AC 2011-1472: DERIVING ORIGINAL SYSTEMS OF EQUATIONS AS AN ASSIGNMENT IN ENGINEERING AND TECHNOLOGY COURSES

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Deriving Original Systems of Equations

As an Assignment In Engineering and Technology Courses

I. ABSTRACT

Course assignments typically include a variety of library research papers, lab reports and homework. Rarely do these assignments call for deriving original mathematical equations. Yet such derivations are a critical component of creating new systems and concepts. The development of this vital skill set is afforded minimal time in undergraduate coursework. Instead, students are encouraged to memorize or mimic a derivation from the literature. An example of this is the derivation of the equation for work by a spring. This derivation is found by combining Newton's Second Law of Motion and Hooke's Law. In this paper, we describe the authors' approach to adding original derivation assignments to the curriculum of engineering and technology courses in order to ensure the genesis of this creative skill set at the undergraduate level. The goal is to develop in undergraduate students learning patterns that will facilitate the ability to write for any system, a set of equations that describes the system.

II. INTRODUCTION

Mathematical modeling entails finding a series of steps that define all the relationships in a system. An example of a system is an energy system, a power system, an electronic circuit, a manufacturing process or a cancer cell. Each of these systems is an ongoing subject for mathematical modeling.[1-4] Students can use a similar equation pattern and apply it to any of these systems.

Each step in the model is derived from what is known empirically (observed data) and what is known theoretically (the laws and accepted equations).[3] When all the steps are assumed to be found, a verification and possible confirmation of a complete set occurs. The model accounts for all the observed data and all the known laws and equations. If any data is unexplained, then all the relationships have not been found or included and more are needed for confirmation. After the completeness is validated, the model is revised to remove any redundancies. The final model contains the simplest relationships and rules that govern the system.[1] The model is designed to be applicable to all systems in a wider class of systems.

Mathematical modeling does not replace other engineering methods. Rather, it assists in making the methods of designing systems general enough so that that the process can be applied to the wider class of systems. The goal of modeling is to find the simple rules that apply to this wider class. Any equations found should agree with what is already known. The model is based on equations that define the principles and guiding relationships and parameters.

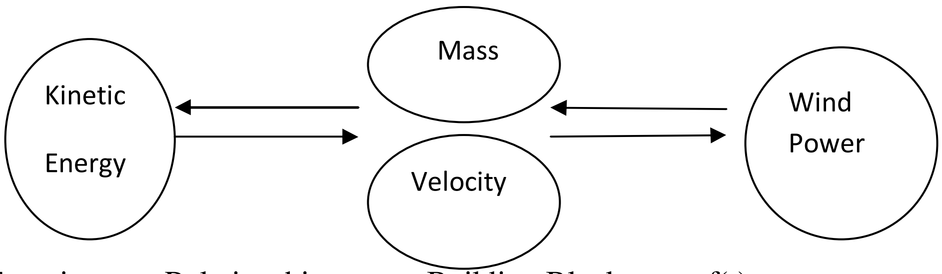
III. THE MODEL

When engineers create schematics, a high level of detail and component specificity is inclusive in the creation or design. The designs are specific for the requirements as outlined by the specifications. Engineers follow the rules and structure that are known and practiced through education and experience. Engineers do not always try to find the more general design principles nor do they try to define the general rules. Engineers are given the input and output as a starting point and they design a system that can receive the input and produce the output. Often, the input can be received and the output can be produced in several different ways. What is needed is a more general design that is more efficient, easier to build, and more cost effective and can be applied to a whole general class of systems. Mathematical modeling helps in achieving this goal.

In the authors' course in mechatronics, the students were given assignments to derive the mathematical equations for an energy system. Examples of the energy systems addressed in the course were wind, solar, thermal, hydrothermal, nuclear, and fuel cell systems. The student assignment was to describe the energy system in equations. In particular, the student was to account for the power production from the system. The equations define the process inside the system from input to output. Below is a summary of the derivations resultant of the class assignments.

To derive an equation for a mechatronics system, three natural starting points are the input, the transformation and the output. If the input is a signal, then the first and subsequent transformations are called signal conditioning. Another task is to itemize the boundary conditions. These boundary conditions include the range for the input values and the range for the output values, the value of the zero point, the duration in time and frequency of the input and output, and known maxima and minima for intermediate values.

