Design and Comparison of Various Controllers for a Two-Tank Liquid-Level System

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Abstract

In this paper, we studied various methods of control system design for a two-tank liquid level process. A mathematical model for the system in terms of a set of differential equations is derived and the system parameters are allowed to vary within 50% from their nominal values during operation. Proportional Derivative (PD), Proportional, Derivative and Integral (PID), State-Feedback, and State Feedback with Integral Control actions are designed using the principles of control theory. Comparisons between performances of controllers are made. Robustness of controllers is also studied. MATLAB and Simulink are used for analysis and design of controllers as well as simulations. Based on the system parameters and their variations, 16 extreme cases are determined along with a nominal case. Simulations are performed for all extreme cases with a nominal case.

Introduction

In the design of controllers, there are many areas that must be looked at to insure that the desired response is obtained from the system. Since our system dealt with the changing of volume at any given instant in time; we had to have a thorough understanding of fluid transport phenomena. We began with the modeling step since it is the most crucial part of control design. The modeling process begins with linking the instantaneous height in a governing tank to the volume change with respect to time; better known as volumetric flow rate. Once the dynamic equations of the two fluid heights were complete, the dynamics of the proportional control valve could be constructed. By generating the dynamics of the system we were able to represent the motion by a system of first-order ordinary differential equations, known as state-space representation.

An important concept that should be inspected is the controllability of a system. A system is completely output controllable if the construction of an unconstrained control vector will transfer any given initial state to any final state within a finite time interval. By having prior knowledge that a system is controllable and/or observable signifies that the systems design process will not result in an unsatisfactory manner.

Mathematical Model

The discussions of compensators in cascade with the plant and feedback stream were combined to determine the desired output. Here we will begin with a brief discussion of a control scheme that utilizes a frequency domain method known as state-space controller design. In this scheme additional poles and zeros are added to the system that modifies the response strongly. This design process makes great use of specific poles in second-order systems. The addition of an integral gain to the PD-controller will drive the error to zero and a gain value will be generated from the state matrix gain assembly. To clearly adopt this pole-placement design procedure a mandatory requirement must be taken into consideration; that is all state variables must be known or estimated for successful feedback control. If any state cannot be measured, observers can be introduced into the system to estimate the uncertain states [1, 2, 3].

A supplementary technique, which takes the performance criteria into consideration, is based upon the settling time and overshoot of the system. The performance criteria employed for leveling the system were fixed to a settling time of six seconds (6.0 sec) with a maximum overshoot of five percent (5.0 %). This technique is commonly used to reshape the root locus of the system to pass through a desired pole location that will generate the desired output of the system [4].

The addition of a zero in the open-loop transfer function is utilized to speed the response of the system. This procedure is recognized as PD-control *(Proportional* plus *Derivative).* However, this method may not reveal the desired output at all times, but this method is recommended before further control design is implemented. The procedure of this conceptual design process will give a better understanding of the root-locus analysis and dynamics of the system. It is also considered a strong base for the up coming approach in controller design.

In most systems, steady-state error can be reduced by introducing a pole into the denominator of the open-loop transfer function, which is recognized as the characteristic equation of the system. This addition of a pole essentially drives the steady-state error to zero. By simply combining the PD- controller design with the addition of a pole, the completion of what is know as PID- control *(Proportional* plus *Integral* plus *Derivative)* will be established, not only with the performance criteria embedded, but with the steady-state error driven to zero as well.

Neglecting the nonlinearities [5, 6], a mathematical model of a two-tank liquid-level system with actuator dynamics produces a third order linear differential equation. The system is depicted in Figure (1) and parameters are defined next:

