

Delivering Lectures in Introductory Graduate-Level Continuum Mechanics Courses using *Mathematica*

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Abstract

In the authors' experience, having been on both the giving and receiving ends of the "lecturing process," some of the topics covered in introductory (as well as advanced) continuum mechanics courses turn out to be particularly hard to communicate and to grasp. Representative examples might involve subjects such as the polar and the spectral decompositions of a tensor and their physical significance. In this paper we discuss how the authors make use of the software package *Mathematica* to create lectures allowing one to expose key concepts by taking advantage of the programming, manipulation, visualization, and animation capabilities of the computer program *Mathematica*, developed by Wolfram Research. The emphasis of this paper is as much on the way a lecture can be delivered using *Mathematica* as it is on the specific examples presented. In other words, we want to illustrate that software such as *Mathematica*, by combining symbolic manipulation, computation, and visualization, allows one to turn "a laptop and a projector" into an "electronic *color-board*" which makes for effective and dynamic presentations of even the most sophisticated topic.

Introduction

The third author of this paper has been actively teaching mechanics courses at the Pennsylvania State University for the last seven years. During these years, he has made an effort to make the students excited about mechanics as a way to characterize the behavior of materials at various scales and for a variety of applications, including topics such as the descriptions of phase transitions and the derivation of nonlinear continuum properties for molecular dynamics simulations. Behind this effort there is a strong belief that theoretical mechanics has much to offer to the solution of today's technological challenges, which, with a strong emphasis on nano-scale phenomena, require a deep exploration of subjects such as nonlinear deformations, material inhomogeneity, evolving micro- and nano-structure.

As nonlinearity and inhomogeneity are at the core of the authors' research activities, they have struggled to acquire the necessary familiarity, intuition, and instincts that allow one to "think nonlinearly." In turn, this struggle has made the authors meditate on how to transmit their experience to new generations of students. As a response to this need,

a project called the Virtual Laboratory" (VL) was initiated four years ago to bring new tools to the *theoretical* mechanics classroom that better illustrate how the mathematical formalisms used in continuum mechanics, elasticity, and strength of materials model the deformation of materials. In other words, the VL project primary goal is not that of describing real materials, but that of illustrating how a material is forced to behave when described by a given mathematical model. Therefore, in a way, the VL project intends to place the material characterization problem "on its head," that is, to make students better modelers by making them pay attention to what the mathematical language they end up using is actually allowing them to say. The main motivation for this approach is that the VL is intended to primarily support the training in the theoretical aspect of material characterization. For example, the VL is intended to force students to confront the problem concerning the number and kind of experiments that one would need to perform to characterize a given material system. Furthermore, it is intended to make students aware of what information can actually be "squeezed" out of a certain model so that they are aware of why certain predictions or experiments turn out not to match the expectations that one may have about a certain model or material system.

Although our experience with the VL has been limited, it has been positive and very encouraging. We therefore see this paper as a way to share our experience, for what it's worth, by describing specific examples of what the authors have developed and tried in class. The VL project has components addressing a didactic need at both the undergraduate (specifically for undergraduate strength of materials) and graduate levels.

The examples presented herein are intended for use in an introductory course in Continuum Mechanics, in which emphasis is placed on the tensorial nature of the concepts of stress and strain and on the mathematical definition of material symmetry properties. In the authors' experience, some of the topics that are particularly hard to communicate and to grasp involve subjects such as

- the polar decomposition of a tensor along with its implication, that is, the separation of rigid body rotations from deformations, where *deformation* is being used here to denote a *change of shape*;
- the spectral decomposition theorem and its implication in the definition of principal stresses and strains.

We will therefore present examples on these topics and we will then present a discussion concerning the students' response to the new material and on the future developments that these tools will allow one to explore.

From an historical viewpoint, the examples presented in the following sections are the results of ideas the authors have dealt with in undergraduate courses such as sophomore level dynamics (for a description of the developments in dynamics see^{2,3}) and strength of materials (these developments have yet to be published as they have yet to be tested in class). Furthermore, as said earlier, it is the result of a learning process on the part

of the authors which ended up being codified and made available to the students in the graduate continuum mechanics course. This premise is made to point out that the need for the material presented here did not arise from a review of the graduate curriculum or by the enrollment level in the course. Instead, it was an initiative taken by the authors inspired by the belief that a “picture is worth a thousand words” as well as the belief that, although challenging at times, mathematics is the language of science. In other words, the use of visualization aids in mathematical physics is not viewed not as substituting but as supporting mathematics learning. This learning is important in that its lack would result in the inability to acquire a solid physical understanding of any phenomenon.

The course: introduction to Continuum Mechanics

This section is devoted to a description of the course in which the examples described later were presented. The course in question is called “Introduction to Continuum Mechanics” and its catalogue number at Penn State is “EMch 540.” This course is offered by the authors’ department, namely the Department of Engineering Science and Mechanics, which, at Penn State, is charged with the teaching of a wide variety of mechanics courses, at both the undergraduate and graduate levels.

