

## **Deriving Closed Formula Design Restrictions in Electronics Labs**

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# Deriving Closed Formula Design Restrictions in Electronics Labs

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## **Abstract:**

The Electronics circuit design course is a required course for undergraduate Electrical Engineering students. The course covers oscillator circuits based on Schmitt trigger comparators with the hysteresis effect and multivibrators such as the 555 Timer circuits. The course introduces students to applications, such as timers and frequency dividers. The course is offered as a 3-credit course. Two credits are for lecture and one credit for lab. As part of lab assignments, students need to come up with closed formulas for the output signal and design restrictions. The objective of this step is to assess how well they understand the circuit's concept of operation. It challenges their creative thinking and develops their insightful analysis skills. The goal is also to help them formulate the design limitations and rules in a reusable format. The obtained design is tested in PSpice simulation, and the formula is presented using MATLAB animation to see the effect of changing different design parameters on the output signal. As a final step, the circuit is built and tested.

Thorough investigation of the theoretical design limitations helps in getting better hardware implementation designs. Being able to precisely explain circuit designs and define design restrictions and specifications is an important proficiency for practicing engineers. This paper shares lab designs and results that can be used in Electronics labs to strengthen the fundamentals of circuit design and analysis.

## **Introduction:**

Effective teaching guidance is attained when students can *capture, structure, and reuse* the information that they learn in class. Learning new concepts is most effective when grounded in personal experiments especially by direct hands-on implementation.

The objective of electronics courses is to explain the idea of operation of widely used circuits. If the theoretical concept is well understood, students can add modifications to the circuits and implement them as needed in design projects or later in their jobs. The main design concepts are explained and proven in class. In the lab, students are required to use the basic circuits for different applications. They need to select values for circuit parameters to meet given design specifications. Random selection of circuit elements without defining the restrictions of the required design leads to a circuit that will fail to meet the specifications.

To obtain a deeper understanding, students are required to investigate the relation of different design parameters from different aspects, visualize the dependency of circuit performance on varying different parameters, and comparing design simulations with built circuits.

In order to obtain a final circuit design that satisfies the requirements and meets the required functionality, students will be practicing the Kolb's Experimental Learning model [1] shown in figure 1. Students go through a ring process between mathematical derivations, parameter setups and calculations, circuit simulations, model testing, circuit building, signal

observation and measurements. All steps contribute to a better understanding of the theory and circuit analysis.

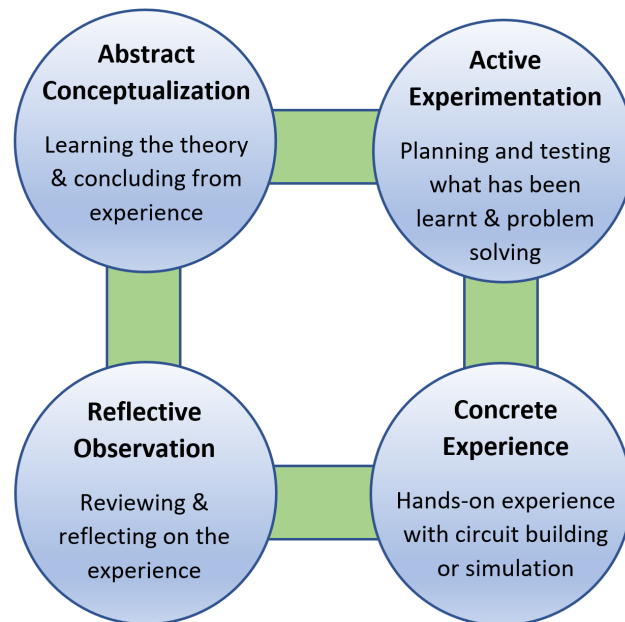


Fig.1 Summary of the Kolb's experimental model adapted from [1].

The learning lab-experience should apply the 3H approach integrating Head, Hand and Heart [2], [3]. The head, hands and heart concept is an organizing principle by which transdisciplinary study (head); practical skill sharing and development (hands); and translation of passion is integrated. They use their theoretical understanding and mathematical skills in setting the design parameters and restrictions, while using simulation tools, MATLAB modeling and circuit building skills. The assignment work targets their passion as engineers to complete the work successfully and be proud when the circuit operates well and meets the requirements.