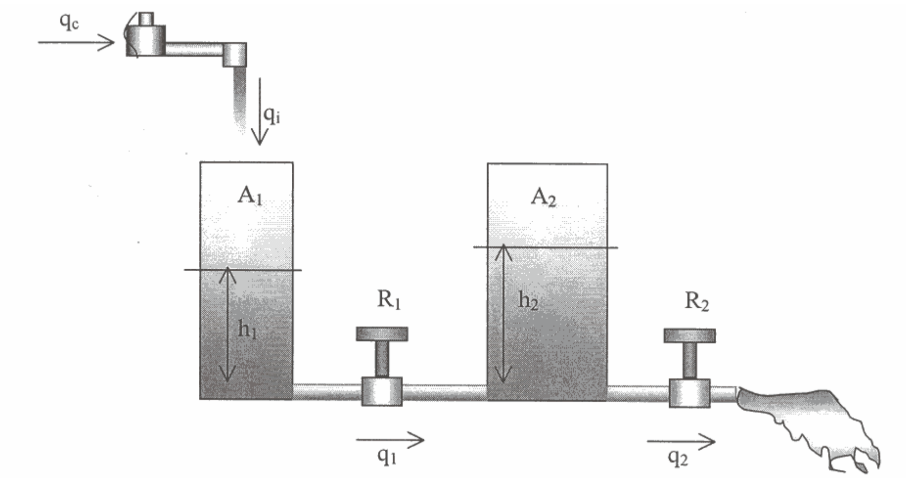

Figure 1: A two-tank liquid-level system.

Nomenclature for the system:

A_1 - Cross sectional area of tank1.
A_2 - Cross sectional area of tank 2.
q_c - Inflow to proportional valve controller.
q_i - Inflow to tank 1.
q_1 - Inflow to tank 2.
q_2 - Outflow of tank2.
R_1 - Valve resistance of valve 1.
R_2 - Valve resistance of valve 2.
τ_1 - Time constant, A_1R_1.
τ_2 - Time constant, A_2R_2.
τ_3 - Time constant, A_2R_1.
T - Actuator time constant.
V_1 - Volume of fluid of tank 1.
V_2 - Volume of fluid of tank 2.
h_1 - height of fluid in tank 1.
h_2 - height of fluid in tank 2.

In order to accommodate for the changing fluid level we must differentiate the volume of the first tank with respect to time.

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Texas A & M University-Corpus Christi

$$A_1 \frac{d}{dt} h_1(t) = q_i(t) - q_1(t) \quad (1)$$

$$q_1(t) = \frac{1}{R_1}\left(h_1(t) - h_2(t)\right) \quad (2)$$

The same would than be applied to the second storage facility which is governing to an outlet.

$$A_2 \frac{d}{dt} h_2(t) = q_1(t) - q_2(t) \quad (3)$$

$$q_2(t) = \frac{h_2(t)}{R_2} \quad (4)$$

$$\frac{dh_1(t)}{dt} = \dot{h}_1 = \frac{q_i}{A_1} - \frac{h_1(t)}{A_1 R_1} + \frac{h_2(t)}{A_1 R_1} \qquad \frac{dh_2(t)}{dt} = \dot{h}_2 = \frac{h_1(t)}{A_2 R_1} - \frac{h_2(t)}{A_2 R_1} - \frac{h_2(t)}{A_2 R_2} \quad (5), (6)$$

The actuator dynamics would be given by,

$$q_c = T \cdot \dot{q}_i + q_i$$

Defining the states of the system as,

$$\begin{aligned} x_1 &= h_2 & \dot{x}_1 &= \dot{h}_2 & \tau_1 &= A_1 R_1 \\ x_2 &= \dot{h}_2 & \dot{x}_2 &= \ddot{h}_2 & \tau_2 &= A_2 R_2 \\ x_3 &= q_i & \dot{x}_3 &= \dot{q}_i & \tau_3 &= A_2 R_1 \end{aligned}$$

Using MATLAB, the state-space representation of the system can be obtained as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{\tau_1 \cdot \tau_2} & -\left(\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3}\right) & \frac{1}{A_1 \cdot \tau_3} \\ 0 & 0 & -\frac{1}{T} \end{bmatrix} \cdot \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T} \end{bmatrix} \cdot q_c$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

From the above representation and use of MATLAB, transfer function for the system would be obtained as,

$$G(s) = \frac{\tau_1 \cdot \tau_2}{A_1 \cdot (s \cdot T + 1) \cdot (\tau_1 \cdot \tau_3 \cdot \tau_2 \cdot s^2 + (\tau_1 \cdot (\tau_3 + \tau_2) + \tau_3 \cdot \tau_2) \cdot s + \tau_3)}$$

Nominal and Extreme Values of the Plant

The plant parameters are assumed to vary with their lower and upper bounds for testing robustness of each controller designed. These values are given in the following tables.

