data could be processed immediately at the end of the collection process, or later, making it possible to compare theory and experiment.

Comparisons: Quasi-steady theory and experiment.

Huge tables of data were collected using our Data Acquisition system. They were used to obtain the following relationships and compare the results to what is predicted by quasi-steady theory.1) the position of the free surface as a function of time was derived in equation (7). Theory and experimental results are compared in Figure 1 for all five tanks described in Table 1.

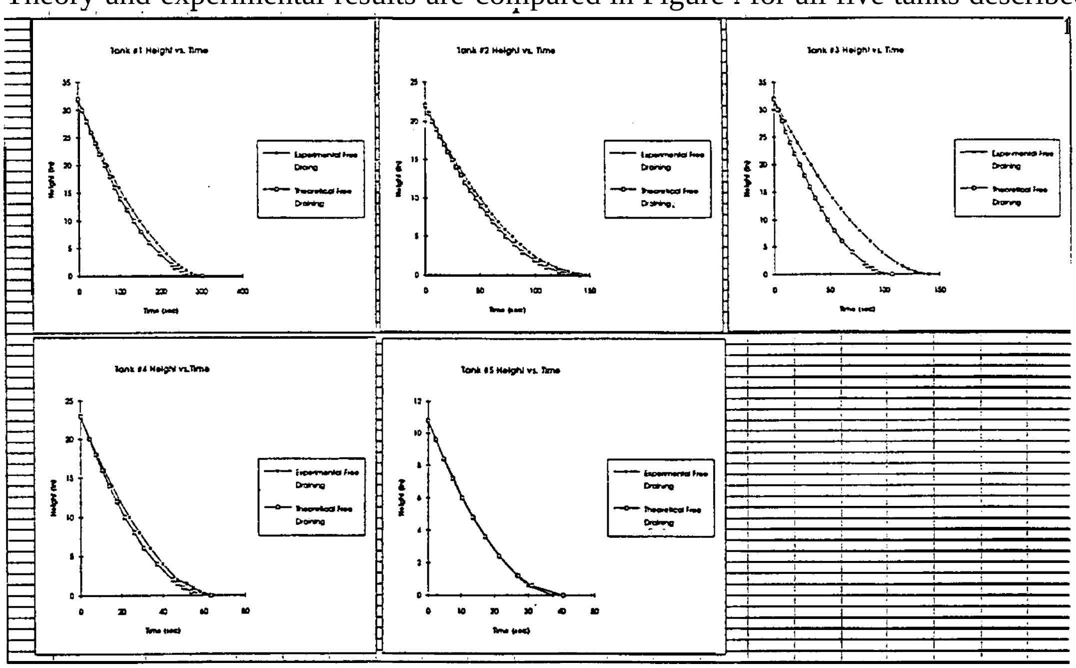

Figure 1. Water Height vs. Time--Plots of Dimensional Quantities.

It can be seen that theory predicts the behavior very well. However, in almost all cases, theory underestimates the time that is necessary to drain a given amount of fluid from the tank. This discrepancy varies with the size, the area ratio and the geometry of the cross section of the tank. Our data show that the discrepancies are larger with the square tank than with the circular tank (Table 1). It is to be noted, however, that when the height is nondimensionalized using the original height and the draining time is nondimensionalized using the total draining time corresponding to each case and the data plotted anew, it is clearly seen then that quasi-steady theory does indeed predict the draining behavior even better in almost all cases (Fig. 2).