EMch 540 is intended to be the first graduate level formal training in continuum mechanics and it is often taught concurrently with Theory of Elasticity. The textbooks typically used in the course are those by Malvern, Gurtin, and Bowen^{1,4,6}. Over the past three years, the course, which is offered every fall semester, has had an average enrollment of approximately twenty students. Before EMch 540, most of these students have never been exposed to any formal training in tensor algebra and calculus. For this reason, and due to the personal preference of the instructor who typically teaches this course (namely the third author), the introductory course in continuum mechanics has become the place where tensor algebra and calculus are taught.* In fact, the first three weeks of the course are entirely devoted to the covering of the mathematical preliminaries. The rest of the course is divided into three parts:

- (i) Kinematics: this part is devoted to a rigorous description of motion in a full non-linear context. Both direct and index notation are used interchangeably. Furthermore, the linear kinematics, that is the description of both rigid and deformable motions with infinitesimal displacement gradient, are derived by linearizing the full nonlinear theory.
- (ii) Balance laws and thermodynamics: in this part the global statements of momentum balance are given and the corresponding local statements are derived via the use of the appropriate divergence and transport theorems. An extremely rigorous

*This part of the course may differ from what is more usually taught by other engineering department at other universities.

derivation of the Cauchy stress is presented (following that in⁴) and a number of applications are covered. Following the momentum balance, the first and second laws of thermodynamics are covered and the Clausius-Duhem dissipation inequality is derived.

- (iii) Constitutive theory: in this section the derivation of constitutive relations for isothermal and non-isothermal elastic bodies are covered, along with the constitutive relations for elastic materials with heat conduction and viscous dissipation.

In the presentation of all these three topics, the students are made aware of the different appearance the theory takes on when formulated according to both the Lagrangian and Eulerian viewpoints. Furthermore, in covering the balance laws and the constitutive equations, students are exposed to Galilean relativity in the form of the Material Frame Indifference Principle. Finally, when dealing with the constitutive part, the derivation of the constitutive equations is always followed by a discussion of symmetry properties and the derivation of the equations of motion for the material system at hand.

Second order tensors

Students in continuum mechanics are told that things such as stress and strain are tensors (second order tensors in particular). Emphasis is also placed on the fact that the Cauchy stress and virtually every strain measure discussed in the course take on the form of *symmetric* tensors. Finally, depending on the treatment, emphasis may be placed on the fact that the deformation gradient is a second order tensor with positive determinant. However, it is often difficult to make the students *visualize* the relation between the aforementioned mathematical constructs and the physical reality these are meant to embody.

In the authors' opinion much can be learned about tensors by giving graphic form to their definition. Provided that tensors can be presented in various ways, the authors have found the following definition to be the most easily presentable in graphic form: a tensor is a linear transformation from a vector space to another vector space. Hence, with reference to Fig. 1, tensors are introduced as functions returning an output vector whenever an *input* vector is given. However, to better convey the nature of the transformation expressed by a tensor, it is not sufficient to look at the action of a tensor on just one vector. For this purpose, it is better to consider the action of a tensor on a densely populated set of vectors. The solution devised by the authors has been the depiction of the action of a tensor, in its corresponding matrix form, onto the set of vectors forming, for example, the unit sphere (circle in 2D) or the unit cube (square in 2D). Figure 2 shows how this idea is presented to the students taking advantage of some of the formatting and manipulation commands available in *Mathematica*. The visualization of the idea in question was realized by defining two different graphics functions, one for closed line domains consisting of perimeters of ellipses and squares; the other for the interior of such domains. The output of the first type of function can be seen in Figs. 3 and 4. The visualization strategy

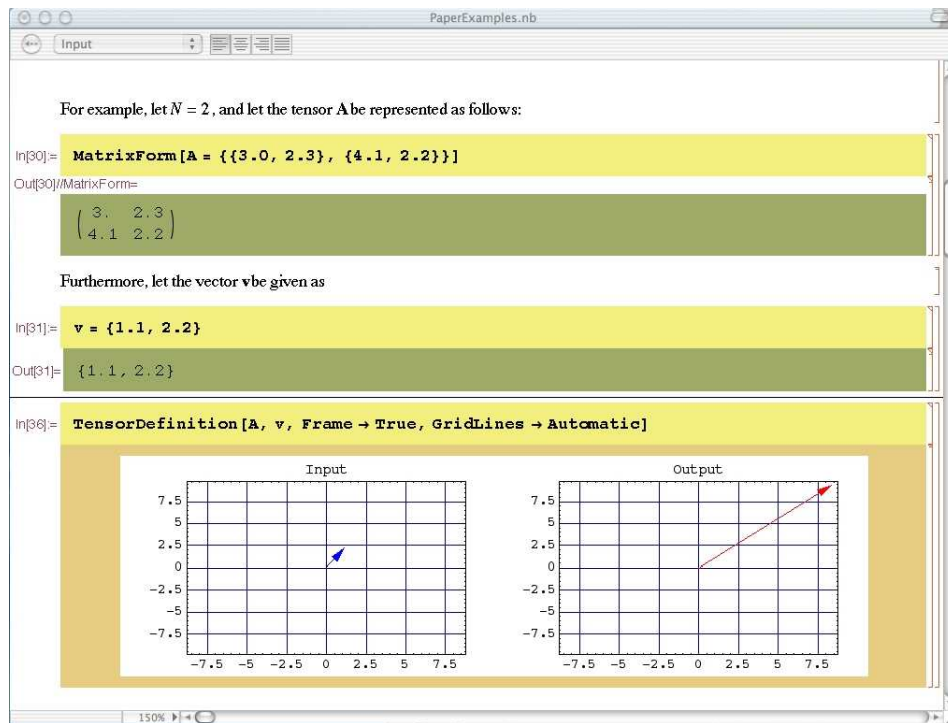


Figure 1. Screen shot of the *Mathematica* graphic display in a notebook discussing the definition of a 2nd order tensor over a 2D real vector space.

