# 2006-21: DEDUCTIVE PROBLEM SOLVING STRATEGY APPLIED TO THE OPTIMIZATION OF WALL INSULATION

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# Deductive Problem Solving Strategy Applied to the Optimization of Wall Insulation

## Introduction

The current approach to problem solving in undergraduate engineering education allows the students to use a combination of inductive and deductive methods. Problems are well defined with all the required input information provided in the problem statement. Students are able to hone their analytical skills well with these exercises. Unfortunately, this does not help them with problem formulation skills that they will need when they enter the work force. There when they are given a problem they will be fortunate if they have a well-defined objective.

This paper provides instructors and students with a method that will help them better formulate problems. Problems are presented without specifying the inputs needed for the solution. The student then applies the deductive approach to lead them to identify for themselves which inputs are required to solve the problem. Although the problems are open-ended the method provides the structure the student needs to break a large problem into manageable pieces.

The deductive approach begins by determining how the objective of the problem can be quantified. In design problems this is often an optimization. Most optimizations require a cost analysis to compare the competing forces on an equivalent basis. After that the laws of nature (e.g. conservation of energy, conservation of mass, Fourier's Law of Conduction, etc.) are used to connect the desired result to variables that can be measured directly or specified.

The deductive approach was used extensively in a graduate level course on heat transfer in the summer of 2005 and is being used in the second semester of thermodynamics during the spring semester of 2006. The feedback from students has been positive. One graduate student stated in his course evaluation that the deductive approach is a wonderful tool for engineers. Out of a class of 27 thermodynamic students 21 thought it was beneficial to track the equations and unknowns, 19 found the systematic nature of the approach beneficial, 10 stated it was easier to follow the work and seven found it beneficial to solving open-ended problems when compared to standard textbook problems.

In this paper, an example is provided that shows how the method is applied to determining the optimal thickness of insulation in a building. For wall insulation the major competing forces in the optimization are initial costs of the insulation and energy costs. Since there is a high degree of uncertainty in many of the input variables a non-dimensional sensitivity analysis was useful to prioritize the data collection process. The example problem is relevant for undergraduate and graduate courses in heat transfer and optimization as well as air conditioning and refrigeration design courses. However, any textbook problem can be adapted to accommodate the deductive strategy by removing the specified inputs for the problem.

Literature Review

The McMaster Problem Solving Program (Woods, et. al., 4/97)[1] found the most effective way to teach problem solving skills to students is to use a workshop approach. The key components to the workshop are an introduction, pre-test, application, and immediate feedback. Students are then asked to reflect on what they learned in a journal. In the McMaster program they had four different workshops. The first two helped the students develop the analytical skills they needed to solve well-defined, typical homework problems. The third workshop concentrated on team problem solving. The fourth and final workshop dealt with solving open-ended problems. The deductive problem solving strategy presented here would be most applicable to this fourth workshop in the solution of open-ended problems.

Suliman (2004)[2] introduces a new format to teaching engineering based on problems as opposed to a lecture format. Small groups of students are given a problem each week. A faculty tutor is assigned to guide the students to identify the key issues related to the problem. Rather than have the faculty provide the facts to the students through a lecture the students need to learn on their own the content at the appropriate breadth and depth. The deductive strategy will provide the students with the structure needed to succeed with the open-ended nature of this problem-based format.

Problem solving is a key to a holistic approach presented by Jordan et. al. (2000)[3]. They present the importance of open-ended problems to assist future teachers in understanding the connection between the principles of science and math and the physical world. The deductive problem solving strategy is a valuable tool that can be used to systematically solve a wide variety of science and math problems.

Hill (1998)[4] discusses how creativity can be developed through open-ended problem solving. Unfortunately in these problems both order and disorder coexist. The deductive strategy presented here can provide a framework to channel the disorder associated with open-ended problems.