An example of a mathematical model is the power of the wind hitting a wind mill.[5, 6] The overall relationship in this case is the equation for the kinetic energy. Kinetic energy depends on the mass and the velocity. Wind depends on air. Air has mass and density with density being the amount of mass in a unit volume. The blades of the wind mill have area that is hit by the air molecules. Power is the change in energy per unit of time.



Equation ← Relationships ← Building Blocks → f(t) → output

f(t) is the operator transformations

Figure 1. Model Relationships and Building Blocks

For the wind mill, the governing equation is for the kinetic energy:

kinetic energy = $(1/2)\, m * v^2$ in which the

velocity = air speed

mass = velocity * area * density * time

The starting point for the derivation is the building blocks. The building blocks in the above example are the mass of air hitting the blade of the wind turbine. The velocity or wind speed and the direction of the wind is another building block. In the diagram above, the relationship between these building blocks leads from the center left to the relationships between them and the known equation for kinetic energy. To the right from the building blocks leads to the transformations through the operators that act on the building blocks over time to produce the final output.

IV. RESULTS

Students were given the following instructions in the assignments

- **Derivation of Equations for Power Productio**n: These equations should use letters with numbers as coefficients. All symbols should be explained. The equations should cite known principles and lead to the equations utilized in the calculation. Explain each equation and cite the source. The final equations(s) should be utilized in the calculation.
- **Calculation of Power/Energy Production**: State the given starting conditions. These should correspond to the variable in the final equations(s) from the derivation and the starting equations(s) for the calculations.

The following derivation process was outlined for the students to follow.

TABLE I

DERIVATION PROCESS

Identify the system
Collect data about the system
Identify the building blocks
Identify the relationships between the building blocks
Identify the transformation operators
List the transformation steps
Order the steps from input to output
Form all the connections between the steps

Match the steps to the data
Match the steps to equations
Remove redundancies
Remove inconsistencies
Validate model on known and unknown systems

The students were told to identify the following types of building blocks:

TABLE II

IDENTIFICATION OF BUILDING BLOCKS

Identify: The boundaries of the system all inputs to the system all outputs from the system all data known about the system all parameters (variables) measured by the data all connections between the parameters all defined relationships between the parameters all known equations that define the relationships

Below summarizes the student equation derivation for three energy systems: fuel cell, nuclear reactor,and hydroelectric.

Student Equation Derivation:

Fuel Cell Relationships ⟶ Equations:

- Fuel required for required output
- Gas volume at operating temperature
- Resulting Natural gas input over time
- Conversion rate of Natural Gas to Electricity
- Power Efficiency % = Power out/Fuel in

Nuclear Reactor Relationships ⟶ Equations:

- Nuclear Fuel for required output
- Nuclear Fuel volume ⟶ fissionable products plus energy over time

- Resulting heat output over time and conversion to steam
- Conversion rate of Steam to Electricity
- Power Efficiency % = Power out/Fuel in

Hydroelectric System Relationships ⟶ Equations

- Potential Energy (falling water) for required output
- Volume of falling water over time
- Resulting heat output over time and conversion to steam
- Conversion rate of Steam to Electricity
- Power Efficiency % = Power out/Fuel in

When the student completes all the steps and includes all relationships and equations, the evaluation process begins. The evaluations are shown in Table III.

Table III

DERIVATION EVALUATION

1. All data
A set of observed test data is available.
2. Building blocks:
Based on the data, all building blocks are included.
3. Connections:
All connections and constraints are included.
4. Relationships:
Confirm all known relationships are included.
5. The order:
Confirm the order of the steps is correct based on the observed intermediate steps observed.
6. Redundancies and Inconsistencies
No redundancies or inconsistencies exist.
7. Simulate:
Test model and show it can receive input data and Process the intermediary data to produce the output.
8. Matches the data, input and output:
Simulation of the model with input data produces intermediary data and output as observed.

Each energy system has different building blocks and relationships (connections). However, similar patterns can be outlined as shown in the table below. Table IV represents an encapsulation of the student derivations for the five types of energy systems: wind, solar,thermal, hydroelectric, nuclear, and fuel cell.