This paper has a methods section, that describes a procedure for teaching students new electronic application circuits, and includes examples of commonly used circuits, such as Schmitt Trigger relaxation oscillators and 555 Monostable Multivibrator used for frequency division. The examples should describe how the learning method is implemented in class and lab for different application circuits. The paper then includes the MATLAB code example that is used to elaborate the effect of design parameters in the derived closed formulas. The evaluation section has reflective observation from the instructor as well as survey results as feedback from students.

### Methods:

When students are introduced to new concepts, it is useful to review the basic foundations that they need to build on the new applications. Figure 2 shows the learning steps used in the

electronics course. For a relaxation oscillator circuit, the output signal has main specifications such as amplitude, frequency  $f$ , and duty cycle  $D$ . In most of the textbooks, the saturation levels of the Schmitt Trigger comparator are given, the resistance values are supplied, and it is required to solve for the frequency and duty cycle of the output signal, or to vary the relaxation time constant to meet design specifications for the output signal frequency. We use these examples to sketch the output signal and use it to derive an expression for  $f$  and  $D$  as in step (c). In the lab, to develop critical thinking skills, students are asked to find a closed formula for the duty cycle and frequency after they have practiced the derivation process with numerical parameters. They are required to find restrictions on the resistance ratios to assure oscillation sustainability in step (e).

