Design and Construction of a Low-Cost 'Hydrostatic Force on a Submerged Surface' Experiment Setup

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Abstract
Development of a new laboratory for an engineering program is always challenging and the fact that public universities must constantly deal with tight budgets makes it even more difficult. With the approval of the new engineering program at Southern Arkansas University, it was incumbent to develop two upper level laboratory courses one being the Thermal-Fluid Science Lab. The Thermal-Science Lab will be offered for the first time in the Spring of 2016; so the great challenge was to develop several low cost experimental setups related to fluid mechanics and thermal sciences. In the current project a detailed description of the design and construction of a 'Hydrostatic Force on a Submerged Surface' is provided. The main purpose of the current project was to make an experimental setup at a low cost while giving engineering students an opportunity to obtain hands-on experience through the construction process. The design and the construction of the experimental setup were completed as a project in a manufacturing class. The current study is intended to be shared with people in academia so that others can build it when facing tight budgets and/or wanting to provide hands-on experience to their students or just want to do it for their own satisfaction. One interesting aspect of this project is the use of 3D printing technology to make the main component of the setup. The study also includes sample data, repeatability plot, and the lab procedure for the students to perform this experiment. The details of the setup and the experimental methods are discussed in this paper.

Introduction
Budgetary constraints and limited resources always create challenges when developing a new engineering lab. The initial cost to setup a new teaching lab for an engineering program is usually very high. Different programs use different approaches to minimize these costs. Some engineering programs [1] teach laboratory courses through web based tools to minimize the cost. Jamie Douglas et al reported the development of inexpensive hands-on experiments related to mechanics of materials for distance education. However, distance or web based experiments are not suitable for all engineering programs. Due to the huge expense of lab equipment some programs are even moving to a 'Lab in a box' approach to minimize costs [3]. The cost depends on the type of lab being developed as well. Usually the equipment in a typical fluid mechanics or in a thermal-fluid science lab is reasonably expensive due to the nature of the equipment. A well-equipped laboratory that covers different areas of thermal science may easily cost several hundred thousand dollars. Acquiring bench-top or cart-mounted equipment could save money and space. David Torick et al developed an entire fluid mechanics lab for under $6000.

The Engineering program at Southern Arkansas University is going to offer a Thermal Fluid Science lab for the Spring 2016 semester. Some of the equipment that will be used for this laboratory class were designed by engineering faculty and made by students as part of their project work in another class to obtain hands-on experience. In the current article the details of designing and fabrication along with a detailed budget of an experimental setup, a hydrostatic force measurement, for the new thermal-fluid science lab is discussed. The motivation behind this work was to reduce the cost of developing the experimental setup as well as to engage engineering students in hands-on projects.

While taking the Thermal Fluid Science Lab, through this experiment, students will be introduced to the demonstration of a hydrostatic force on a vertical submerged surface. Students will determine the force experimentally though the static equilibrium of the system and then compare it with the theoretical one. The corresponding lab handout for this experiment is also prepared and included in the appendix section of this paper.

Planning and Design

When a body is submerged in a static fluid, the fluid will apply forces onto the surfaces of the body. These forces can be determined using simple static analysis. Determination of such forces is important in designing dams, storage tanks, hydraulic equipment, etc. In the planned experimental setup the main component is a plastic quadrant; the detailed design of it is shown in Figure 1. The quadrant has two concentric cylindrical surfaces, two parallel surfaces, and two rectangular surfaces. The outer radius of the quadrant is 10 inches and the rectangular surfaces are of 5 in x 5 in. The quadrant is planned to be hinged at the center of the concentric surfaces.

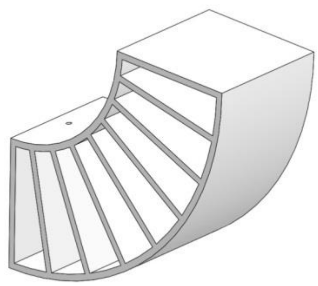

Figure 1a: The partially hollow interior of the quadrant

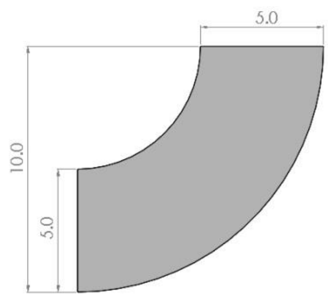

Figure 1b: The quadrant cover with dimensions

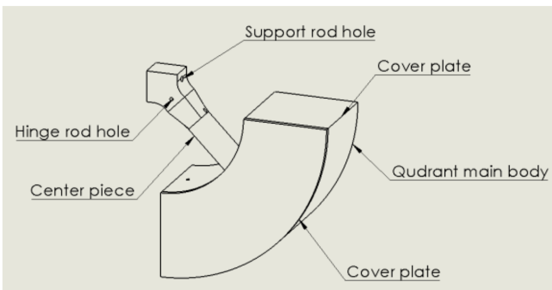

Figure 1c: The quadrant assembly including the body, the cover, and the centerpiece