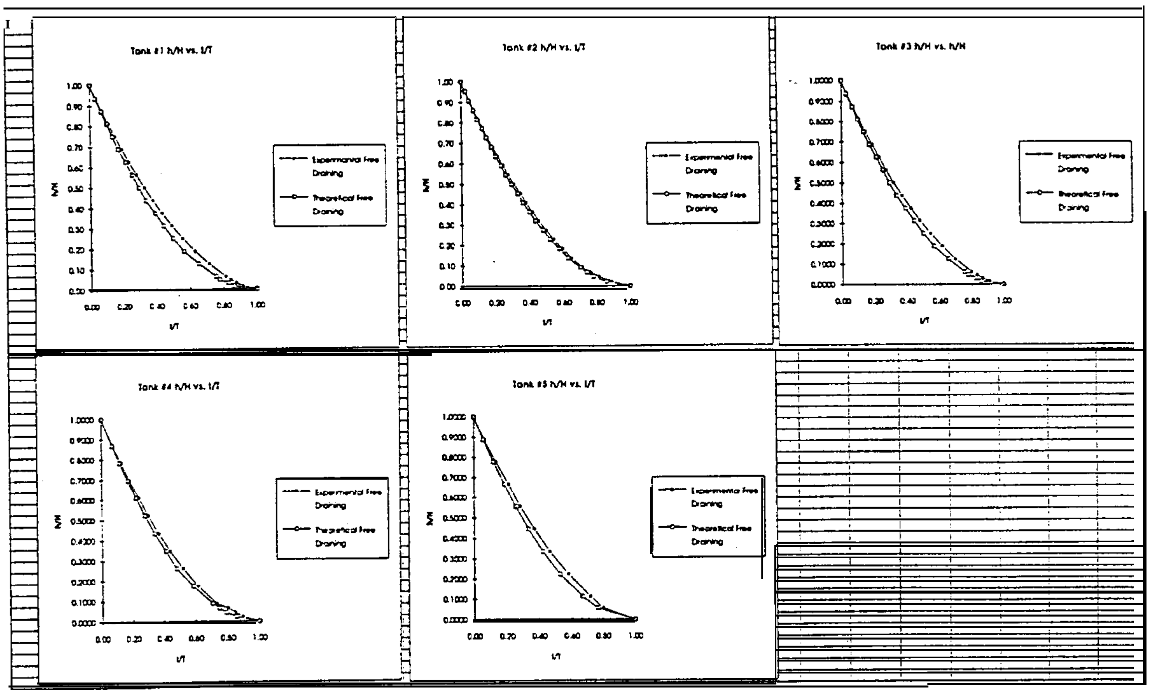

Figure 2. Water Height vs. Time--Plots of nondimensional quantities.

Also, the discrepancies between theory and experiment are smaller. Now, the effects of size of the tank, area ratios, and shape of cross sections become smaller still.

2)The instantaneous velocity of the free surface as a function of position is given by equation (5); in this case, theory and experiment are compared in Figure 3.

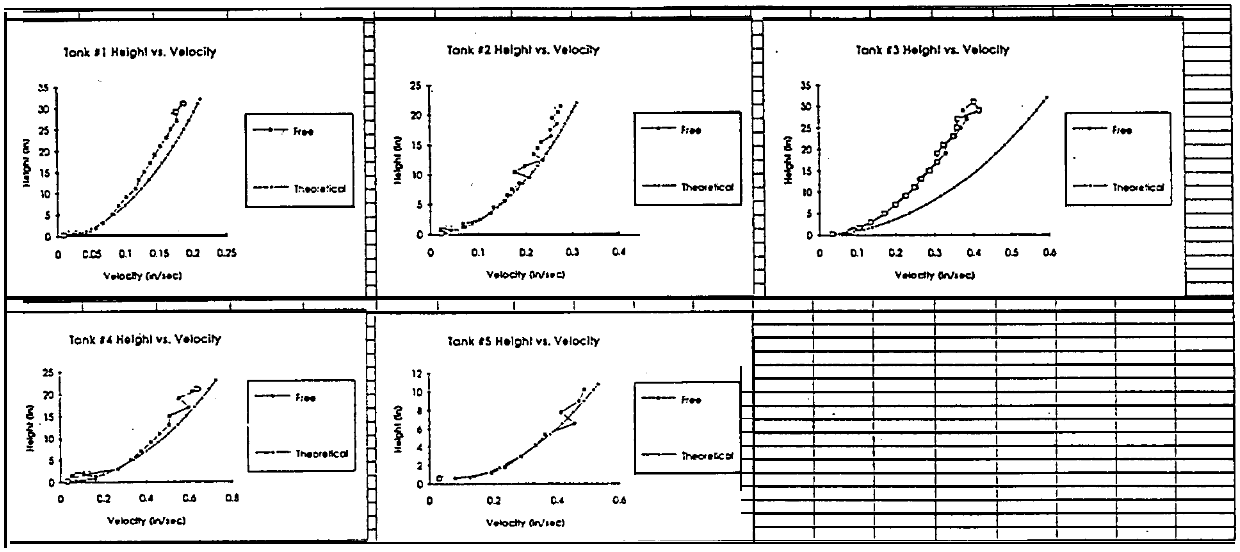

Figure 3. Water Height vs. Velocity of the free surface of water.