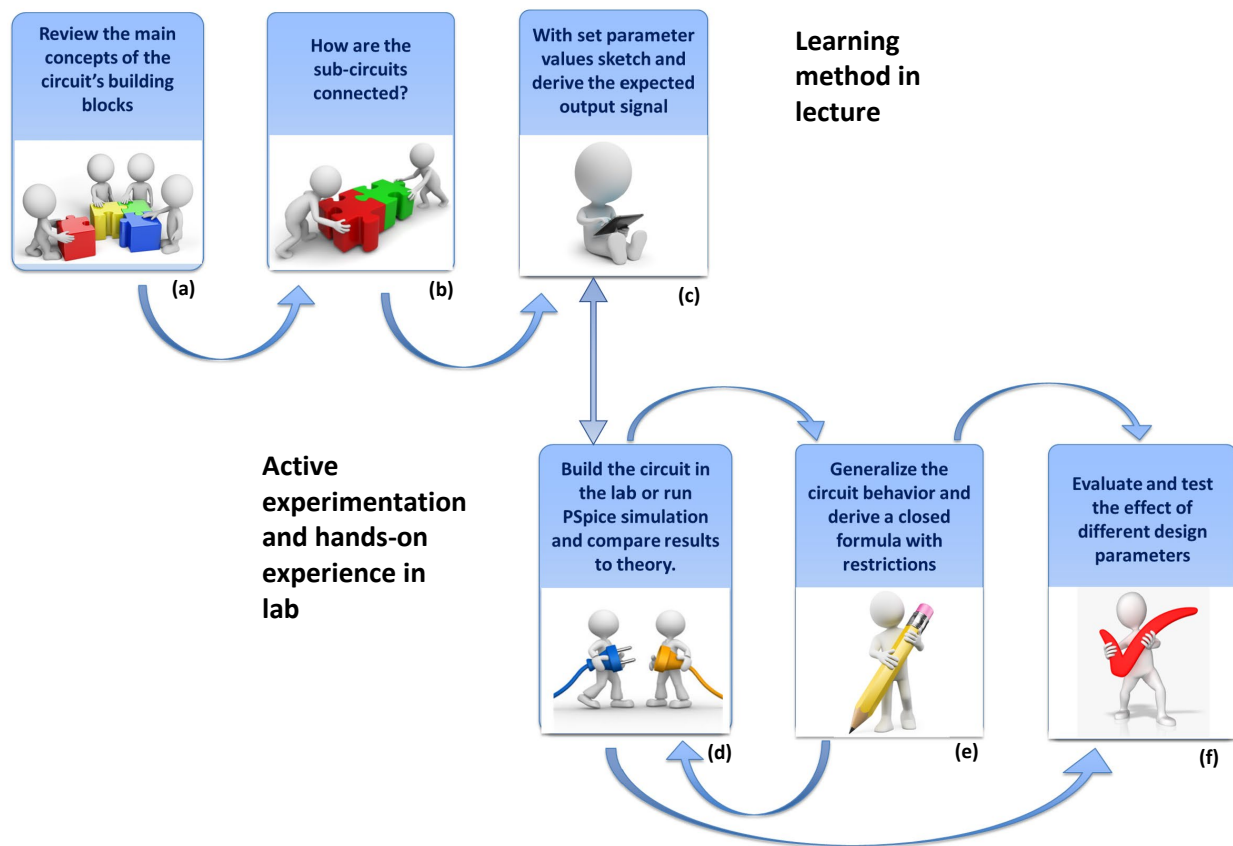


Fig.2. Steps (a) and (b) with the circuit layout explanation are completed in lecture. Step (c) is introduced in lecture and two example circuits are solved. Different design parameters are given, and the output signal is sketched and used for frequency and duty cycle calculations. Number of design variables is limited. Derivation steps are clear with numerical values. In the lab step (c) is completed by students with different setups compared to the lecture examples. Students reuse the solution strategies they gained in lecture. In step (d) they build the circuit and verify the signal with your sketched signal. This process is repeated for another setup and verified by circuit measurements. Step (e) and (d) cover generalization of the concept with a mathematical equation and more testing.

The following are lab examples are included to give a better understanding of the suggested learning process. The lab experiments cover an Inverting Schmitt Trigger Oscillator, a Non-inverting Schmitt Trigger oscillator with an integrator, and a monostable 555 multivibrator circuit used as a frequency divider.

### A. Inverting Schmitt-Trigger Relaxation Oscillator:

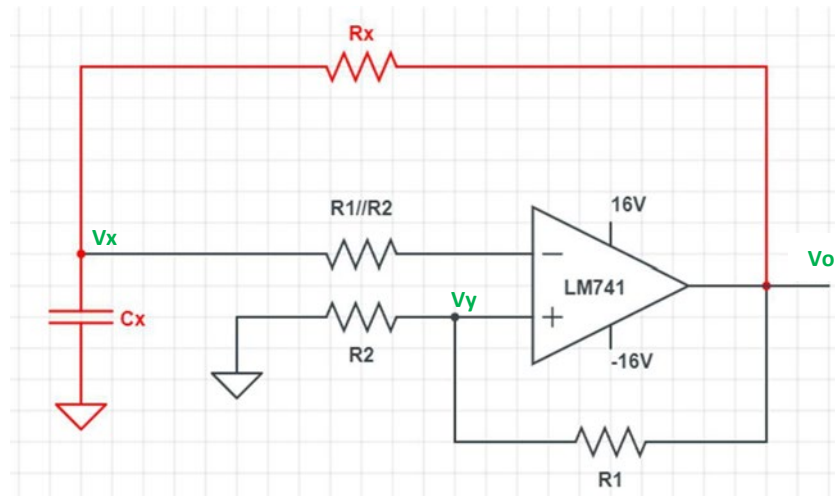


Fig.3 Inverting Schmitt Trigger Relaxation Oscillator. Schematics built using [www.CircuitLab.com](http://www.CircuitLab.com).

In lecture, the Schmitt Trigger comparator and the RC transient response are reviewed, and then the connection of the sub-circuits is explained. For given resistance values and saturation levels, the output signal is sketched based on the expected idea of operation. An expression for the charging time and the discharging time of the capacitor voltage is derived in terms of the time constant. Using the timing equations, the expressions for frequency and duty cycle are derived. The following procedures are listed in the lab handout following the method section.