	Nominal Value	Upper Limit	Lower Limit
A_1 (m^2)	.0285	.0428	.0143
A_2 (m^2)	.0457	.0686	.0229
R_1 (sec/m^2)	50	75	25
R_2 (sec/m^2)	50	75	25
T (sec)	1	1	1

Table 1. Nominal, Upper, and Lower Limits of uncertain parameters

Upper and lower limit

	a_1	a_2	R_1	R_2
$f_1(s)$	max	max	max	max
$f_2(s)$	max	max	max	min
$f_3(s)$	max	max	min	max
$f_4(s)$	max	max	min	min
$f_5(s)$	max	min	max	max
$f_6(s)$	max	min	min	max
$f_7(s)$	max	min	min	min
$f_8(s)$	max	min	max	min
$f_9(s)$	min	max	max	max
$f_{10}(s)$	min	max	max	min
$f_{11}(s)$	min	max	min	max
$f_{12}(s)$	min	max	min	min
$f_{13}(s)$	min	min	max	max
$f_{14}(s)$	min	min	min	max
$f_{15}(s)$	min	min	min	min
$f_{16}(s)$	min	min	max	min

Actual values of upper and lower limits

a_1	a_2	R_1	R_2
.043	.069	75	75
.043	.069	75	25
.043	.069	25	75
.043	.069	25	25
.043	.023	75	75
.043	.023	75	25
.043	.023	25	75
.043	.023	25	25
.014	.069	75	75
.014	.069	75	25
.014	.069	25	75
.014	.069	25	25
.014	.023	75	75
.014	.023	75	25
.014	.023	25	75
.014	.023	25	25

Table 3. Table of 16 extreme cases.

Open-Loop Simulation

To compare the controlled system response to uncontrolled system response, it is helpful to perform the simulations in open-loop via MATLAB and Simulink. In an open loop system, the output is neither measured nor fed-back for comparison with the input command and as a result, the output of the system has no effect on the control action.

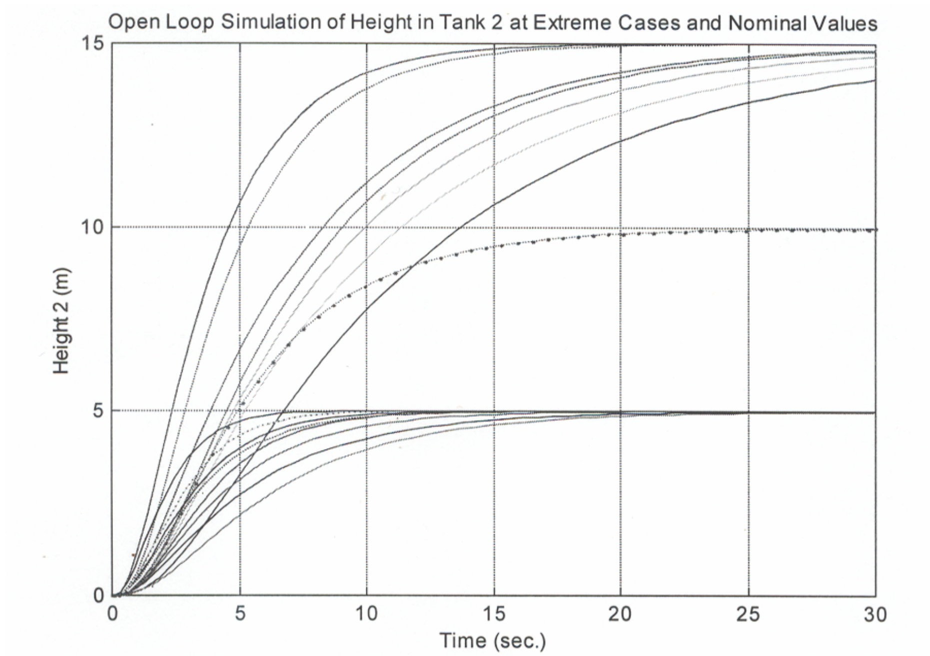

Figure 2: Responses of an open-loop system for a unit-step input for 16 extreme cases