of the second type can be seen in Figs. 5 and 6. The coloring of the unit circle and of the unit square is done so to allows students to match the input with the output by matching colors. The *linear* nature of the tensor mapping is readily apparent by noticing that the lines in Figs. 3 and 4 are simple *scalings* of one another. Furthermore, the linearity of the operation is also visible in the fact that, when mapping the unit square as in Figs. 4 and 6, the sides of the output geometrical figure remain straight. Furthermore, the comparison between input and output shows that the action of a tensor can be described in terms of stretching, pinching, and something that looks like a rotation.

The display of how the input domain is transformed by a tensor also allows the instructor to distinguish between three fundamentally different classes of tensors, namely those with null, positive, and negative determinants. With reference to Fig. 7, we see that the action of a tensor with null determinant is to take the unit input domains and map them into a line (or a point in the case of the zero tensor). The graphic display of this effect is usually very effective in getting across the idea that a lower dimension in the output, with respect to the dimension of the input, yields the impossibility of reconstructing the input domain from the output, that is, it says that the transformation is not invertible. As far as the effect of the sign of the determinant is concerned, the graphic output shows that for tensors with positive determinants the color sequence in the output is the same as that in the input. By contrast, when a tensor has negative determinant, the color sequence in the

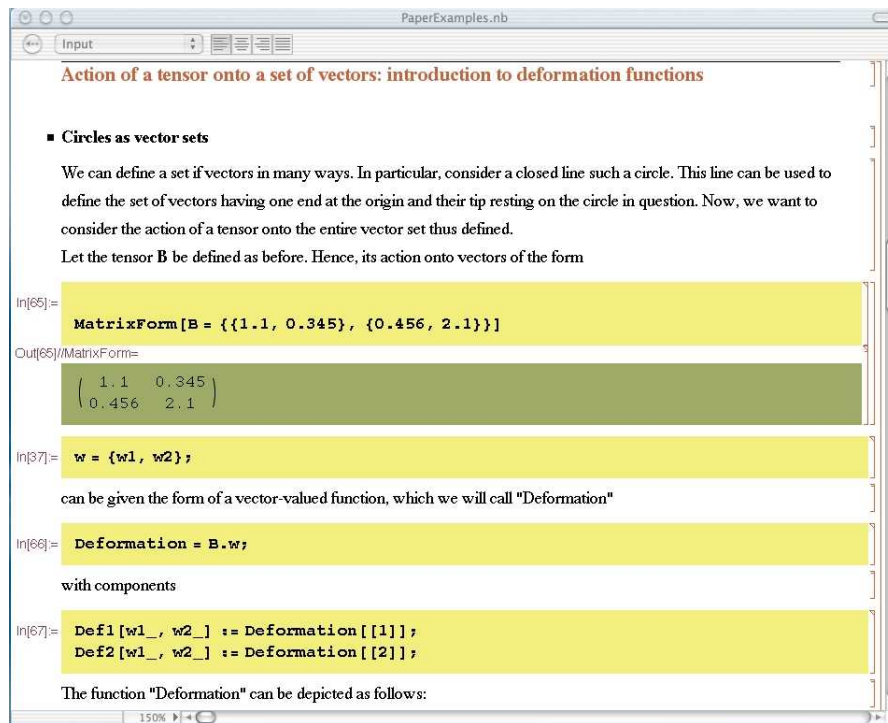


Figure 2. Screen shot of a section of a *Mathematica* notebook discussing the action of a tensor **B** onto the various sets of vectors defined in terms of closed lines.

output is reversed with respect to that of the input, as it can be seen in Fig. 8. The concept that the above observations are meant to get across is that strain is a tensor because the action of a tensor *deforms the input domain into the output domain*. Furthermore, the graphics depiction of the deformation caused by a tensor seems to be *instinctively* decomposable into physical actions such as stretching, pinching, and rotating. Finally, the observations concerning the sign of the determinant of a tensor are meant to illustrate the idea that tensors with null determinants squeeze a finite volume domain into a domain with zero volume and they are therefore not capable of describing physically meaningful deformations. As far as the distinction between tensors with positive and negative determinants, one can argue that "reversing" the color scheme sequence, with respect to that given in the input domain, in a 2D deformation is tantamount to having to disassemble the input domain and re-assemble it into a mirror image (with respect to one of the coordinate axes) of itself, which, in turn, gets deformed into the final output domain. If one accepts this interpretation, one is led to the conclusion that, again, tensors with negative determinants cannot be used to represent deformations which do not imply the *disintegration* of the original body. In the end, the concept that only tensors with positive determinants are "used" for the description of regular deformations (cf.¹⁴) is emphasized.