- (1) Using  $R_1$  and  $R_2$  equal to  $1\text{K}\Omega$ , and the output saturation levels at  $\pm 16\text{V}$ , sketch the expected output signal, and find an expression for the frequency and the duty cycle in terms of the time constant  $R_x C_x$ . Does the duty cycle depend on the time constant?
- (2) Select the time constant parameters ( $R_x$  and  $C_x$ ) to obtain an output oscillation frequency of  $1\text{KHz}$  at  $50\%$  duty cycle.
- (3) Build the circuit on your breadboard using the design values selected in step 2. Observe  $v_o$ ,  $v_x$  and  $v_y$  on the oscilloscope, and in your report explain the oscillation operation.
- (4) Change  $R_2$  to  $20\text{K}\Omega$ , how does that affect the output signals. Support your explanation using mathematical equations. Does this change affect the duty cycle?

In this step, student should notice that the duty cycle for the circuit with symmetric saturation levels, is independent on the resistance values. This will be proven from the closed loop formula later.

- (5) Add voltage limiters at the output using Zener diodes to change the output voltage levels to  $+6\text{V}$  and  $-12\text{V}$ . What is the frequency and the duty cycle if we keep the same  $R_x$  and  $C_x$  values? Verify with circuit testing and measurements.

- (6) Sketch the general output signal. Clearly show what determines your output voltage levels, the limits of the  $V_y$  signal, and the oscillating capacitor voltage. Using your sketch derive a general closed formula for the frequency and duty cycle. Simplify your expression and verify that it is valid for the two cases you tested in the lab.
- (7) Modify the provided MATLAB code to apply graphical animation that elaborates the effect of changing the positive saturation level, the resistor values at symmetric and non-symmetric output saturation levels on the output signal.

```

%-----
% Studying the effect of R1/R2 at non-symmetric saturation levels
%-----
Rx = 1000;          % Rx = 1KOhm
Cx = 100*10^(-9); % Cx = 100nF
R2 = 10;           % R2 = 10KOhm
Vlow = -5;         % The lower output saturation level
Vhigh = 15;        % The upper output saturation level
tau = Rx*Cx;

for R1 = 1:1:100    % R1 tested over the range 0.1R2 to 10R2

    t1 = tau*log(1 + (R1/R2)*(1-Vlow/Vhigh)); % t1 = Charging time
    t2 = tau*log(1 + (R1/R2)*(1-Vhigh/Vlow)); % t2 = Discharging time
    f = 1/(t1 + t2); % f = frequency of the output signal
    T = 1/f; % T = period of the output signal
    t = 0:0.01*T:30*T; % t = time vector scaled by the signal period

    for i = 1 : length(t)
        n = floor(t(i)/T);
        if (t(i)-n*T)<=t1    vo(i) = Vhigh;    else    vo(i) = Vlow;    end
    end

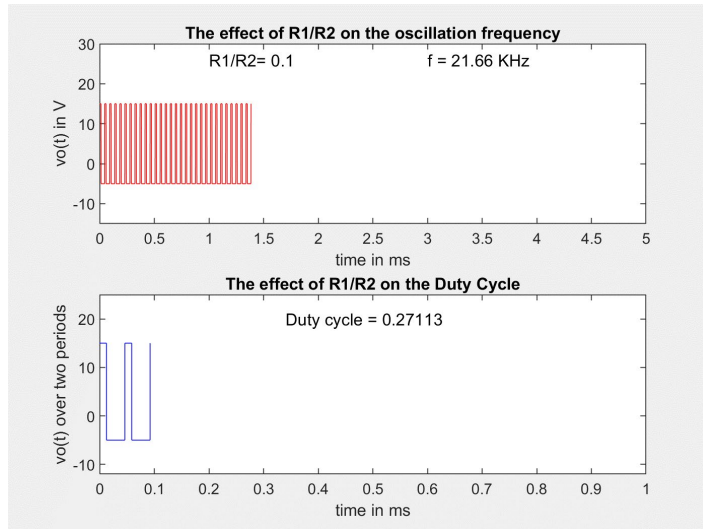
    plot(t*1000,vo,'r'); % plot the output voltage
    axis([0 5 -15 30]) % axis limits
    title('The effect of R1/R2 on the oscillation frequency');
    xlabel('time in ms'); % label('vo(t) in V');
    text(1,26,sprintf('R1/R2= %.1f',(R1/R2)))
    text(3,26,sprintf('f = %.2f KHz',f/1000))

    drawnow
    F(R1) = getframe(gcf);
    pause (0.01)
end

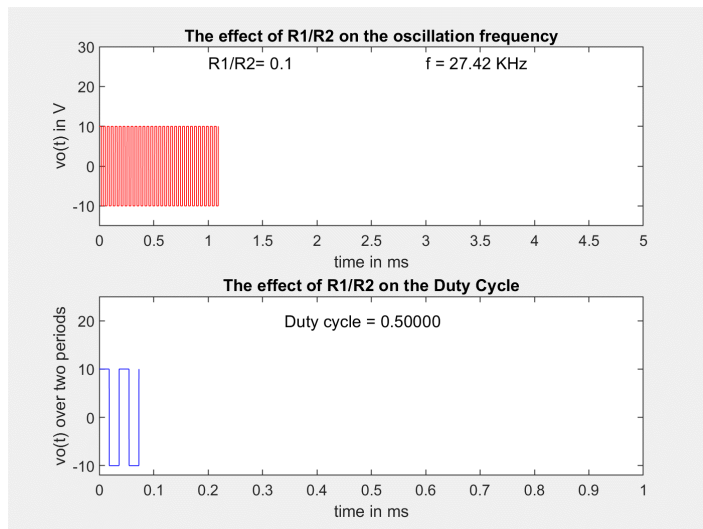
video = VideoWriter('R1_effect_SymmetricDCrail.avi','Uncompressed AVI');
video.FrameRate = 5;
open(video)
writeVideo(video,F)
close(video)