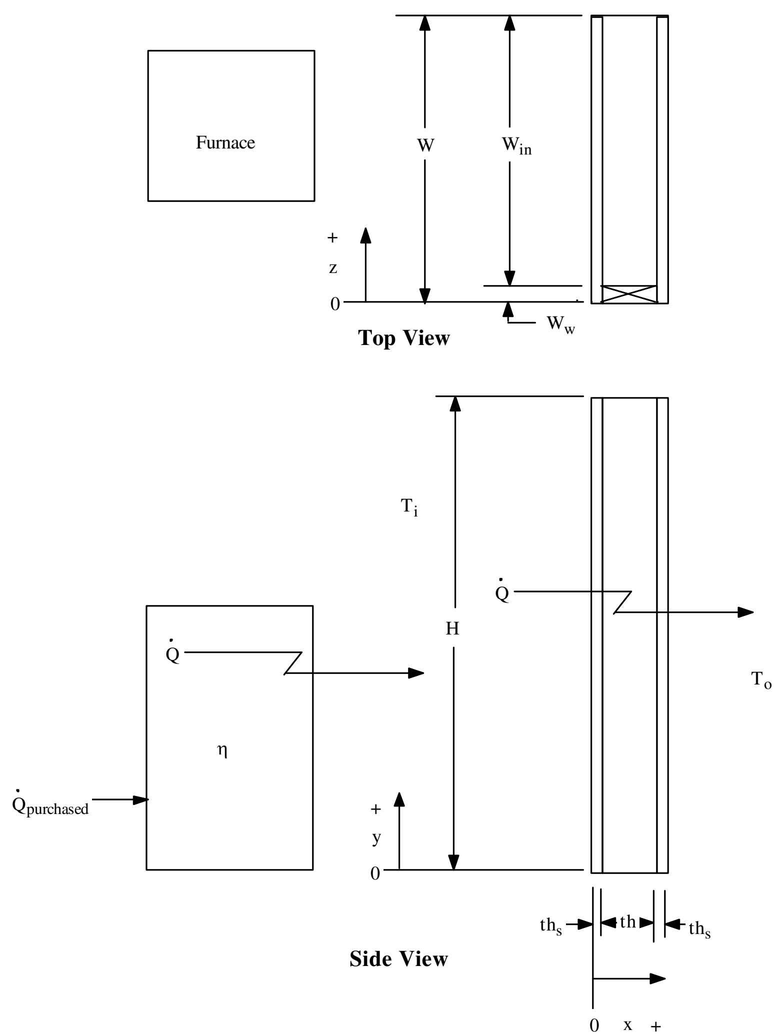
Systematic Innovative Thinking (SIT) is a method that provides a balance between order and disorder that is essential in fostering creativity. Barak (2002)[5] provides a brief overview of this method and how it was applied to produce a valuable product innovation. Current textbook problems are too structured and do not provide the students with the level of disorder they will face in the work place. The deductive problem solving strategy moves the student closer to disorder in the problem solving process when compared to typical textbook problems. This brings the student closer to a creative balance between order and disorder.

Procedure

Even though the deductive problem solving strategy is presented in a linear manner in this paper it is inherently an iterative process. For example when this problem was first presented to the class a student asked if the cost of the furnace should be included. Let us think about what happens when the thickness of insulation increases, the heat loss drops. This, in turn, reduces the size, and correspondingly the cost, of the furnace required to keep the occupants comfortable. Therefore the cost of the furnace should be included in the analysis.

The first step in applying the deductive strategy is to determine the primary objective of the problem. This needs to be clearly defined and quantifiable. For our example the primary objective is to determine the optimum insulation thickness for a wall in a building that only required heat. For an optimum to occur there needs to be two forces that compete with each other. To assist in the formulation of the objective function for the problem a system schematic, as shown in Figure 1, is an essential tool. The system schematic is an item that will likely evolve with the problem. However the more thorough and complete the schematic is at the onset the sooner a solution will be obtained. All key variables need to be identified on the schematic.

Figure 1. System schematic of wall section and furnace.




In order for the system schematic to be effective the key features of the system need to be identified. This can be done by asking some basic questions. What is the function of wall insulation? How does the insulation influence the initial cost of the system? How does it influence the operating cost of the system? The function of insulation is to resist the transfer of heat from a heated space to the outdoors. As the thickness of the insulation increases its cost also increases. Additionally a wall is needed to hold the insulation in place so as the thickness of the insulation increases so does the thickness and cost of the wall. The heat which passes through the wall needs to be replaced. This is accomplished with a furnace. As the insulation increases the heat transfer rate decreases therefore the size of the furnace can be reduced along with its initial cost. Finally, as the heat transfer rate drops the amount of energy needed throughout the year decreases along with its associated costs.