Physically, the time response of the system is not acceptable since we require a settling time of the desired height, h2, to be approximately 5-10 seconds. The height of water in both reservoirs also exceeds the maximum heights of the tanks at both the nominal and extreme values. As shown in figure 2, the height of the second tank, h2, fluctuates between 5 and 15 meters as the plant parameters change. By the design criteria, the maximum height in both tanks is 24 inches, or approximately .6 meters. In reality, the water cannot fill any of the tanks past 24 inches or else the liquid will start to overflow and create undesired effects. The open loop system does not correct for any difference or measurement of h2 and is simply commanded by the step input of .2 m3/s. Since no signal is generated for the actuator to regulate any liquid, the input flow does not change. No control of the height in the second tank is possible since no measurement is taken into account by any controller. If we wish to maintain a desired height of 20 centimeters in tank 2, measurements of h2 need to taken and compared to the desired height of 20 cm. The controller then needs to produce a commanded flow to compensate for the difference between the actual height and the desired height to maintain the level in tank 2. For the next simulation, feedback will be introduced in attempt to maintain a desired height in tank 2.

PD Control

As a first and simple control action, PD control is used. Using root-locus design and MATLAB Simulink, A PD controller is obtained with the following transfer function [7, 8]

$$G_{PD}(s)=(s+1.059)$$

Closed-loop system transfer function with PD control will then be

$$G_{COMP} = G(s)_{NOM} \cdot G(s)_{PD} = \frac{(s+1.059)\cdot 15.36}{s^3 + 2.58 \cdot s^2 + 1.88 \cdot s + .31}$$

It was observed that the PD control action was not able to produce desired response for all the extreme values of the system. The following figure shows only two cases that the PD controller was able to control the system. It seems that for each set of parameters new PD controller needs to be designed.

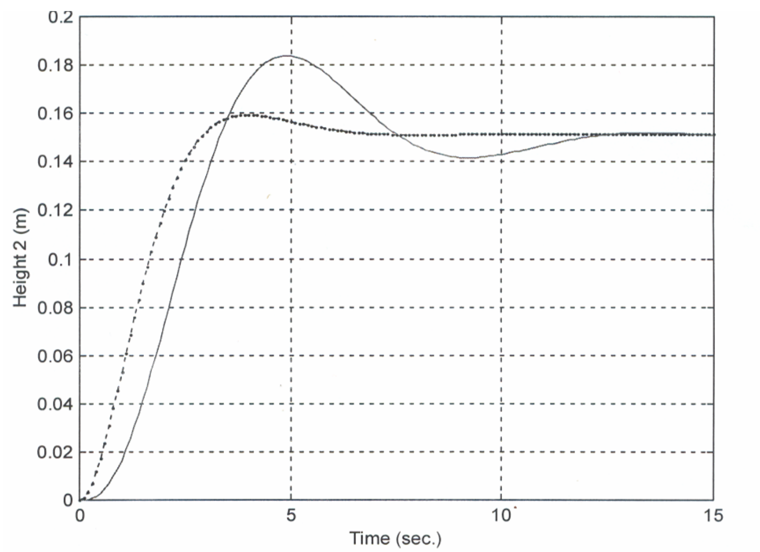

Figure 3: Height in Tank 2 using a PD-Controller

PID Control

The steady state error of the liquid-leveling system can be reduced adding an additional pole that requires an integrator to the forward path. This increases the system type and drives the associated steady state error to zero. The PD-controller is designed to meet the transient response and with the new integrator added to the controller, the steady state error will be zero [9, 10].