```

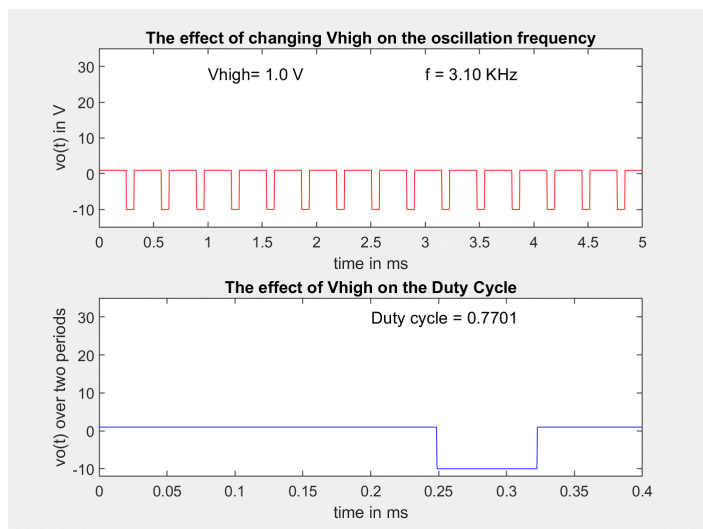
Fig. 4 MATLAB code showing the implementation of graphical animation to show the effect of changing  $R_1$  on the output signal of the Schmitt Trigger oscillator. The animation is saved as a video file.



(a) With non-symmetric output saturation levels of the Schmitt Trigger comparator, the duty cycle changes with varying R1/R2.



(b) With symmetric output saturation levels of the Schmitt Trigger comparator, the duty cycle is always 50%.



(c) Increasing Vhigh/Vlow ratio decreases the duty cycle and increases the oscillation frequency.

Fig.5 Screenshot of the animated graphs elaborating the effect of design parameters on the output signal.