The experimental velocity is actually the average velocity between two consecutive positions of the free surface. The graphs indicate that theory predicts velocities that are higher than those obtained experimentally. Here again, the largest discrepancies were observed in the square tank.

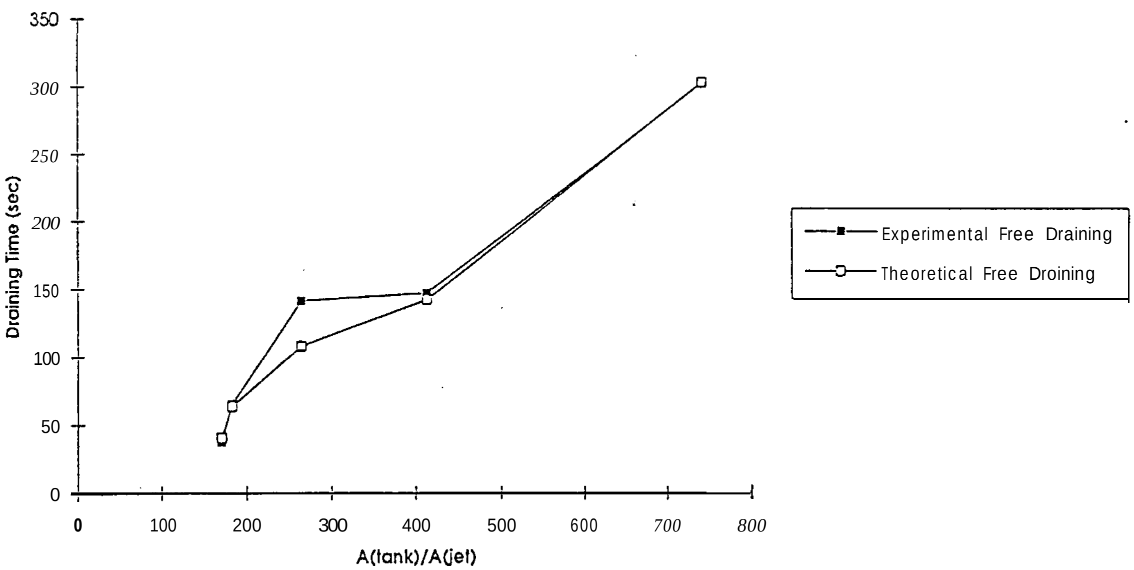

Figure 4. Total Draining Time vs. Area Ratio.

3)The acceleration of the free surface as a function of position was computed from the data on velocities. Quasi-steady theory assumes that the acceleration of thee free surface is constant. However, it was observed that the acceleration computed from experimental data did not remain constant as the draining process took place. Instead, they fluctuated with the change in heights. In general, however, experiments yielded magnitudes that were between 30%and 50% higher than theory, although in some cases, they fluctuated considerably about the value indicated by the theory. There appeared to be three different regions for the variation of the acceleration. One was at the beginning of the draining process; the second one covered the middle of the tank and the last one was near the exit. The fraction of the height of the tank covered by each region varied with the tank. Roughly, they corresponded to the top quarter of the tank, the middle half of the tank and the bottom quarter, respectively. The behavior of the acceleration appeared to be different in each of these regions.

4) Draining Time as a function of the area ratio is given in Figure 4.

It can be seen that agreement with theory is very good except for the case of the square tank, where theoretical draining is about 28% shorter than was obtained experimentally.

5) Pressures that were measured in the fluid during draining followed hydrostatic patterns although their magnitudes varied somewhat from them.

Conclusions

This paper illustrated one way in which computer data acquisition systems and supporting software are being utilized in the fluid mechanics laboratory. We designed five different cylindrical tanks to measure the time it takes fluid to drain. Positions of the free surface and pressures at the bottom of the tank were measured with respect to time during draining. These data were also used to compute the velocity and acceleration of the free surface during draining. The results were compared with what quasi-steady analysis of this problem predicts. It was found that the quasi-steady analysis yields results that are accurate, generally, although the sizes of errors varied with the specific quantities being examined as well as with how the quantities were arranged. Nondimensional ratios showed smaller discrepancies with theory than dimensional quantities. These experiments were made practical with the availability of data acquisition systems and software to process huge amounts of data in a short time. Enough details were presented herein to allow faculty at other institutions to reproduce these experiments in their laboratories.

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