Therefore the two opposing forces are the costs which increase with increasing insulation thickness [the initial cost of the insulation ($IC_{in}$) and wall ($IC_w$)] versus the costs which decrease with increasing insulation thickness [the initial cost of the furnace ($IC_f$) and operating cost ($OC$) for maintaining comfortable conditions for the occupants of the building]. These costs are combined in the present worth ($PW$) of the system which is the objective function for this problem.

$$PW=IC_{in} + IC_w + IC_f + OC \qquad (1)$$

Once the objective function has been defined each unknown of this function is systematically evaluated. The relationship between each of these unknowns and the independent variable has already been qualitatively evaluated in the formulation of the system schematic and objective function. To quantify these relationships it is necessary to ask how to go from each of the unknowns in the objective function to a measurable quantity. Moving from left to right in Equation 1 the first term to be evaluated is the initial cost of the insulation. To be systematic it is highly recommended to follow each unknown until all the variables needed to determine the unknown can be measured or specified. Equation 2 shows how the dimensions and unit cost of insulation are used to calculate the initial cost of insulation.

$$IC_{in}=H \times W_{in} \times th \times N_s \times Ciu \qquad (2)$$

where,
$H$ is the height of the wall [m]
$W_{in}$ is the width of the insulation [m]
$th$ is the thickness of the insulation [m]
$N_s$ is the number of sections of wall in the building [#]
$Ciu$ is the cost of insulation per unit volume [$\$/m^3$]

It is useful to keep track of the number of equations and unknowns. This will help in determining if any additional equations need to be applied to the problem. An unknown is a variable that can not be measured or specified directly. In Equation 1 there were five (5) unknowns with one equation. Therefore, at least four more equations are needed to solve the problem. Equation 2 provides one of these equations. All the new variables introduced in Equation 2 are measurable therefore the work with this unknown is complete. Table 1 demonstrates a helpful way to track this information.

Moving to the next variable from Equation 1 an equation is developed for the initial cost of the wall as shown in Equation 3.

$$IC_w = H \times W_w \times th \times N_s \times Cwu \qquad (3)$$

where,
$W_w$ is the width of the wood [m]
$Cwu$ is the cost of the wall per unit volume [$/m$^3$]

Up to this point anyone with a high school education can develop the equations. The next term in Equation 1 will require knowledge of heat transfer. The rate of heat loss under design conditions is needed to determine the size of the furnace. The initial cost of the furnace can then be calculated from Equation 4.

$$IC_f = \dot{Q}_{design} \times Cfu \qquad (4)$$

where,
$\dot{Q}_{design}$ is the rate of heat loss at design conditions [kW]
$Cfu$ is the unit cost for the furnace [$/kW]

In Equation 4 the heat transfer rate can not be measured directly so it will be added to the list of unknowns. The unit cost of the furnace can be determined from supplier data. To determine the rate of heat transfer under design conditions a thermal resistance network is applied to the wall. One-dimensional, steady state conditions are assumed with heat flow in the direction perpendicular to the wall.

$$\dot{Q}_{design} = \frac{T_i - T_{o,\min}}{R_i + R_{s,i} + R_{eq} + R_{s,o} + R_o} \qquad (5)$$

where,
$T_i$ is the indoor temperature [°C]
$T_{o,min}$ is the minimum outdoor temperature under design conditions [°C]
$R_i$ is the thermal resistance of the air on the inside of the wall [°C/kW]
$R_{s,i}$ is the thermal resistance of the sheetrock on the inside of the wall [°C/kW]
$R_{eq}$ is the equivalent thermal resistance of the insulation and wood combination [°C/kW]
$R_{s,o}$ is the thermal resistance of the sheetrock on the outside of the wall [°C/kW]
$R_o$ is the thermal resistance of the air on the outside of the wall [°C/kW]