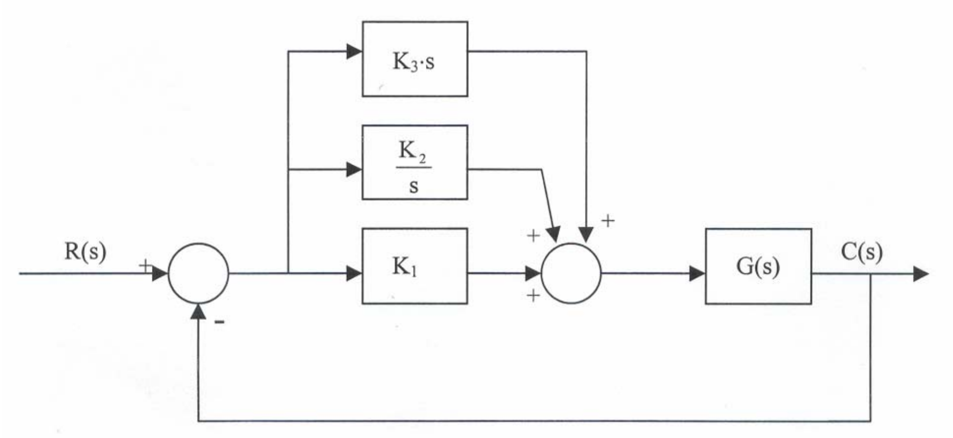

Figure 4: Closed-loop system block diagram with PID Controller

Transfer function of a PID controller is given by

$$G_{PID}(s) = \frac{K \cdot (s + 1.059) \cdot (s + .2)}{s} = \frac{.056 \cdot (s + 1.059) \cdot (s + .2)}{s} = \frac{.056 \cdot s^2 + .071 \cdot s + .012}{s}$$

Then the closed-loop system transfer function with PID controller is given by

$$G_{SYS}(s) = G_{PID}(s) \cdot G_{NOM}(s) = \frac{K \cdot (s + 1.059) \cdot (s + .2)}{s \cdot (s^3 + 2.557 \cdot s^2 + 1.884 \cdot s + .307)}$$

From the previous MATLAB Simulink simulations, it is evident that the steady state error for each extreme case is zero. The desired height in tank 2 can be reached despite any change in the system parameters within +/- 50%; however, our performance

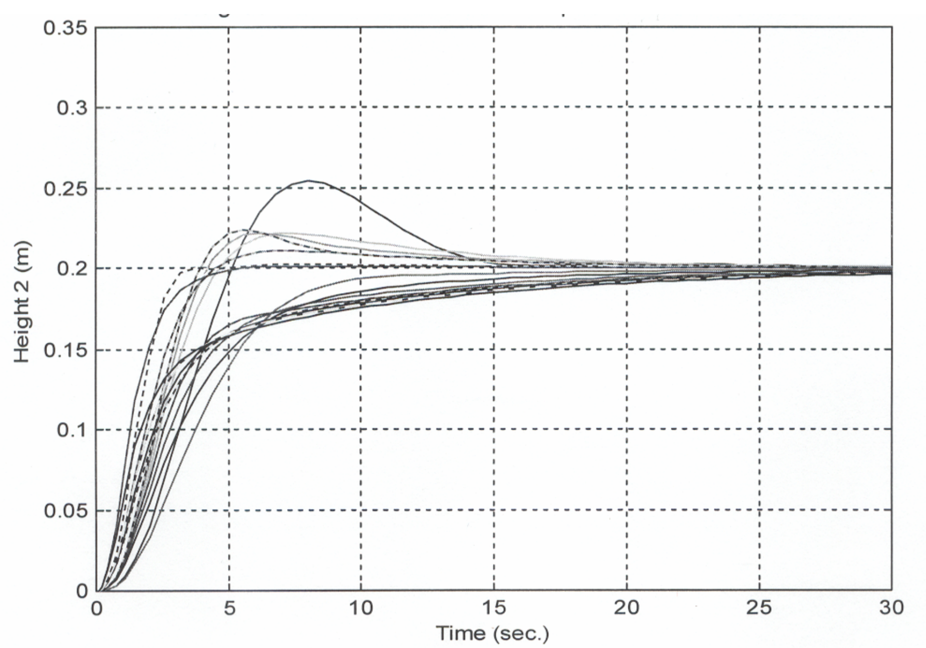

Figure 5: Closed-loop system response with PID controller for 16 extreme cases

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specifications are only half met. It is required that the height in tank 2 reaches its steady state value (settling time) within 4-7 seconds. In figure 5, the settling time varies from approximately 4 to 30 seconds depending on which extreme values the plant parameters are set to. Since the desired response of the extreme cases is unattainable for any PID controller, other control methods need to be considered to meet the desired performance specifications.