B. Non-Inverting Schmitt Trigger oscillator:

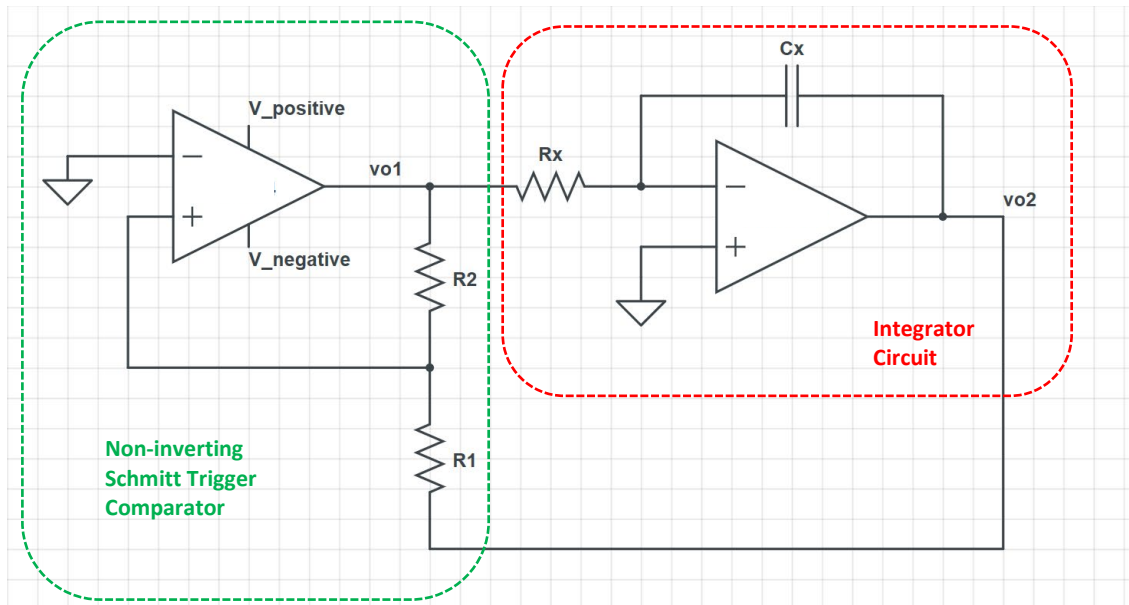


Fig.6 Noninverting Schmitt Trigger Oscillator. Schematics built using [www.CircuitLab.com](http://www.CircuitLab.com).

(1) Find an expression for the time  $t_1$  during which  $v_{o1}$  is high, and the time  $t_2$  at which the voltage  $v_{o2}$  is low. The expressions should be in terms of  $R_1$ ,  $R_2$ ,  $R_x$ ,  $C_x$  and the  $v_{o1}$  output saturation levels  $V_{high}$  and  $V_{low}$ .

(2) Prove that the duty cycle expression is:  $V_{low} / (V_{low} - V_{high})$

(3) Prove that the oscillation frequency expression is given by:

$$f = (-V_{low} * V_{high}) / ((R_x C_x)(R_1/R_2)(V_{low} - V_{high})^2)$$

(4) Based on the recommended supply voltage levels of the LM741, select  $V_{low}$  and  $V_{high}$  values that would generate a signal with 75% duty cycle.

(5) Ideally, what is the recommended range for the ratio  $R_1/R_2$  for the 75% duty cycle? Explain. Based on the understanding the idea of operation, the threshold voltages of the Schmitt trigger comparators must stay less than the output saturation levels of  $v_{o2}$ .

(6) Let  $R_1 = 1K\Omega$  and  $R_2 = 6K\Omega$  (2 series  $3K\Omega$  resistors), calculate  $R_x$  and  $C_x$  to obtain an oscillation frequency of 250Hz.

(7) Connect the circuit and verify that the output signals meet the design requirements. Observe  $v_{o1}$  and  $v_{o2}$  and explain the operation of the oscillator circuit. Modify your design to get a duty cycle of 25% at the same frequency 250Hz. Verify your design.

(8) Modify the circuit to obtain a duty cycle of 25% at the same frequency. Again, verify your design using simulation in PSpice or circuit building and measurements.



Using the MATLAB code that applies changes to one parameter over a *for* loop and the *drawnow* and *pause* commands, students can visualize mathematical operations, and clearly see the effect of one parameter on the output signal. For this Schmitt trigger and integrator circuit, and with the equations that students have proven for the charging and discharging times  $t_1$  and  $t_2$ , the output square wave and ramp waveforms can be constructed. Keeping the negative supply voltage  $V_{low}$  of the operational amplifier at -10V and gradually changing the value of the positive supply voltage  $V_{high}$  to values between 10V and 20V, can show the effect of the saturation voltage on the duty cycle and the amplitude of the output signal. Students need to also include the fact that if the lower threshold voltage of the Schmitt Trigger  $(-R_1/R_2)V_{high}$  passes the value of the  $V_{low}$ , the oscillation will fail. This must be added in the design restrictions and the validity check in the MATLAB animation graph.