Table 1. Accounting of Equations and Unknowns

| Equation | Unknown(s) | Measurable or specified variables | Minimum remaining equations (Unk-Eq) |
|---|---|---|---|
| 1 | $PW, IC_{in}, IC_w, IC_f, OC$ | | 4 |
| 2 | | $H, W_{in}, th, N_s, Ciu$ | 3 |
| 3 | | $W_w, Cwu$ | 2 |
| 4 | $\dot{Q}_{design}$ | $Cfu$ | 2 |
| 5 | $R_i$, $R_{s,i}$, $R_{eq}$, $R_{s,o}$, $R_o$ | $T_{i,}$ $T_{o,min}$ | 6 |
| 6 | $A$ | $h_i$ | 6 |

Table 1. Accounting of Equations and Unknowns (continued)

| Equation | Unknown(s) | Measurable or specified variables | Minimum remaining equations (Unk-Eq) |
|---|---|---|---|
| 7 | | | 5 |
| 8 | | $th_s$, $k_s$ | 4 |
| 9 | $R_w$, $R_{in}$ | | 5 |
| 10 | | $k_w$ | 4 |
| 11 | | $k_{in}$ | 3 |
| 12 | | | 2 |
| 13 | | $h_o$ | 1 |
| 14 | $\dot{Q}_{avg}$, $PA$ | $Dur, Ceu, \eta$ | 2 |
| 15 | | $T_{o,avg}$ | 1 |
| 16 | | $i,n$ | 0 |

Equation 5 introduced five new unknowns with the thermal resistances. Therefore, at least four additional equations will be needed to solve the problem. In Table 1 this is added to the two equations that were previously needed. Systematically moving from left to right in Equation 5, Newton's law of cooling will provide one of the equations for the inside thermal resistance of the air as shown in Equation 6.

$$R_i = \frac{1}{h_i A} \quad (6)$$

where,

$h_i$ is the convective heat transfer coefficient on the inside surface of the wall [kW/m$^2$-°C]

$A$ is the area the wall normal to the direction of heat transfer [m$^2$]

For Equation 6 it is possible to specify the area but since the area is not directly measured it is better to count it as an unknown and write an additional equation as shown in Equation 7. The convective heat transfer coefficient was not counted as an unknown since the American Society of Heating, Refrigerating and Air Conditioning Engineers Handbook of Fundamentals[6] has a table which handles this geometry and operating conditions.

$$A = H \times (W_w + W_{in}) \times N_s \quad (7)$$

All the variables for first unknown in Equation 5 have been specified. Now the second unknown, $R_{s,i}$, can be evaluated. Fourier's Law of Conduction provides the necessary equation for relating variables which are able to be measured or specified to this unknown.

$$R_{s,i} = \frac{th_s}{k_s A} \quad (8)$$

where,

$th_s$ is the thickness of the sheetrock [m]

$k_s$ is the thermal conductivity of the sheetrock [kW/m-°C]

For the third unknown from Equation 5 there are two different materials, insulation and wood, through which the heat is being transferred in parallel. The electrical resistance analogy for two resistances in parallel provides an equation to account for this case as shown in Equation 9.

$$R_{eq} = \frac{R_w R_{in}}{R_w + R_{in}} \tag{9}$$

where,
$R_w$ is the thermal resistance of the wood studs [°C/kW]
$R_{in}$ is the thermal resistance of the insulation [°C/kW]

Since both of these resistances are conduction resistances Fourier's Law of Conduction applies. This produces Equations 10 and 11.

$$R_w = \frac{th}{k_w A} \tag{10}$$

where,
$k_w$ is the thermal conductivity of the wood [kW/m-°C]

$$R_{in} = \frac{th}{k_{in} A} \tag{11}$$

where,
$k_{in}$ is the thermal conductivity of the insulation [kW/m-°C]

The next unknown in Equation 5, $R_{s,o}$, can now be evaluated, because all the variables related to the equivalent resistance, $R_{eq}$, can be measured or specified. Just as with $R_{s,o}$ Fourier's Law of Conduction is applied to the outside layer of sheetrock.

$$R_{s,o} = \frac{th_s}{k_s A} \tag{12}$$

To complete Equation 5 the last unknown, $R_o$ is evaluated using Newton's Law of Cooling.

$$R_o = \frac{1}{h_o A} \tag{13}$$

where,
$h_o$ is the convective heat transfer coefficient on the outside surface of the wall [kW/m$^2$-°C]