State Feedback (Pole Placement)

Pole-placement specifies all closed loop poles of a system, which are based on the performance specifications: settling time and percent overshoot. By a series of matrix calculations, in MATLAB, a gain matrix can be chosen to force the system to have the closed loop poles at the desired pole locations. However this method requires that all state variables must be available for feedback. If any of the states are not available for feedback, observers need to be introduced into the controller to estimate or observe the unknown state variables. Based on performance specifications, desired poles are determined to be placed at s_1=0.6665+0.699i, s_2=0.6665-0.699i, and s_3=20. The block diagram of the system is given as follows:

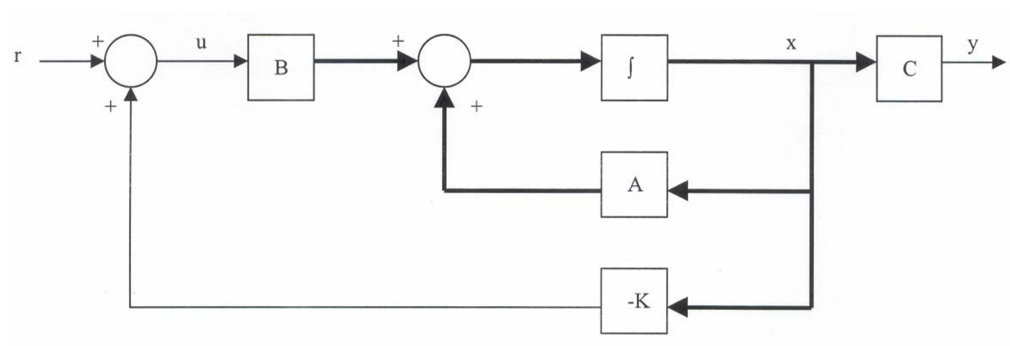

Figure 6: Plant with State Feedback

The Ackermann’s formula is used [11, 12] to determine the state feedback gain matrix with the following numerical values:

$$\mathbf{J} = \begin{bmatrix} .66654 + .699i & 0 & 0 \\ 0 & .66654 - .699i & 0 \\ 0 & 0 & -20 \end{bmatrix}$$

Simulation results are seen in figure 7. As expected, the simulation of the nominal case, figure 7 reveals that the settling time is approximately six seconds and the percent overshoot is 5%. This type of response is expected because the gain matrix is forcing the system to have the closed loop poles at the desired locations. However, the steady state error of the second tank is approximately 3.5 centimeters and is not acceptable. The solution to this problem will be handled in the next section.