Similarly, an animation graph can be created for the design value of  $(R_1/R_2)$ . This ratio controls the peak-to-peak value of the ramp function at the output of the integrator circuit. It also plays a role in sustaining the oscillation along with the saturation levels.

### C. Frequency Divider

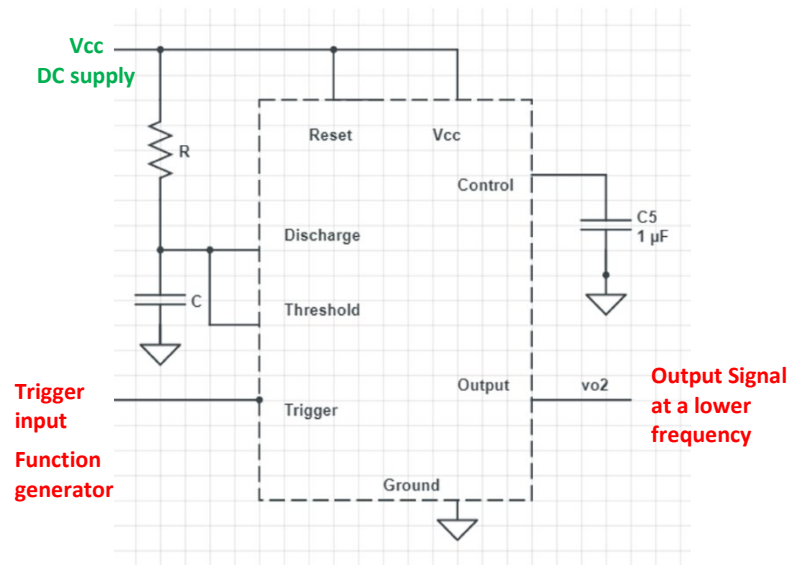


Fig.7 Monostable multivibrator using a 555 timer circuit.

- (1) Use a function generator to generate a square wave with  $f = 250\text{Hz}$  and a duty cycle of 75%. It is required to design a monostable circuit that would divide the signal frequency by 2. What is the range of possible duty cycle values of the output signal?
- (2) Design your circuit to obtain a 75% duty cycle at the output. Observe the input and the output signals on the oscilloscope and verify the frequency divider operation. Observe the output signal on Channel 1 on the oscilloscope and observe the voltage across the capacitor on channel 2. In your lab report, explain the monostable operation using an oscilloscope screenshot.

- (3) Use a function generator to generate a square wave with  $f = 200\text{Hz}$ . Design a frequency divider to divide the frequency by 2. The output signal should have a controllable duty cycle using a pot to obtain variable values between 60% and 80%. The duty cycle should not exceed 80% and it should not fall below 60%. Define your circuit parameters and select a suitable duty cycle for the input signal. Verify your design and explain your design specifications.

In this lab report, students need to think about the limitations controlling the duty cycle of the output signal. If the idea of operation of the monostable circuit is not well understood, it will not be possible for students to derive an expression that summarizes the limitations on the duty cycle of the output signal. If it is required to divide the frequency by a factor of  $n$ , then the pulse generated by the monostable multivibrator has to extend over at  $(n-1)$  times the period of the input signal and stay below  $n$  times  $T$ . This limitation dictates the range of the duty cycle that the output signal may have. The repeated excitation of the monostable multivibrator is the main idea of the frequency divider, but the design restrictions need to be clearly identified by students.

### Assessment and Evaluation:

The labs have been offered with the assignment to derive a closed formula and a statement for any design restrictions for the last 3 years. In fall 2020, the lab used PSpice simulation as a replacement for circuit building. The MATLAB animation was added in fall 2022.