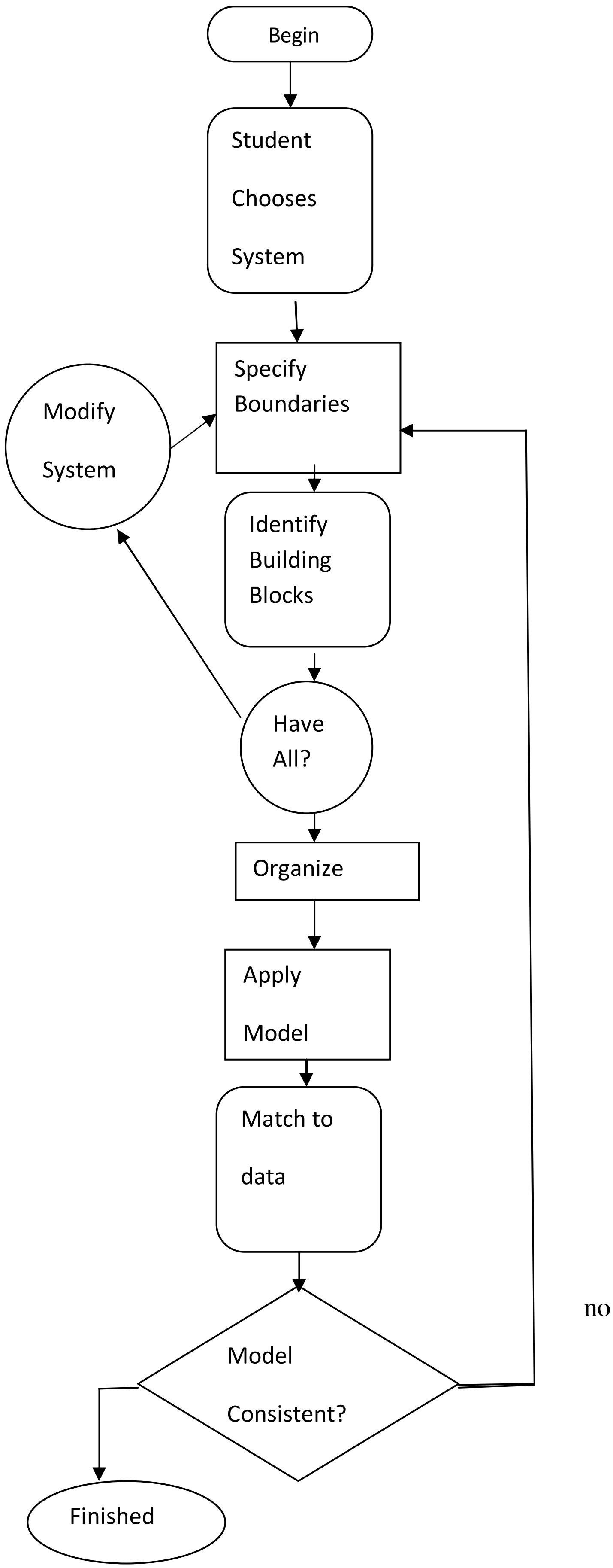
TABLE IV

SYSTEMS

System	Inputs	Outputs	Data	Connections	Laws
Wind Mill	wind	Power	Wind speed Power output solution.	Area of blade Density of air Air speed Direction	KE = .5 * M * v
Solar Cell[7] or Solar Thermal	Solar energy	Power	Sunlight per area and time	Efficiency of PV cell, transmission of heat to water.	Power = light/area x electricity conversion/light / time
Hydrothermal	Falling Water	Power	Height of dam Rate of water flow Mass of water per time	Changing water flow rates	P.E. = mgh
Nuclear[8,9] Reactor	Radioactive fuel	Power	Mass of fuel Heat Efficiency	Changing mass & changing energy	$E = M*C^2$
Fuel Cell	O2 H2 Similar reactants	Power	Natural gas Efficiency	Conversion of natural gas to electricity	$Work_{electrical}$ = charge x voltage

The flow diagram for the equation derivation process is illustrated in Figure 2.

Figure 2. Derivation Flow :



V. CONCLUSION

Results indicated that students can successfully derive equations and construct a mathematical model for an energy system that can be applied to a wide range of systems and thusly can be extended to any type of system including electronic circuits, computers, cancer cells, and manufacturing processes. By finding the simple relationships and laws that govern systems, it leads to innovations, new concepts, and better products. Undergraduate engineering students who become adept at making mathematical models through derivation will be able to conceptualize and create new and more efficient products and designs.

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