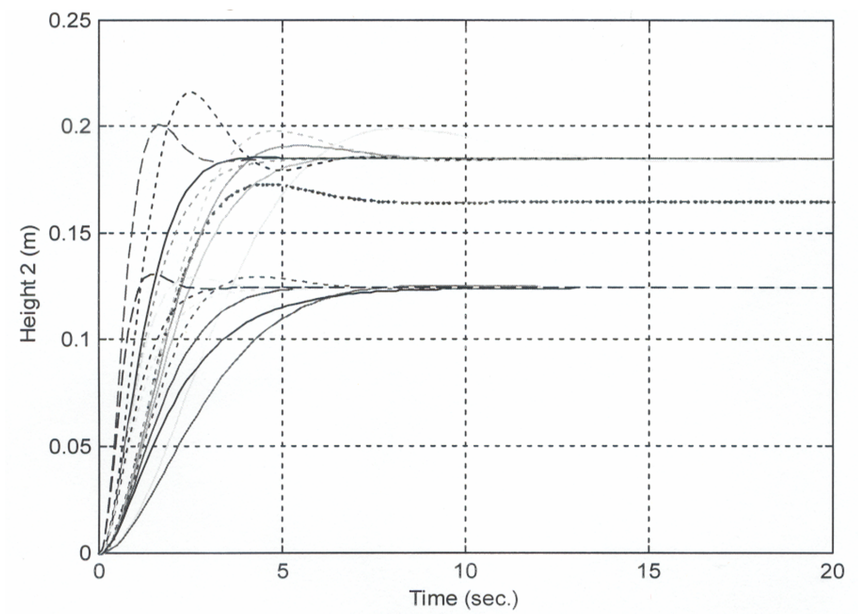

Figure 7: Simulation of plant with state-feedback for nominal and extreme cases.

State Feedback with Integral Control

With the PID controller design, an integrator was inserted in the feed-forward path of the error comparator and the plant to reduce the steady state. This insertion increases the system type from a Type 0 plant to a Type I plant and reduces finite error to zero. The same technique can be applied in the state space form to drive the height of tank 2 to the desired height as shown in figure 8. Consequently, we will design a state-feedback controller for a steady state error of zero and the desired response for our two-tank system.

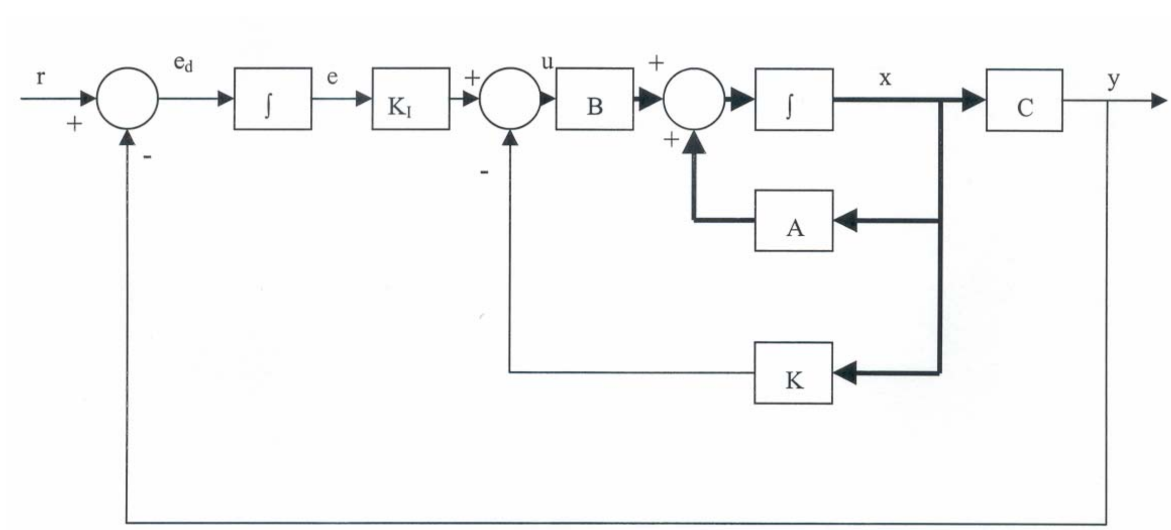

Figure 8: Block Diagram of Plant and State Feedback with Integral Control

A modified Ackermann's equation [11, 12] has been applied to determine the gain matrix with the following numerical values.

$$\mathbf{J} = \begin{bmatrix} 66654 + .699i & 0 & 0 & 0 \\ 0 & .66654 - .699i & 0 & 0 \\ 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & -20 \end{bmatrix}$$

The MATLAB Simulink simulations of the state-feedback controller with the integral gain have produced acceptable responses of our liquid level system. The controller is designed for the nominal case but has also reduced the steady-state error to zero for the parameter variations of the plant. In all extreme cases, the final value of the height of tank 2 has reached the desired level of 20 centimeters. The peak time and settling time of the system is also favorable since most of the settling times are approximately 6-7 seconds with a percent overshoot of about 5%. This type of response is expected since the controller is forcing the plant to have its closed loop poles at the desired locations. If the desired height of the second tank was 50 centimeters, the settling time will still be approximately 6 seconds and the steady state error will also be zero. However, the flow rate coming into tank 1 will have to increase due to the increase in the desired height. Again, the controller is driving the plant to have the specified poles. This is shown in the following figure.