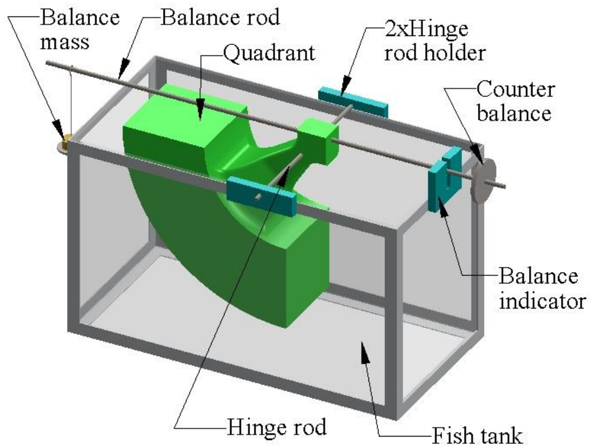

Figure 2: The planned setup [5]

The planned schematic is shown in Figure 2[5]. The quadrant was to be placed in a fish tank. Initially the quadrant will be kept balanced by making the top rectangular surface parallel to the ground when there is no water in the tank. When water is added into the tank, it will apply forces onto all surfaces of the quadrant. The forces on the cylindrical surfaces will pass through the pivot axis and will cause zero moments. The forces on the side parallel surfaces will counteract each other. The only unbalanced force will be the one acting on the vertical submerged rectangular surface. The quadrant will be brought back to the balanced position by adding restoring masses onto the balance pan. The experimental hydrostatic force can be calculated using simple moment analysis. The detailed moment analysis and the equations are shown in the lab manual for this experiment that is given in the appendix. The theoretical force will be determined from $F_R = \rho g h_c A_{wet}$, where F_R is the hydrostatic force on the submerged area of the vertical face, ρ is the density of water, g is the gravity, h_c is the distance to the centroid of the wetted area from the free surface, and A_{wet} is the submerged area of the vertical surface.

A plan of building the experimental setup was first given in the paper that was presented at the ASEE Midwest conference held in Springfield – Missouri in September 2015. The cost was expected to be under \$200 [5] as shown in Table 1. It is noted that a fully functional such setup would easily cost at least \$3000 if purchased commercially.

Table 1: Estimated cost for hydrostatic force on a submerged surface setup [5]

Equipment/Materials	Estimated cost	Source
3D printed quadrant	\$50 (filament cost)	Would be 3D printed
5 gallon fish tank	\$40	Amazon
Other parts/materials	\$100	Will be 3D printed or purchased

Construction

A group of students in the manufacturing class was responsible for the construction of the setup as part of their project work for the class.