Now the heat transfer rate in Equation 5 can be determined from measured or specified variables. This in turn allows for evaluation of the initial cost of the furnace. This leaves the last unknown in Equation 1 the operating cost. It can be calculated using the definition of efficiency and the time value of money. The time value of money is necessary since the operating costs are incurred at some future time when the currency has less value. Therefore, the operating costs need to be discounted to today's value. This discounting then provides homogenous dimensions, present dollars, for all the variables in Equation 1.

$$OC = \left( \frac{\dot{Q}_{avg} \times Dur}{\eta} \right) Ceu \times PA \tag{14}$$

where,

$\dot{Q}_{avg}$ is the average rate of heat loss over the heating season [kW]
*Dur* is the duration of the heating season [hrs/yr]
*Ceu* is the unit cost of fuel [$/kWh]
*PA* discount factor to convert an annual payment to present worth [$\$_{present}/\$_{annual}$]
$\eta$ is the efficiency of the heating system [$kW_{out}/kW_{in}$]

To determine the average rate of heat transfer over the heating season a thermal resistance network is applied again to the wall.

$$\dot{Q}_{avg} = \frac{T_i - T_{o,avg}}{R_i + R_{s,i} + R_{eq} + R_{s,o} + R_o} \tag{15}$$

where,

$T_{o,avg}$ is the average outdoor temperature during the heating season [°C]

From Engineering Economics the discount factor for an annual series is given in Equation 16.

$$PA = \frac{[1+i]^n - 1}{i \times [1+i]^n} \tag{16}$$

where,

*i* is the time value of money [$\$_{interest}/\$_{principal}$]
*n* is the number of annual payments over the life of the insulation [#]

The problem can now be solved since each unknown has an independent equation associated with it. All the remaining variables are able to be measured or specified. In summary, for the deductive problem solving strategy, start with the objective and systematically analyze each of the unknowns until all the independent equations have been identified and all the remaining variables can be measured or specified. An important aspect of the method is separating the unknowns from variables which can be specified or measured as displayed in Table 1. It takes experience and common sense to be able to separate these. Knowledge of instrumentation and sources of reference information is also helpful.

Results

Now that a solution is possible for every unknown in the problem it is vital to formulate the problem in a useful manner. The client is interested in what thickness of insulation will provide the minimum overall costs over the life of the system. They know the answer lies somewhere between zero and infinity. With the set of equations developed here the design engineer can provide a reasonable estimate of the optimum thickness. The first solution to determine is called the base case. This is the most probable case based on the data and assumptions at the time of the solution. Table 2 gives the values and sources for all the measured or specified variables for the example problem. The only variable excluded from this table is the thickness of the insulation, *th*, which is the variable of interest.

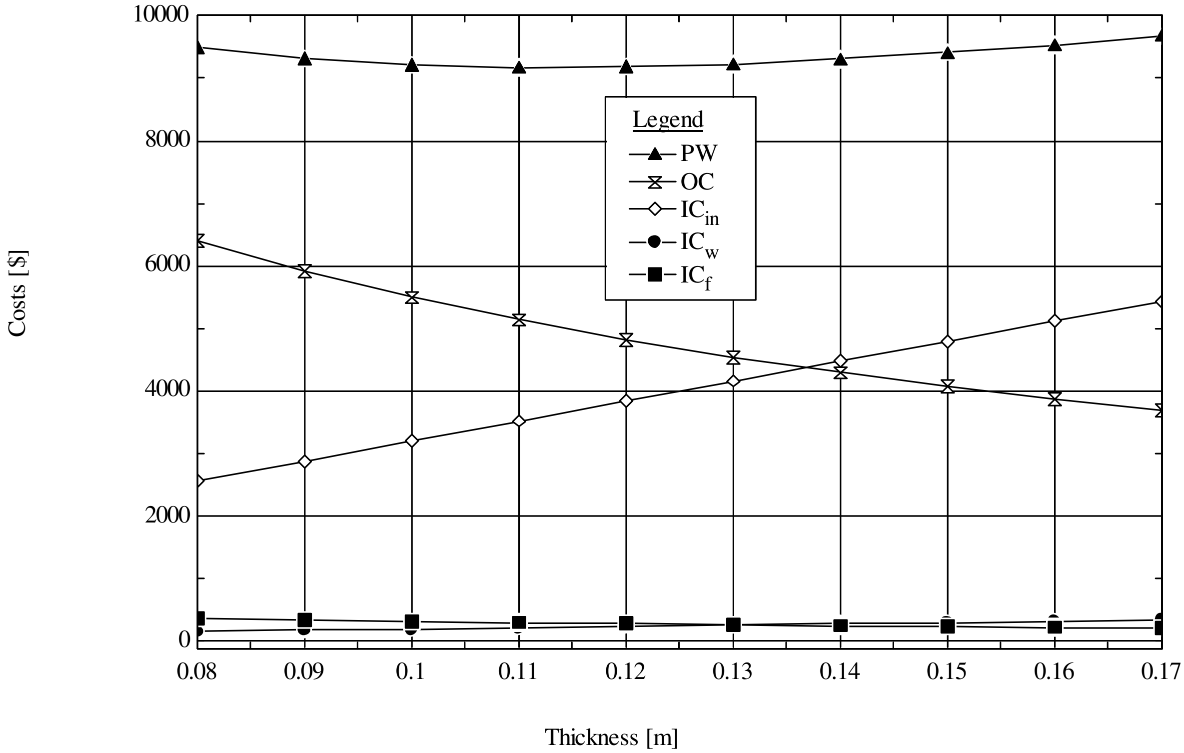
Table 2. Values for Specified Variables in Base Case