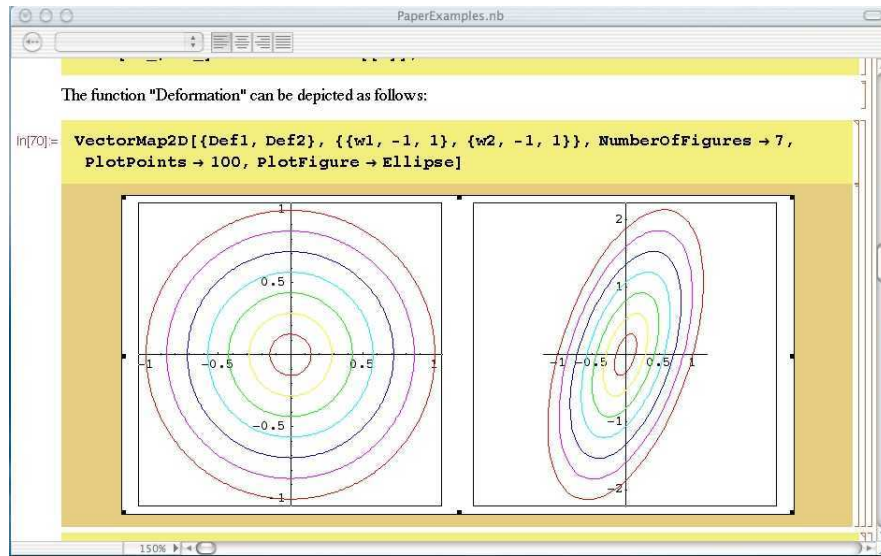


Figure 3. Screen shot of a section of a *Mathematica* notebook displaying the mapping of circles by a tensor.

Symmetric tensors and their spectral decomposition

The discussion of this particular class of tensors is done after the previous demonstration has been presented. The fundamental objective that the authors have tried to achieve in their examples on symmetric tensors is, in a manner of speaking, the activation of a “reflex” in the students by which the words “symmetric tensor” are translated into “sequence of perpendicular projections in mutually orthogonal directions.” The translation from “perpendicular projection in a direction” into “stretch along that direction” is then achieved in a subsequent presentation on the Polar Decomposition theorem.

The graphic display offered in the previous example is here specialized on symmetric tensors. In particular, to the previous demonstration is added the plotting of the directions corresponding to the tensor’s eigenvectors. This process is meant to link the process of “diagonalization of a matrix” with the spectral decomposition theorem for tensors, and, in turn, to link said decomposition with its geometrical interpretation by graphically displaying the outcome of the individual components of the decomposition. These components can be extracted directly from the spectral decomposition, the latter taking on the form:

$$\mathbf{A} = \sum_{i=1}^3 \alpha_i \mathbf{e}_i \otimes \mathbf{e}_i, \quad i = 1, \dots, 3,$$

where \mathbf{A}^* is a second order symmetric tensor, α_i is the i -th eigenvalue of \mathbf{A} , \mathbf{e}_i is the corresponding normalized eigenvector, and the symbol \otimes denotes the tensor (or external, or

* Boldface symbols represent second order tensors defined over a three-dimensional vector space. These tensors can therefore be represented via 3×3 matrices.

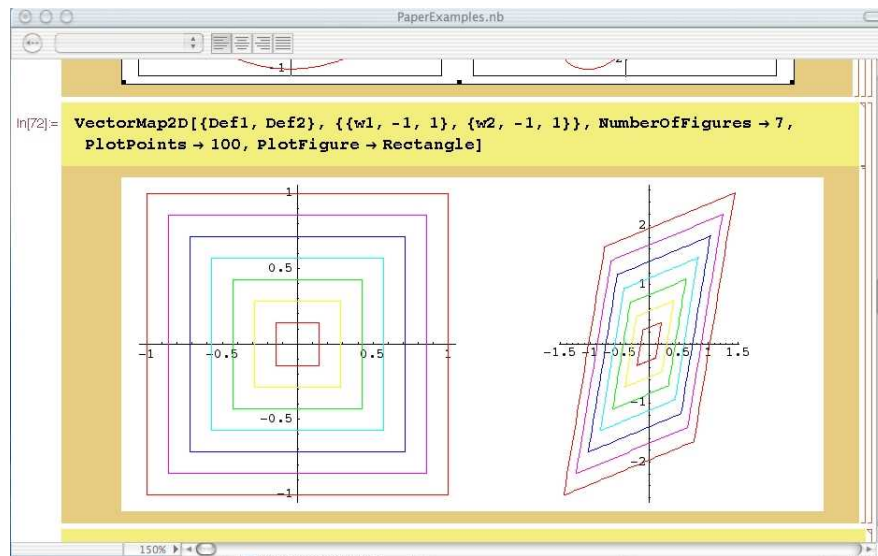


Figure 4. Screen shot of a section of a *Mathematica* notebook displaying the mapping of circles by a tensor.

dyadic) product. What is important about the spectral decomposition is the idea that the individual components of the decomposition represent stretches of an amount α_i in the direction identified by \mathbf{e}_i . Furthermore, the stretching directions turn out to be mutually orthogonal. Hence, what is emphasized in the in-class presentation is the orthogonality of the principal stretches, as the components of the decomposition are usually referred to. This can be seen in Fig. 9, where the same plotting function used in previous example is applied along with the option that causes the principal direction of the stretches to appear.