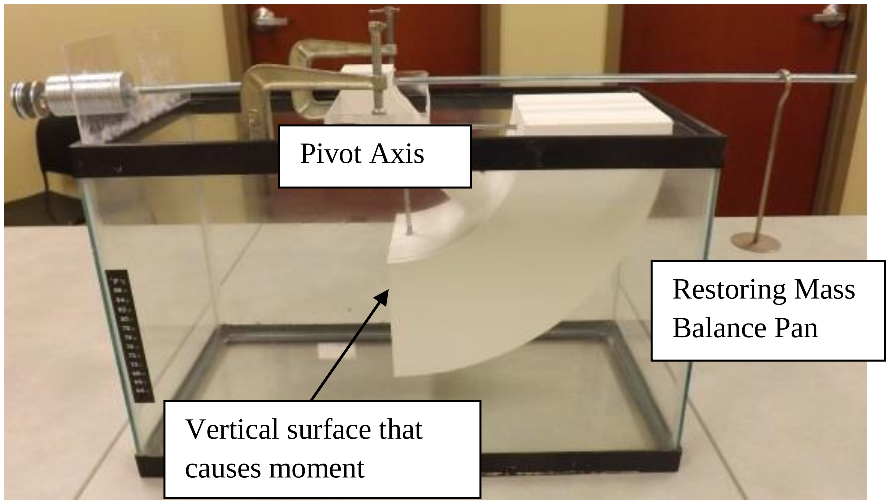

Figure 3: The final setup

The main component in this setup is a quadrant made of plastic. The quadrant is composed of a main body, a cover, and the centerpiece. The main body was made partially hollow inside as shown in Figure 1a. All pieces of the quadrant were 3D printed using a Gigabot 3D printer. The slicing of the models was performed using Simplify3D software. Rather than making them entirely solid the interior infill was kept to 20% to save money and time while printing them. The honeycomb interior pattern provides structural reinforcement to the quadrant. The total print time for the entire thing was approximately 48 hours. The cover and the centerpiece were glued to the main body (Figure 1c) making sure that all gaps were sealed watertight. A thin metal rod passes through a hole that is located at the center of the cylindrical surfaces and the rod acts as the axle for the axis of rotation. The axle rod is placed in plastic fixtures at the ends that allow for free rotation. The axle along with the plastic fixtures sits on the fish tank and is orientated parallel to the surface of the fluid. Thus, the quadrant hangs from and can rotate about this axle inside the tank. Two C-clamps were used to hold the plastic fixtures securely to the tank. Slightly higher on the quadrant is a little larger threaded rod that pierces through the quadrant exactly perpendicular to the axle. At one end of the threated rod a counterweight is attached and at the other end a balance pan for the restoring mass is attached. The threaded rod is oriented in such a way as to be parallel with the surface of the fluid when the quadrant is in equilibrium. The counter weight on the threaded rod is used to achieve this orientation while no fluid forces are acting on the quadrant.

One of the goals of this work was to keep the cost at a minimum level. The total cost of construction of the setup was less than sixty dollars which is a lot less than the anticipated cost. The cost for all 3D printed pieces was less than $30. The significant portion of the cost was supposed to be the procurement of a fish tank where the quadrant would be placed in. An old fish tank was actually donated and thus the budget significantly decreased. The other supplies such as plastic, rods, and glue cost about $30.

Experimentation and Data Collection

As the fluid is added into the tank, the plastic quadrant becomes submerged and it experiences a moment about the axle rod. To make the system balanced a counter moment can be applied by adding weight to the end of the threaded rod opposite of the counter weight. When the threaded rod has once again become perfectly horizontal, the measurement can be taken. The equilibrium of moment can be used to calculate the force that the plastic quadrant is experiencing on the vertical surface. The measurement of the depth of the object into the fluid can then be used to calculate the hydrostatic force the fluid applies onto the surface.

Table 2: Trial 1 data

Partially Submerged											
h, cm	h, m	A_{wet}, m^2	h_c, m	h_p, m	y, m	d_R, m	m_{CW}, gm	W_{CW},N	Experimental F_R, N	Theoretical F_R, N	% Error
2	0.02	0.0025	0.010	0.013	0.230	0.243	12	0.118	0.17	0.25	29.97
4	0.04	0.0050	0.020	0.027	0.210	0.237	60	0.589	0.88	0.98	10.00
6	0.06	0.0075	0.030	0.040	0.190	0.230	130	1.275	1.97	2.21	10.82
8	0.08	0.0100	0.040	0.053	0.170	0.223	230	2.256	3.59	3.92	8.60
10	0.1	0.0125	0.050	0.067	0.150	0.217	355	3.483	5.71	6.13	6.94
12	0.12	0.0150	0.060	0.080	0.130	0.210	505	4.954	8.37	8.83	5.15
Completely Submerged, Trial 1											
14	0.14	0.0156	0.078	0.078	0.110	0.188	625	6.131	11.61	11.88	2.28

The setup was tested for the partially submerged depths of 2 cm, 4 cm, 6 cm, 8 cm, 10 cm, and 12 cm and the completely submerged depth of 14 cm. The data was taken in six trials. The calculations were performed based on the theory and the equations that were presented in detail in the lab manual that is given in the appendix section of this paper. The data table for Trial 1 is presented here in Table 2. The data for the other trials are presented in the appendix section of this paper. For each depth, both experimental and theoretical forces as well as the absolute %error were calculated. For the repeatability purpose the absolute %error for trials were plotted in Figure 4. The error is observed to be very high at a depth of 2 cm and the error decreases with the increasing depth of the submerged surface. The high error observed at 2 cm and 4 cm is mainly due to reading errors of the submerged height. A ruler with centimeter marks was inserted into the water to measure the submerged height. This practice probably caused reading errors at the low depths. In the regular thermal science lab classes, an accurate submerged height will be read using a ruler fixed to the vertical glass surface of the tank. Also, for a given depth there is a slight variation of the %error from trial to trial, however, this variation is acceptable.