| Variable | Base Value | Units | Source |
|---|---|---|---|
| $H$ | 3 | [m] | Drawings |
| $W_{in}$ | 0.4 | [m] | Drawings |
| $N_s$ | 167 | [#] | Drawings |
| $Ciu$ | 100 | [$\$/m^3$] | Supplier |
| $W_w$ | 0.05 | [m] | Drawings |
| $Cwu$ | 50 | [$\$/m^3$] | Supplier |
| $Cfu$ | 150 | [$/kW] | Supplier |
| $T_i$ | 20 | [°C] | Thermostat setpoint |
| $T_{o,min}$ | 0 | [°C] | Weather data |
| $h_i$ | 0.005 | [$kW/m^2$-°C] | Handbook |
| $th_s$ | 0.06 | [m] | Drawings |
| $k_s$ | 0.00017 | [kW/m-°C] | Handbook |
| $k_w$ | 0.00011 | [kW/m-°C] | Handbook |
| $k_{in}$ | 0.000034 | [kW/m-°C] | Handbook |
| $h_o$ | 0.01 | [$kW/m^2$-°C] | Handbook |
| $Dur$ | 4000 | [hrs/yr] | Weather data |
| $Ceu$ | 0.07 | [$/kwh] | Supplier |
| $\eta$ | 0.8 | [$kW_{out}/kW_{in}$] | Supplier |
| $T_{o,avg}$ | 5 | [°C] | Weather Data |
| $i$ | 0.1 | [$\$_{interest}/\$_{principal}$] | Stock Market |
| $n$ | 40 | [#] | Handbook |

With these values the thickness of the insulation was varied and the costs given in Equation 1 were plotted as shown in Figure 2.

For the base case the optimum thickness for the insulation is 0.114 meters. This is found by locating the minimum present worth of the entire system in the plot shown in Figure 2. Numerically it is found by using a uni-variant optimization routine which searches for the minimum present worth by varying the thickness of the insulation. The two costs which make up the majority of the present worth are the initial cost of insulation and the operating cost of furnace. An astute observer realizes there is a significant amount of uncertainty in each of the variables which are specified in Table 2. For example it is impossible to predict what fuel costs will be for the next forty years. Or for that matter, will the furnace and insulation for any particular building endure for that length of time? Therefore, another useful tool for the design engineer is the sensitivity analysis.

Figure 2. Costs in Today's Dollars as a Function of Insulation Thickness.