Deformations and their polar decomposition

The examples listed so far are building blocks for a presentation which usually has taken the third author a whole ninety minute period to go through. The purpose of the presentation is to bring to life a concept which is heard often in solids mechanics courses, namely every deformation process can be viewed (at least locally) as a sequence of three mutually orthogonal stretches followed or preceded by a rigid rotation. In reality, this simple statement contains a number of ideas, some of which are both deep and subtle. For this reason, it is useful to review the application of the polar decomposition theorem to deformation tensors. Hence, given an *admissible* deformation tensor \mathbf{F} , that is, a tensor such that $\det \mathbf{F} > 0$, there exist three unique tensors \mathbf{R} , \mathbf{U} , and \mathbf{V} such that

1. both \mathbf{U} and \mathbf{V} are symmetric with positive determinants;

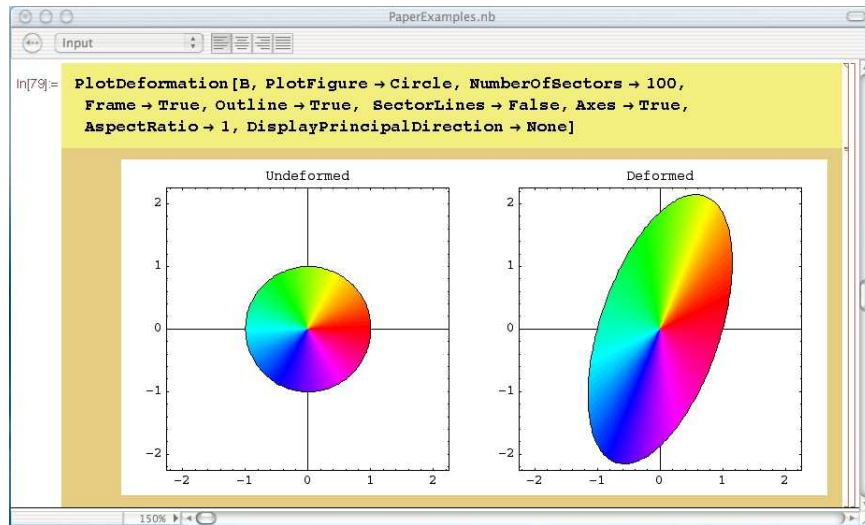


Figure 5. Screen shot of a section of a *Mathematica* notebook displaying the mapping of circles by a tensor.

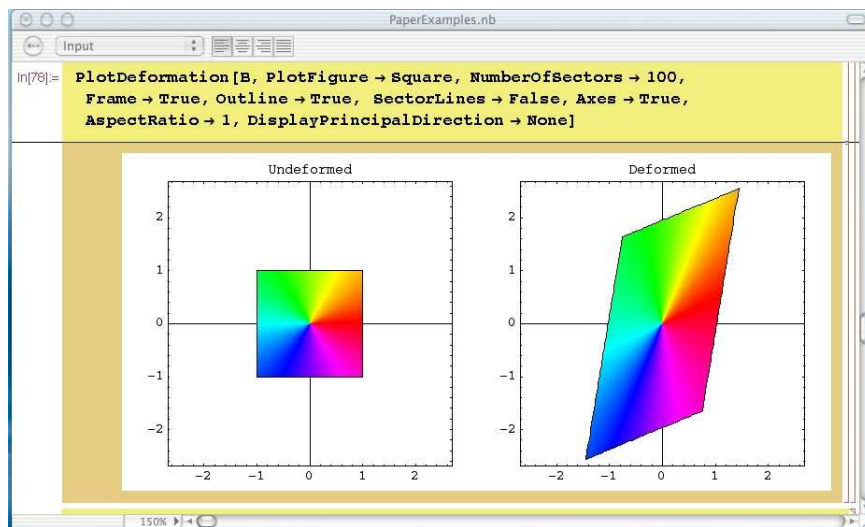


Figure 6. Screen shot of a section of a *Mathematica* notebook displaying the mapping of circles by a tensor.

2. \mathbf{R} is an orthogonal transformation, i.e., a proper rotation tensor; and
3. $\mathbf{F} = \mathbf{R}\mathbf{U} = \mathbf{V}\mathbf{R}$.

The tensors \mathbf{U} and \mathbf{V} are referred to as the right and left *stretch* tensors.

Now, since all of the above tensors have positive determinants, each one of them can be interpreted as a deformation tensor in its own right. In addition, the *multiplicative* nature