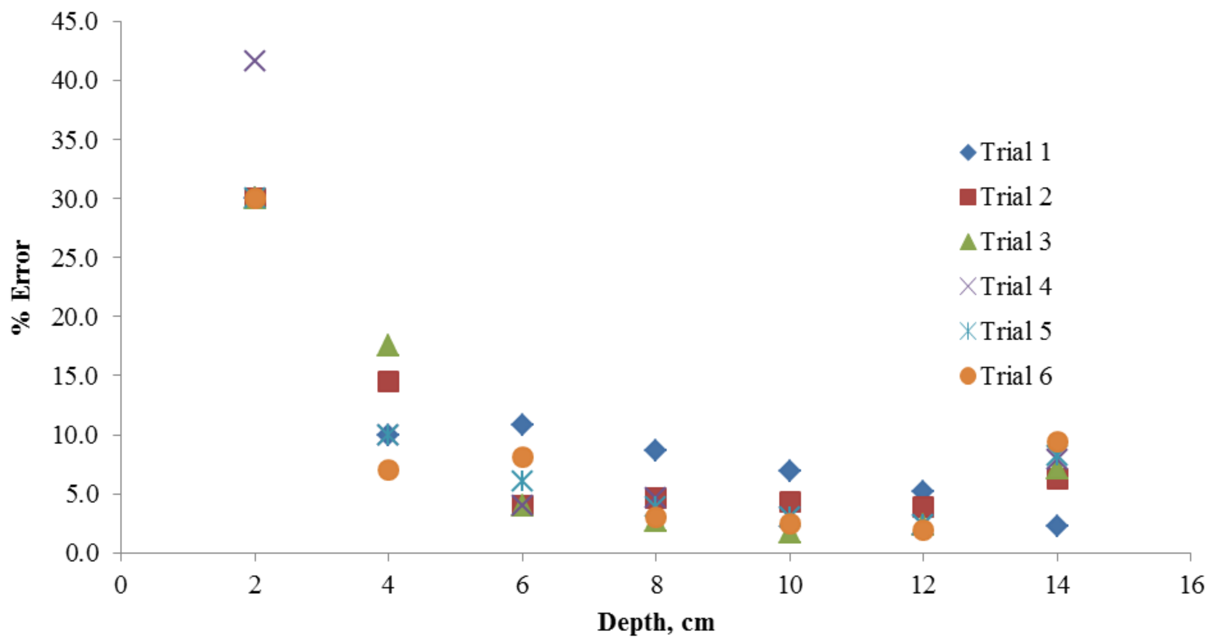

Figure 4: Repeatability plot

Conclusions and Recommendations

This paper covered the design and building of a lab experimental setup related to hydrostatic force measurement on a submerged surface. The design and construction was very simple and straightforward. 3D printing technology was used to make the main component of this setup. The cost of construction was extremely low compared to the similar setups available on the market to conduct a similar experiment. Data was taken for five trials and found to be reasonably repeatable among the trials for a given depth. A large amount of %error was found at a depth of 2 cm due to the height reading error; however, the %error decreases as the depth increases.

The main use of this device is for engineering students to measure the hydrostatic forces applied by a static fluid. The setup was built in accordance with that scope and satisfied initial fulfillment. It is also apparent from the feedback of the students who constructed the setup that they derived a great amount of satisfaction through this project work. This low-cost, yet fully functional setup will be used to conduct a full length laboratory class in the Thermal Fluid Science Lab. It is noted that a corresponding lab handout is also included in the appendix section.

The device met initial expectations and has the potential for further improvements. The device could greatly benefit from the use of a level placed on the plastic above the axle rod in order to increase the ability to discern when the threaded rod has become perfectly horizontal. The current plastic fixtures should be replaced with the 3-D printed ones to increase the rigidity. Lastly, a ruler needs to be placed permanently on the side of the tank to read submerged depths easily.