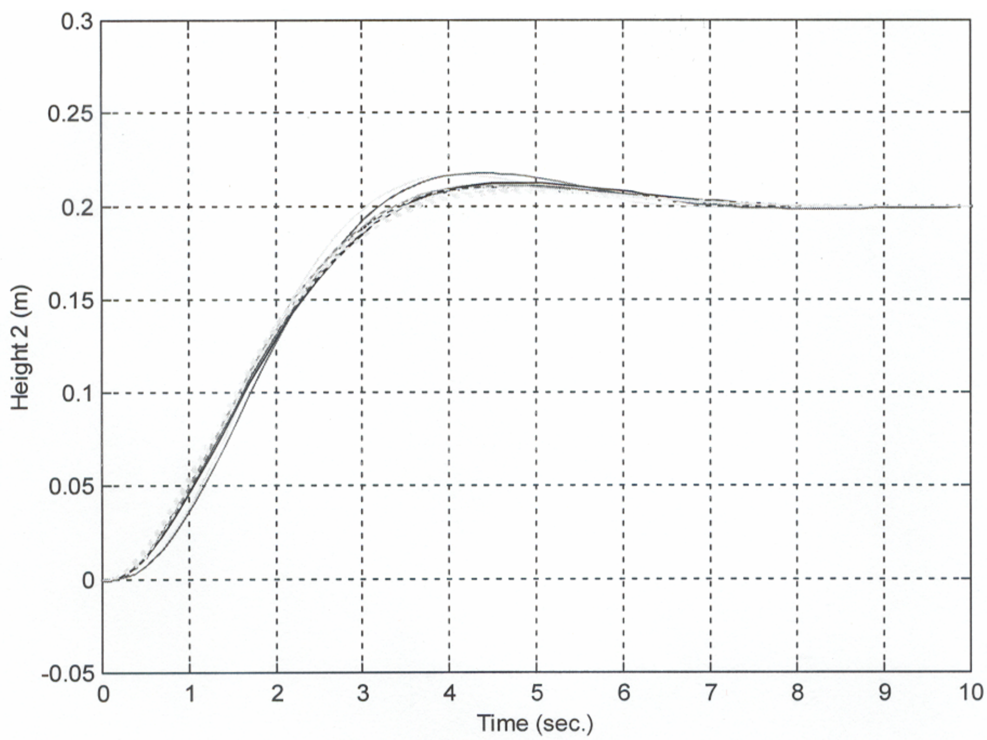

Figure 9: Responses of plant and state feedback with integral control for nominal and extreme cases.

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Conclusions

Several controllers have been designed for the liquid leveling tank in an attempt to maintain a desired height in tank 2 despite variations in plant parameters. These controllers were designed using several methods in both the frequency domain and in state space representation. In the frequency domain, various controllers were proposed to achieve the desired response of the system by assigning a pole location of only the dominant pole. Other poles were neglected since the higher order poles do not affect the response. The PID controller was well suited to reach the desired response for the nominal values of the plant parameters in the frequency domain. However, the extreme values of the plant were simulated using MATLAB Simulink with this PID controller and produced undesired responses. The settling time of the extreme cases ranged form 15 to 25 seconds; however the overshoot of each extreme case was negligible. Since the PID controller did not yield the preferred response, another method utilizing state space representation was employed to ensure the desired response of the system. The fundamental idea of the design in state-space is the pole-placement. This approach specifies all closed loop poles of the system, which are directly calculated from the frequency response requirements. By calculating a gain matrix, it is possible to force the system's closed loop poles at the desired pole locations. Pole placement requires that the liquid-leveling system be completely state controllable and that all three states are available for feedback. A state-feedback controller with an integrator was designed to meet the required performance specifications. This controller was chosen since the response at each of the extreme values met the performance specifications.

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