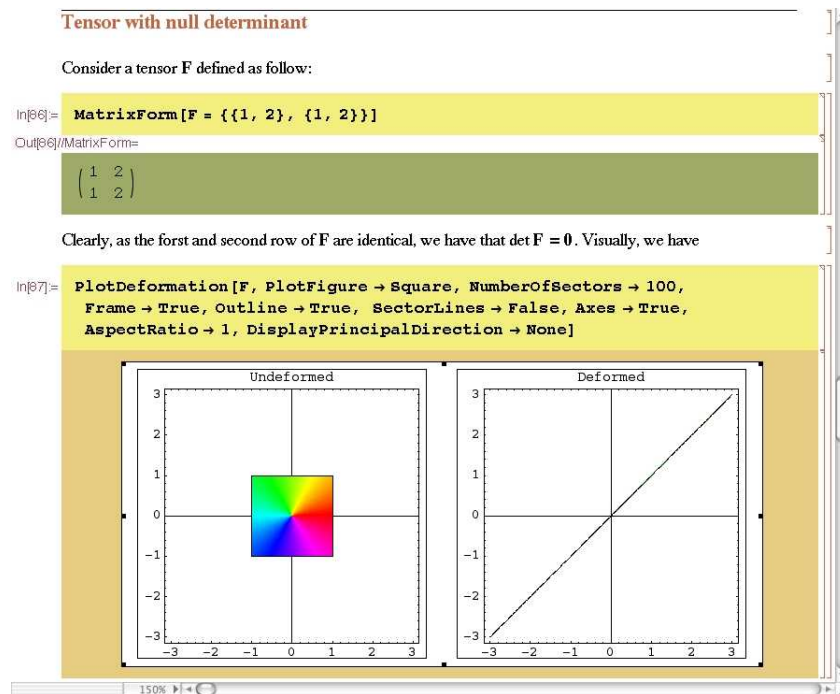


Figure 7. Screen shot of a section of a *Mathematica* notebook displaying the action of a tensor with determinant equal to zero.

of the above decomposition suggests that the tensor **RU** consists of a stretch **U** followed by a rigid rotation whereas the tensor **VR** consists of a rigid rotation followed by a stretch. But there is more! Since both **U** and **V** are symmetric, they can be further decomposed using the spectral decomposition mentioned earlier. However, a problem arises in that, when we say that a deformation is a *sequence* of deformations, said deformation must be decomposed in a multiplicative way.* Unfortunately, the spectral decomposition of symmetric tensors only offers an additive decomposition, which, therefore, cannot be readily taken advantage of. To overcome this obstacle one needs to resort to a little known (and difficult to justify to the students) representation theorem stating that the spectral decomposition can not only be given the traditional additive form, but a multiplicative form as well. That is, given a symmetric tensor **U**, letting its eigenvalues be denoted by λ_i , with $i = 1, \dots, 3$, and its corresponding normalized eigenvectors be denoted by \mathbf{e}_i , with $i = 1, \dots, 3$, we have that

$$\mathbf{U} = \sum_{i=1}^3 \lambda_i \mathbf{e}_i \otimes \mathbf{e}_i = \mathbf{U}_1 \mathbf{U}_2 \mathbf{U}_3,$$

*This is due to the fact that the chain rule of calculus forces the overall deformation gradient of a sequence of deformations to be the product of deformation gradients of each individual deformation written in inverse order with respect to the order of the deformations (for details see the "Proposition" on page 43 of⁴).

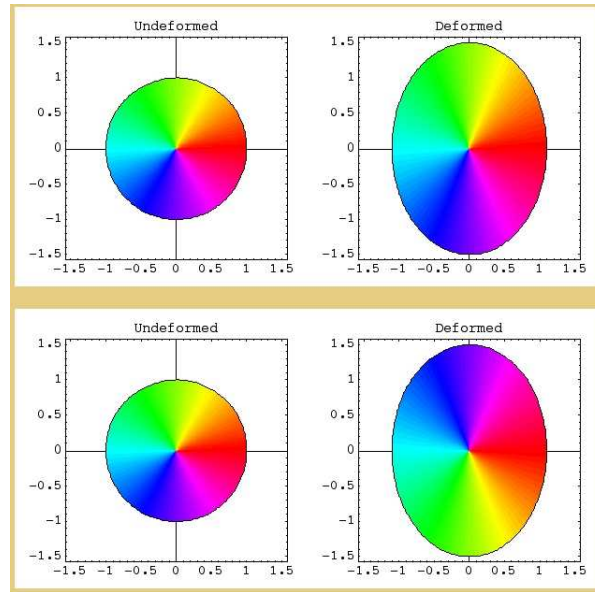


Figure 8. Screen shot of a section of a *Mathematica* plot displaying the action onto the unit circle of a tensor $\mathbf{F1}$ with components $F1_{11} = 1.1$, $F1_{12} = 0.0$, $F1_{21} = 0.0$, and $F1_{22} = 1.5$, i.e. with positive determinant, and of a tensor $\mathbf{F2}$ with components $F2_{11} = 1.1$, $F2_{12} = 0.0$, $F2_{21} = 0.0$, and $F2_{22} = -1.5$, i.e. with negative determinant.

where, for $i = 1, \dots, 3$,

$$\mathbf{U}_i = \mathbf{I} + (\lambda_i - 1)\mathbf{e}_i \otimes \mathbf{e}_i,$$

and where \mathbf{I} denotes the identity tensor (for a proof see page 45 in⁴). Furthermore, each of the tensors \mathbf{U}_i , using the demonstrations mentioned earlier, can easily be shown to represent simple extensions (contractions if $\lambda_i < 1$) in the direction identified by the unit vectors \mathbf{e}_i . The final outcome is a multiplicative decomposition with important physical and geometrical meanings, which can be easily animated using *Mathematica*:

$$\mathbf{F} = \mathbf{R} \mathbf{U}_1 \mathbf{U}_2 \mathbf{U}_3 = \mathbf{V}_1 \mathbf{V}_2 \mathbf{V}_3 \mathbf{R}.$$

Due to the limitations of this written presentation, in Fig. 10 we simply present “snapshots” of the animation the authors have developed for the in-class presentation. In the animation in question, the students see a movie in which the elements of the above decomposition are animated and in which the direction of each extension is clearly indicated by straight lines, each of which becomes visible only when the extension corresponding to it becomes visible on the screen.