References

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3. Jay A. Weitzen, Alan Rux, Erin Isabel Webster, “UML Laboratory in a box, a new way of teaching ECE labs”, 121st ASEE Annual Conference and Exposition, Indianapolis, IN, June 15-18, 2014
4. David Torick, Dan Budny, “Adjusting the Curriculum in the Fluid Mechanics Course by Modifying the Laboratory Setting”, American Society for Engineering Education, AC 2009-1159, 2009
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Appendix

Data tables for the rest of the data are given below:

Partially Submerged, Trial 2											
h, cm	h, m	A_{wet}, m^2	h_c, m	h_p, m	y, m	d_R, m	m_{CW}, gm	W_{CW},N	Experimental F_R, N	Theoretical F_R, N	% Error
2	0.02	0.0025	0.010	0.01333	0.230	0.243	12	0.118	0.17	0.25	29.97
4	0.04	0.0050	0.020	0.02667	0.210	0.237	57	0.559	0.84	0.98	14.50
6	0.06	0.0075	0.030	0.04000	0.190	0.230	140	1.373	2.12	2.21	3.96
8	0.08	0.0100	0.040	0.05333	0.170	0.223	240	2.354	3.74	3.92	4.63
10	0.1	0.0125	0.050	0.06667	0.150	0.217	365	3.581	5.87	6.13	4.31
12	0.12	0.0150	0.060	0.08000	0.130	0.210	512	5.023	8.49	8.83	3.83
Completely Submerged, Trial 2											
14	0.14	0.0156	0.0775	0.0775	0.110	0.188	680	6.671	12.63	11.88	6.32

Partially Submerged, Trial 3											
h, cm	h, m	A_{wet}, m^2	h_c, m	h_p, m	y, m	d_R, m	m_{CW}, gm	W_{CW},N	Experimental F_R, N	Theoretical F_R, N	% Error
2	0.02	0.0025	0.010	0.01333	0.230	0.243	12	0.118	0.17	0.25	29.97
4	0.04	0.0050	0.020	0.02667	0.210	0.237	55	0.540	0.81	0.98	17.50
6	0.06	0.0075	0.030	0.04000	0.190	0.230	140	1.373	2.12	2.21	3.96
8	0.08	0.0100	0.040	0.05333	0.170	0.223	245	2.403	3.82	3.92	2.64
10	0.1	0.0125	0.050	0.06667	0.150	0.217	375	3.679	6.03	6.13	1.69
12	0.12	0.0150	0.060	0.08000	0.130	0.210	520	5.101	8.62	8.83	2.33
Completely Submerged, Trial 3											
14	0.14	0.0156	0.0775	0.0775	0.110	0.188	685	6.720	12.72	11.88	7.10

Partially Submerged, Trial 4											
h, cm	h, m	A_{wet}, m^2	h_c, m	h_p, m	y, m	d_R, m	m_{CW}, gm	W_{CW},N	Experimental F_R, N	Theoretical F_R, N	% Error
2	0.02	0.0025	0.010	0.01333	0.230	0.243	10	0.098	0.14	0.25	41.64
4	0.04	0.0050	0.020	0.02667	0.210	0.237	60	0.589	0.88	0.98	10.00
6	0.06	0.0075	0.030	0.04000	0.190	0.230	140	1.373	2.12	2.21	3.96
8	0.08	0.0100	0.040	0.05333	0.170	0.223	240	2.354	3.74	3.92	4.63
10	0.1	0.0125	0.050	0.06667	0.150	0.217	370	3.630	5.95	6.13	3.00
12	0.12	0.0150	0.060	0.08000	0.130	0.210	520	5.101	8.62	8.83	2.33
Completely Submerged, Trial 4											
14	0.14	0.0156	0.0775	0.0775	0.110	0.188	690	6.769	12.82	11.88	7.88

Partially Submerged, Trial 5											
h, cm	h, m	A_{wet}, m^2	h_c, m	h_p, m	y, m	d_R, m	m_{CW}, gm	W_{CW},N	Experimental F_R, N	Theoretical F_R, N	% Error
2	0.02	0.0025	0.010	0.01333	0.230	0.243	12	0.118	0.17	0.25	29.97
4	0.04	0.0050	0.020	0.02667	0.210	0.237	60	0.589	0.88	0.98	10.00
6	0.06	0.0075	0.030	0.04000	0.190	0.230	137	1.344	2.07	2.21	6.02
8	0.08	0.0100	0.040	0.05333	0.170	0.223	242	2.374	3.77	3.92	3.83
10	0.1	0.0125	0.050	0.06667	0.150	0.217	370	3.630	5.95	6.13	3.00
12	0.12	0.0150	0.060	0.08000	0.130	0.210	520	5.101	8.62	8.83	2.33
Completely Submerged, Trial 5											
14	0.14	0.0156	0.0775	0.0775	0.110	0.188	692	6.789	12.85	11.88	8.19