Sensitivity analysis holds all the specified variables except one constant. That variable is exercised over a range to determine how sensitive the result is to the variable. It is particularly useful if the changes in the specified variables are presented in a non-dimensional form so the relative sensitivity can be assessed. One way to do this is to calculate the value of the variable as a percent difference from the base case as shown in Equation 17.

$$SV = SV_b \cdot \frac{P}{100} + SV_b \tag{17}$$

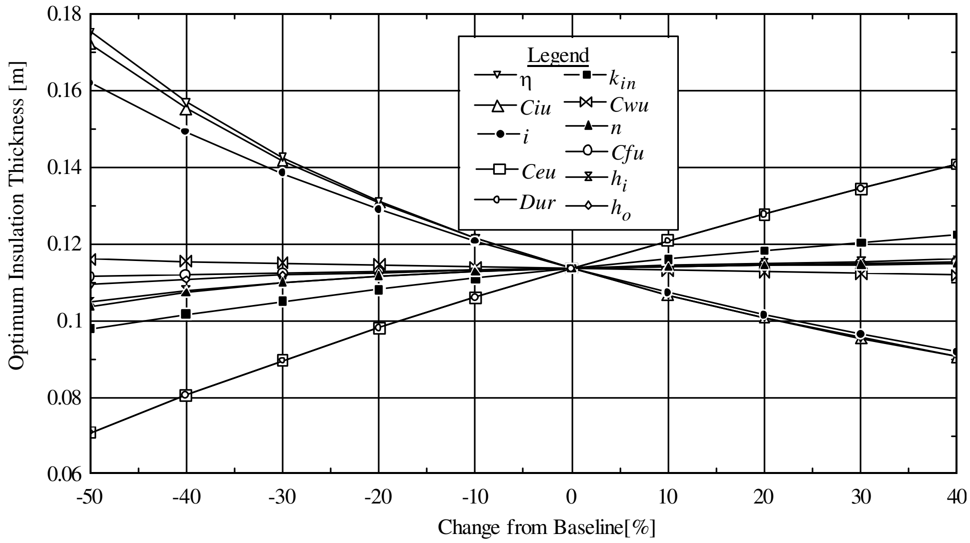
where,
$SV$ is the specified variable (e.g. *Ceu, n*, etc.)
$SV_b$ is the value of the specified variable at the base case as given in Table 2
$P$ is the percentage change from the base case [%]

Figure 3 contains the results of the sensitivity analysis for most of the specified variables in this problem. The steeper the slope of the lines the more sensitive the optimum insulation thickness is to the specified variable. From this plot it is clear the optimum insulation thickness is most sensitive to changes in furnace efficiency, the unit cost of insulation and the time value of money. For a fifty percent reduction from the baseline case each of these variables produced a 54, 51 and 42 percent increase in the optimum thickness. On the other hand unit cost of the furnace and the convective heat transfer coefficients had little influence on the optimum insulation thickness.

Sensitivity of the dependent variable to the independent variables is not the only criterion that needs to be considered when determining the importance of the variable. The volatility of the variable over the life of the system needs to be assessed. When volatility is added two more variables rise in their level of importance. These are the unit cost of energy and the duration of the winter season. The baseline values for these two variables should reflect the designer's best estimate of how these variables will change over the life of the system.

Out of the top three variables the order of volatility would be the unit cost of insulation, the time value of money and the furnace efficiency. Unfortunately, no one knows the future but the design decisions we make today affect the future. Therefore, the values of each of the variables except the initial cost should account for potential changes in the future.

Figure 3. Non-Dimensional Sensitivity Analysis Results




Conclusions

More open-ended problems should be introduced to undergraduate students because these types of problems promote creativity. It also gives students more exposure to the type of problems they will encounter after they graduate. Unfortunately, open-ended problems are more challenging to solve than typical textbook problems where all the necessary inputs are provided. The deductive strategy provides structure to open-ended problems. This approach is a useful tool to aid the students in achieving a successful solution and developing their creativity in the process.

The method is summarized in the appendix and an example is provided in this paper. The method is applied to determine the optimum insulation thickness for the walls of a building requiring only heat. An accurate accounting of equations, unknowns and specified variables is a useful tool to tracking the progress of a solution.

Once a solution is achieved most design engineers realize that the specified variables have a high degree of uncertainty. It is important to assess the impact of this uncertainty on the solution. Sensitivity analysis is one way to compare how each of the variables influence the solution. It allows the specified variables to be prioritized. Then the design engineer can allocate the time necessary to achieve the accuracy required to gain confidence in the solution. For the more volatile variables such as cost and time value of money the design should try to predict the average value over the life of the system.