In the in-class experience of the third author, explaining all of the steps outlined above has always required a tremendous effort which never quite conveyed the physical message “hidden” behind a relatively intricate series of linear algebra theorems. However, the authors are happy to report that they received very good and encouraging feedback from the those students who observed the presentation on the polar decomposition. These

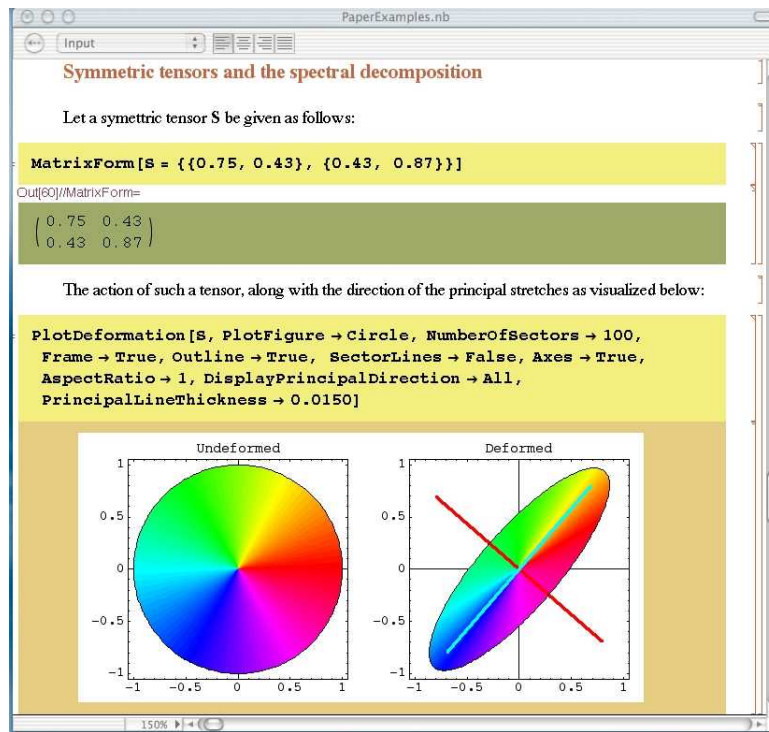


Figure 9. Screen shot of a section of a *Mathematica* plot displaying the action and the direction of the principal stretches of a symmetric tensor.

students seemed to have a very good grasp of this particular topic and have retained a vivid memory of the presentation in question and of the physical ideas it was meant to convey.

***Mathematica* and its presentation capabilities**

In this section the authors want to point out a number of features that make *Mathematica* (and most likely other packages like it) a very valuable tool for teaching intricate topics.

Mathematica offers the possibility of formatting input and output operations using Style Sheets which help in the visual differentiation between the input, the desired output, and the various part of the presentation. In fact, *Mathematica* notebooks (as *Mathematica* files are called) can be given a nice-looking title and can be arranged in sections very much in the same way in which a paper or even a web-page might be arranged. In addition, the text description that one can include in a notebook can be easily navigated with the use of hyperlinks and can be formatted so as to display equations in a proper and effective way, using all of the required symbols to denote the desired operators. Finally, it is possible to hide much of the low-level programming governing the number crunching and

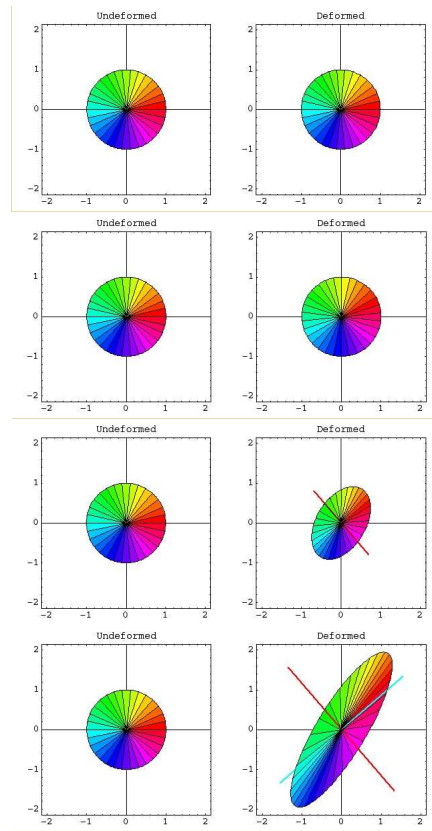


Figure 10. Screen shot of a section of four frames out of an animation for the visualization of the polar and spectral decompositions combined. The polar decomposition depicted is of the “left” type. Hence, from top to bottom, the first frame displays the initial configuration, the second frame displays the system at the end of the rigid rotation part of the decomposition, followed by the first and second stretches, respectively.

the graphics output so that the “spectator” will not be distracted by unnecessarily long programming details.

As far as the display of *Mathematica* notebooks is concerned, *Mathematica* can be configured to use a Presentation Screen Style Environment which helps in the projection of *Mathematica* documents. In fact, this was the way in which the authors actually displayed their *Mathematica* work to the students, as most classrooms in recently built buildings at Penn State are equipped with a projection system to which a laptop can be easily connected.

Clearly, as in any programming environment, making the notebook simple to read and navigate, as well as making the graphics algorithms respond quickly to user input, requires a significant effort on the part of the programmer. In fact the authors have spent a total of roughly 500 hours (spread through various semesters) creating and refining the output of the notebooks they have used in class.