Partially Submerged, Trial 6											
h, cm	h, m	A_{wet}, m^2	h_c, m	h_p, m	y, m	d_R, m	m_{CW}, gm	W_{CW},N	Experimental F_R, N	Theoretical F_R, N	% Error
2	0.02	0.0025	0.010	0.01333	0.230	0.243	12	0.118	0.17	0.25	29.97
4	0.04	0.0050	0.020	0.02667	0.210	0.237	62	0.608	0.91	0.98	7.00
6	0.06	0.0075	0.030	0.04000	0.190	0.230	134	1.315	2.03	2.21	8.08
8	0.08	0.0100	0.040	0.05333	0.170	0.223	244	2.394	3.80	3.92	3.04
10	0.1	0.0125	0.050	0.06667	0.150	0.217	372	3.649	5.98	6.13	2.48
12	0.12	0.0150	0.060	0.08000	0.130	0.210	522	5.121	8.66	8.83	1.95
Completely Submerged, Trial 6											
14	0.14	0.0156	0.0775	0.0775	0.110	0.188	700	6.867	13.00	11.88	9.44

Lab Manual

Measurement of Hydrostatic Force on a Vertical Surface Submerged in a Static Fluid

Objective:

The purpose of this experiment is to determine the hydrostatic forces on a submerged vertical surface experimentally and compares them with the theoretical ones.

Introduction:

The resultant hydrostatic force on a vertical surface can be determined using the following formula -

$$F_R = \rho g h_c A_{wet} \qquad (1)$$

Where ρ is the density of the static fluid, g is the acceleration due to the gravity (9.81 m/s^2), h_c is the vertical distance from the centroid of the wetted surface to the level of the free surface of the fluid (water), and A_{wet} is the area of the surface that is submerged in the fluid.

The experimental setup is shown in Figure 1. The 3D printed white quadrant is suspended in a fish tank and can rotate about its pivot axis (pivot from the front view). The forces acting on the cylindrical curved surfaces pass through the pivot axis causing zero moments. Forces on two of the surfaces counteract each other. The only unbalanced force that causes moment about the pivot axis is the force that acts on the vertical wall at the end as shown in Figure 1.