Students were able to derive the closed formula for oscillation frequencies, and duty cycle. They were also able to identify design restrictions, either on their own, or with some instructor guidance. Building the circuit and comparing results with theory helped them correct mistakes either in the design or in the derivation. Students used similar strategies for subsequent labs.

A short survey was conducted in fall 2022 to get direct feedback from students.

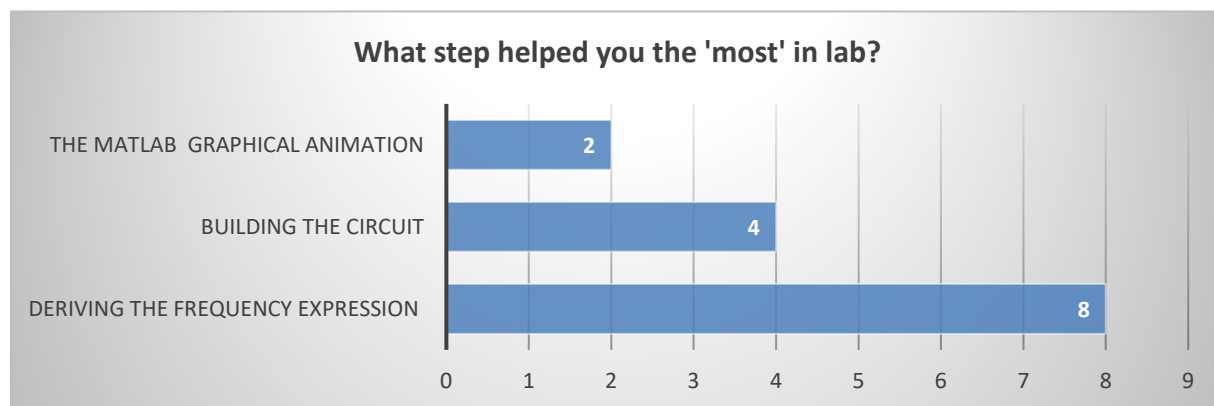


Fig.8 The results of a short survey question completed by 14 students in fall 2022.

The second question in the survey was an essay question: “ If you work in R&D, and you want to present a circuit design and its idea of operation, what approach would you use?”

The following answers were mentioned in their answers with the order of highest-to-lowest rate:

- MATLAB dynamic graphs in a presentation
- Step-by-step derivation of output specifications
- Tested data and output signals
- Simulated circuits
- A video with measurements and explanations

### **Conclusion:**

Having a lab offered for the electronics class is very helpful. Exploring different teaching steps accommodates the different learning styles for students. Being able to precisely explain circuit designs and define design restrictions and specifications is an important proficiency for practicing engineers. When students are encouraged to derive closed formula expressions, create simulation results, or investigate and explain design limitations, they start thinking about the theoretical concept of a circuit implementation from different perspectives. This idea was supported by the student survey outcome shown in figure 8. They realize the difference between a mathematical coefficient in an abstract equation and the physical meaning of a resistance value in a design formula. Adding the MATLAB animation task to the lab procedures may help them to better visualize the effect of different design values on an output signal. It also teaches them new MATLAB syntax that they might have not seen before in other courses, and it gives them presentation ideas that may help them later as practicing engineers and effectively sharing their work with others.

### **References:**

- [1] N. Brown, “Teaching reflective practice in teams: In-person and virtual activities,” in *2022 IEEE Global Engineering Education Conference (EDUCON)*, Tunis, Tunisia, Mar. 2022, pp. 419–422. doi: 10.1109/EDUCON52537.2022.9766457.
- [2] G. Gruhler and T. Fischer, “Learning electronics through head, heart and hands: An unconventional and holistic approach in engineering education,” in *2018 IEEE Global Engineering Education Conference (EDUCON)*, Tenerife, Apr. 2018, pp. 1147–1150. doi: 10.1109/EDUCON.2018.8363359.
- [3] Y. Sipos, B. Battisti, and K. Grimm, “Achieving transformative sustainability learning: engaging head, hands and heart,” *Int. J. Sustain. High. Educ.*, vol. 9, no. 1, pp. 68–86, Jan. 2008, doi: 10.1108/14676370810842193.