Learning how to efficiently program in *Mathematica*, even with the help of manuals such as that by Maeder⁵, requires an investment of time which might be difficult for some to afford. Despite this drawback, the authors have had a good experience with *Mathematica* as a way to present both theory and examples so much so that *Mathematica* has become for them an ever more frequently used class presentation tool. This is not to say that *Mathematica* has been used to replace the traditional exposition of a course's subject matter, but it has become a very nice and effective complement to the traditional lecture.

Student feedback and discussion

The feedback received from the students who have had the opportunity of seeing the presentations prepared by the authors has been very positive and encouraging. Here are some quotes that the authors have collected over several semesters (the text in brackets represents editing made by the authors to improve readability):

While some *Mathematica* presentations were given at the start of [the] sem[ester] (these were very good in understanding), some more could have been used in [the] stress discussions. *This quote was taken from one of the end-of-the-semester anonymous student feedback forms, in answer to the question: "What suggestions do you have for improving the course?"*

I think, particularly for Emch 540, that the visualization of certain relatively abstract concepts is crucial to understanding what is unfolding on the chalkboard. The animations that were demonstrated in class helped me to personally gain a more complete understanding of the topics discussed. All that I could say in the way of improvement would be to provide a few more throughout the semester. *Mr. Richard Schoedel, Penn State graduate student.*

Mailto: rms252@engr.psu.edu.

I found the *Mathematica* presentations very useful for understanding the material presented in class. In particular I enjoyed the animated demonstrations showing rotations and stretching deformations, and it was also useful to see how these changed when you adjusted some of the parameters. It was good to see that stretching/shrinking then rotating yielded the same result as rotating and then shrinking/stretching. I had hoped to receive some instruction on how to use *Mathematica* and how to generate some of the same things as shown in class. *Ms. Jennifer Akers, Penn State graduate student, mailto: jea178@psu.edu.*

The *Mathematica* presentations in the class you made in the class were very brief and for most parts, did not extend the understanding obtained in class, because the class lectures on the basic concepts were good enough for one

to comprehend the concepts. The second presentation you made when Dr. Solomon was also present went a little fast and I did not gain from it, personally. For the amount of detail we dealt in class, the mathematica presentations were not of much help. Probably, they would have been very handy in understanding really advanced and complicated concepts. *Mr. Subash Jayaraman, Penn State graduate student, mailto: sbj110@psu.edu.*

I thought the presentations were very useful. It gave meaning to the 'sea of tensors', which I really appreciated since it is easy to get lost in the math and forget about the application. I do think that either there should have been a couple more presentations or perhaps we should have used it ourselves to really get a good understanding of what was going on. Also, I don't know if it was just the computer that was being used, but timewise it was a little lengthy in terms of the time to reconfigure the input and process the images. I thought the graphics were very good—in particular the variations in color. *Ms. Nakhiah C. Goulbourne, Penn State graduate student, mailto: ncg107@psu.edu.*

I liked the use of Mathematica. It's always easier for me to remember things when I actually see them in front of me. For example: the right and left decomposition. I would have liked to have had more applications with Mathematica for the same reason. Concerning the homework: as a student, I probably would not like to be forced to use it because of the overload of work necessary (even if Mathematica seems really easy to use...); but, I'm sure that from a pedagogical point of view, it would have helped me to assimilate faster. Also, I think each student should be able to use Mathematica by the time they graduate, as it seems to be very powerful and widely used. *Mr. Samuel Tissot, Penn State graduate student, mailto: spt123@psu.edu.*

The quotes offered above are consistent with the verbal feedback given by other students for which no written record is available. With very few exceptions, the message that these quotes provide is that the presentations were effective in giving life to mathematical abstractions which usually remain vague and difficult to memorize. The one dissenting opinion reported above, the only one that the authors have in printed form, suggests that some individuals either do not gain from visualization or require better visualization. The last sentence in the comment by Mr. Jayaraman, indicates that the gain in the use of visualization is linked to the perceived difficulty level of the subject matter. To address this question, the authors will try to expand the use of these presentations to all major topics in the course and survey the students throughout the semester to check what subjects best benefitted from a *Mathematica* presentation. Such an effort would also be in response to the comment made by several students that there were too few presentations. Finally, some of the students suggest that they would have liked to use *Mathematica* and the programming contained in the authors' notebooks to solve problems on their own. This last

observation is somewhat problematic in that, at least at Penn State, students do not normally receive formal training in *Mathematica* (or other symbolic manipulation packages) usage and that the course instructor would then be asked to provide enough training for the students to be able to use *Mathematica* themselves. In principle, the present authors do not oppose this kind of requests and do like the idea of letting students do their own experimentation. However, the danger exists that helping the students become familiar with software such as *Mathematica* when used for advanced material, such as that presented in Continuum Mechanics, might be very time consuming.

Overall the experience has been very positive and the third author will continue to increase the frequency of use of *Mathematica* as a presentation tool.

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Biographies

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FRANCESCO COSTANZO came to Penn State in 1995 and is an Associate Professor of Engineering Science and Mechanics. He earned a Ph.D. degree in Aerospace Engineering from the Texas A&M University in 1993. His research interests include the mechanics of nanostructures and the dynamic crack propagation in thermoelastic materials.