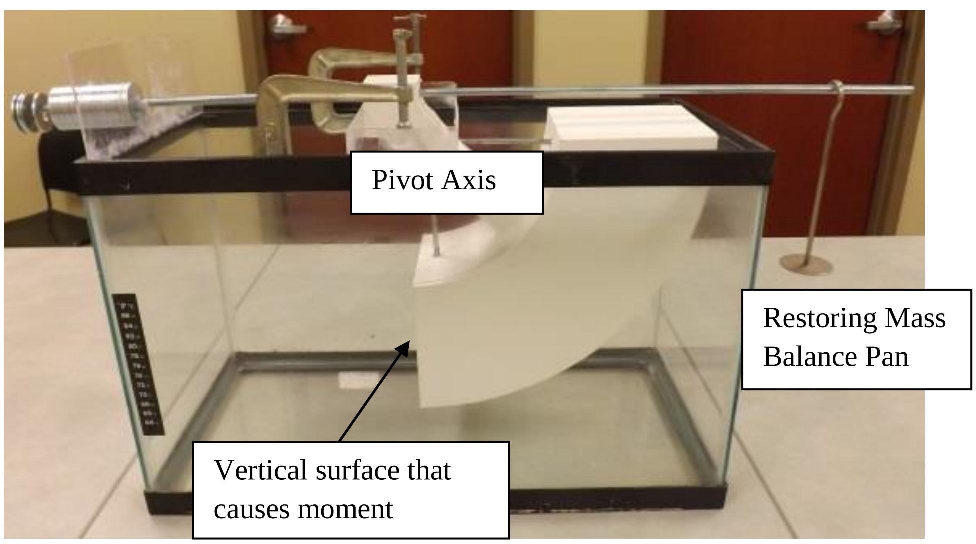

Figure 1: Experimental Apparatus

Initially the quadrant is balanced about its pivot axis without any water in the tank. It was done by putting some counterweight at the end of the rod (at the end face side of the quadrant) that passes through the pivot axis. Once the water is added into the tank the hydrostatic force (F_R) that acts on the vertical end face will make it unbalanced. The quadrant will be brought back to the balanced position again by adding mass to the balance pan that is located at the other end of the rod (at a fixed location) as shown in Figure 2. The weight due to this restoring mass is W_{CW}. The fixed distance (moment arm) between the balanced pan location of the rod and the pivot axis is d_{CW} that is measured horizontally. The vertical distance between the pivot point and the line of action of the resultant force is d_R. The location of the resultant force depends on the depth of the fluid in the tank. Thus d_R varies with the depth of water. Since the resultant moment about the pivot axis will be zero at the balanced position, the experimental resultant force can be determined a follows -

$$\Sigma M = 0 : d_R F_{R(\exp)} - d_{cw} W_{cw} = 0, \text{ or } \mathrm{F}_{\mathrm{R(exp)}} = \frac{d_{cw}}{d_R} W_{cw} \qquad (2)$$

Whereas the theoretical force is determined using Equation 1. The process is repeated at a different depth of water by simply adding more water in the tank. Students will investigate both partially submerged and fully submerged cases in this experiment.

Partially submerged

If the vertical end face is not completely under the water then the depth to the center of the wetted area, h_c, is given by h/2, where h is measured as the vertical distance between the water surface to the bottom edge of the vertical end face. It is considered that the resultant hydrostatic force is acting at a depth of h_p, where h_p is given by 2h/3. Consider that the width of the end surface is w and the height is H. The entire discussion can be summarized as -

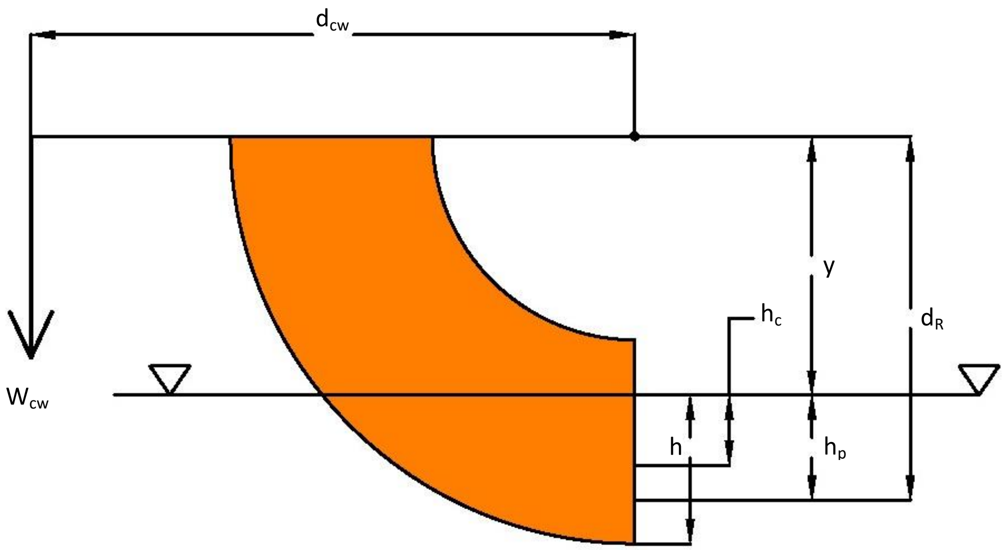

Figure 2: Partially submerged

$$h_c = \frac{h}{2} \tag{3.a}$$

$$h_p = \frac{2h}{3} \tag{3.b}$$

$$d_R = y + h_p \tag{3.c}$$

$$A_{wet} = h * w \tag{3.d}$$

Fully submerged

When the end surface is completely under the water, then h_p will be determined differently as compared to that of the partially submerged case. The moment of inertia of the wetted area about its centroidal axis in involved in calculation of h_p. However, the theoretical force F_R will still be determined using Equation 1. The following equations are needed for the fully submerged case -