Students are finding the deductive approach a useful tool to solve open-ended problems. A majority of the students in classes where this method has been taught recognize the benefits of this systematic approach. These benefits include providing a starting point for the solution of a complex problem, assistance in breaking the problem down into manageable parts, and leading one to the next step in the solution.

## Recommendations

Educators should consider introducing more open-ended problems in the undergraduate experience, especially at the junior and senior levels. Provide the students with instruction on the deductive approach to equip them to solve these challenging problems. If you teach heat transfer or air conditioning classes use this sample problem in the class. For other classes the textbook problems can be adapted for this strategy by removing the specified inputs. Finally, if you design buildings be sure to spend adequate time securing accurate (<10% uncertainty) data for the heating system efficiency, cost of insulation, cost of energy, duration of the heating season and the client's desired rate of return (time value of money).

## References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6) American Society of Heating, Refrigerating and Air Conditioning Engineers (ASHRAE) Handbook of Fundamentals. 2001. ASHRAE. Atlanta, GA. Table 1. Surface Conductances and Resistances for Air. P. 25.2

Appendix 1. Deductive Problem Solving Strategy Handout for Thermal Science Students

Please use this systematic strategy to break a problem down into smaller parts for all your homework. If you discipline yourself to follow all these steps, even for easy problems, you will have a tool to help you when you face a problem where you have no idea where to begin. The deductive approach will assist you when you face problems that are not well defined.

1) Find: Determine the dependent variable(s). This is the objective or goal of the problem. This is the starting point of the deductive problem solving strategy. For optimization problems identify the objective function (a function that has a maximum or minimum as the independent variable(s) are varied over their range(s)). One common objective function is the present worth of a system.

2) Solution:
   a) A system schematic and process diagrams (e.g., P vs. v, T vs. s, P vs. h, T vs length, etc.) are essential to the definition of the problem. In the schematic identify the system boundaries, all energy interactions (i.e., heat and work) which cross the boundaries, identify any relevant reference frames (e.g., position) and the positive direction.
   b) If the dependent variable can be measured directly then measure it. Variables which can often be measured directly with instruments include: mass, length, time, temperature, force and pressure.
   c) If the dependent variable can not be measured directly then determine which law of science or definition can be applied to help connect variables which can be measured with the dependent variable. To be systematic start with an equation that has the dependent variable in it.
      i) For thermal science problems the following laws and definitions may be relevant to the problem. Conservation of mass, momentum and energy. Definition of efficiency or coefficient of performance. In comparisons percent difference is a common objective.
      ii) For heat transfer the following laws may be relevant: Fourier's Law for conduction, Newton's Law of Cooling for convection and Stefan-Boltzmann Law for radiation heat transfer problems. For convection empirical relationships to determine the heat transfer coefficient. For radiation view factor relationships may be helpful. (e.g., reciprocity, summation, superposition, and symmetry.) For multi-mode heat transfer the electrical analogy may be useful.
      iii) For thermodynamics property relationships may be used to provide additional equations. First determine the phase of the substance by comparing the properties with the critical point and the saturation properties of the substance.
      iv) Formulate the equation in terms of the dependent variable. As much as possible the equation should be written in explicit form with the dependent variable by itself on the left hand side of the equals sign.
      v) Use descriptive variable names (e.g., $P_i$ rather than $x_1$ for initial pressure)
      vi) Provide references for all equations (e.g., conservation of energy).
      vii) Note any assumptions used when applying the equations. Justify the relevance of the assumptions to the problem. (e.g., Ideal gas because the compressibility factor (Z) is near 1)
   d) Evaluate each of the variables in the equation. List all the variables in the equation that can either be specified or measured. Circle and account for all new unknowns that have not already been counted. Tally the number of equations and unknowns with each equation and determine how many more equations are needed.
   e) Are there sufficient equations to solve the problem? If not repeat steps b) and c).

3) Summaries
   a) Summarize all the variables which need to be specified or measured. Identify the source of the data. (e.g. handbook, measurement, etc.)
   b) Create a nomenclature section to define all variables and their associated units.
   c) Summarize all assumptions that have been made throughout the solution.
   d) Identify all relevant variables on the system schematic.

4) Perform reality checks
   a) Check equations for dimensional homogeneity
   b) For computer solutions perform sample calculations