$$h_c = h - \frac{H}{2} \quad (4.a)$$

$$h_p = h_c + \left[\frac{I_c}{h_c} * A_{wet} \right] \quad (4.b)$$

$$d_R = y + h_p \quad (4.c)$$

$$A_{wet} = H * w$$

$$I_c = w * \frac{H^3}{12} \quad (4.d)$$

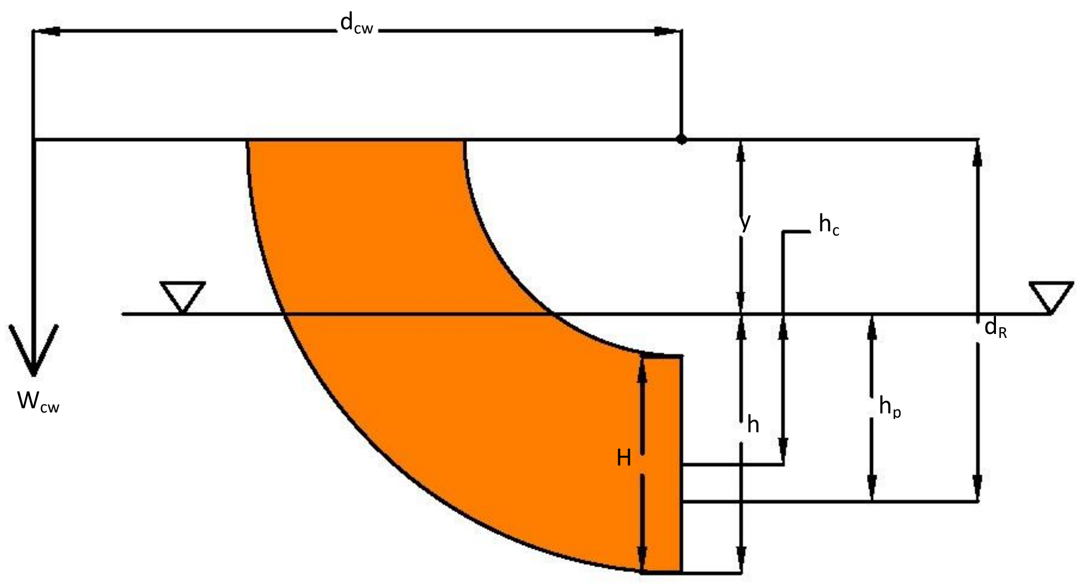

Figure 3: Fully submerged

Procedure:

1. Make sure there is no water in the tank and the setup is balanced.
2. Make sure that the setup is located near a water faucet and a sink. You need a tube to add water into the tank. You may need a small cup as well to make slight adjustments of the water level if needed. You need a ruler to make all your measurements.

3. Measure the radius (R) of the quadrant
4. Measure the moment arm, dcw, from the balance pan to the pivot axis horizontally and record them.
5. Measure the width (w) and height (H) of the vertical end surface of the quadrant and record them.
6. Add water slowly to the tank with the tube until you obtain a depth of 2 cm (h) from the bottom edge of the surface to the free water level. This will make the system unbalanced.
7. Add mass to the restoring pan to bring it back to a balanced position. Record this mass onto your data table. You have to convert the mass to kg and multiply the mass by gravity to get the weight in Newton. This is your W_{CW} for the given h value.
8. Calculate h_c and h_p.
9. Calculate y as R-h and then calculate d_R as $y+h_p$. d_R is the vertical distance between the pivot axis and the line of action of the hydrostatic force.
10. Calculate the theoretical value of the force using Equation 1 and the experimental value of the force using Equation 2.
11. Calculate the percentage error as $\%Error = \frac{\left|F_{R(theo)} - F_{R(\text{expt})}\right|}{F_{R(theo)}} \times 100$
12. Add more water into the tank until you obtain a depth of 4 cm and repeat the above steps to calculate theoretical and experimental hydrostatic forces and the %error.
13. Keep repeating the steps for the rest of the partially submerged depths of 6 cm, 8 cm, 10 cm, and 12 cm
14. Now increase the depth to 14 cm. The end surface will be completely under the water and this is your fully submerged case. Add a restoring mass to bring the system back to the balanced position.
15. Use equations 4.a through 4.d to calculate h_c, I_c, h_p, d_R, etc.
16. Calculate the theoretical and experimental hydrostatic forces and the %error.
17. Drain the water and clean up and organize your work area.
18. Write a report using the proper format.

Table A: Initial data

R, cm	d_{CW}, cm	Surface Width (w), cm	Surface Height (H), cm

Table B: Detail calculation

Partially Submerged											
h, cm	h, m	A_{wet}, m^2	h_c, m	h_p, m	y, m	d_R, m	m_{CW}, gm	W_{CW},N	Experimental F_R, N	Theoretical F_R, N	% Error
2											
4											
6											
8											
10											
12											
Completely Submerged											
14											