2006-1270: DESIGN OF APPLICATION-ORIENTED COMPUTER PROJECTS IN A PROBABILITY AND RANDOM PROCESSES COURSE FOR ELECTRICAL ENGINEERING MAJORS

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Design of Application-Oriented Computer Projects in a Probability and Random Processes Course for Electrical Engineering Majors

Abstract

A course of Probability and Random Processes is regularly taken by many engineering students, because the study of this topic is fundamental to a wide range of disciplines. Usually students recognize that learning probability and random processes is a struggle. The primary reason is that the course materials are abstract and the concepts are difficult to understand. Currently published textbooks include computer exercises/projects to help students learn the concepts, primarily, the development of an intuition for randomness. But most of these computer exercises/projects are pure theoretical, which do not have a clear link with real applications. In this paper, we will report the effort made to design application-oriented computer projects as a complement to current textbook resources for a graduate-level class. Most of these projects are designed based on the applications of probability and random processes in Digital Image Processing and Digital Speech Processing, two major areas for the application of this knowledge. Image and speech signals have very vivid presentation, which can motivate students to learn; image and speech signals can be easily manipulated in MATLAB, the widely used engineering software, so students are able to do these projects without specific background. Some projects are in Digital Communications, where probability and random processes theories are widely used as well. The preliminary outcome is discussed. The extension of this experience to an undergraduate level course is also presented.

I. Introduction

A course of Probability and Random Processes (or courses with similar titles) is regularly taken by many engineering students, because the study of this topic is fundamental to a wide range of disciplines. The goals of such a course are to introduce students the principles of probability and random signals, and to provide tools whereby they can deal with systems involving such signals. In the Department of Electrical and Computer Engineering (ECE), students need to master such tools, in particular, when they want to work in communication and signal processing systems, because they have to deal with signals and systems in a chaotic environment.

Usually students recognize that learning probability and random processes is a struggle. The primary reason is that the course materials are abstract and the concepts are difficult to understand. Motivating students is another challenge in probability courses. Beyond meeting degree requirements, the main motivation of most engineering students is to learn how to solve practical problems. For the majority, the mathematical logic of probability theory is, in itself, of minor interest. What the students want most is an intuitive grasp of the basic concepts and lots of practices working on applications.

The development of an intuition for randomness can be aided by the use of computer exercises. Currently published textbooks include computer projects to help students learn the concepts. For instance, how to generate random samples with a specific probability density function. But most of these computer exercises/projects are pure theoretical, which do not have a clear link with practical applications.

In this paper, we will report the effort made to design application-oriented computer projects as a complement to current textbook resources for a graduate-level class, ECE8803 Random Signals & Systems, in the ECE department at Mississippi State University (MSU). Most of these projects are designed based on the applications of probability and random processes in Digital Image Processing (DIP) and Digital Speech Processing (DSP), two major areas for the application of the knowledge of probability and random processes. Their vivid presentation can motivate students to learn. Image and speech signals can be easily manipulated in MATLAB, so students are able to do these projects without much prior background in DIP and DSP. Some projects are in Digital Communications, where probability and random processes theories are widely used as well.

A dozen of developed projects are documented. Similar projects will be modified for an undergraduate level probability and random processes class, MA4533/6533 Probability and Random Processes, for ECE majors at MSU.

II. Improvement of ECE8803

2.1 Introduction of ECE8803 at MSU

ECE8803 Random Signals & Systems is a graduate level course for electrical engineering (EE) majors at MSU. It consists of three hours of lecture, and offered every fall semester. The prerequisite is MA4533/6533 Probability and Random Processes, an undergraduate course offered by Department of Mathematics and Statistics (MATH)

The objectives of this course are: 1) to teach the students theoretical concepts in probability and random processes, and 2) to teach the students how to manage random signals in electrical engineering systems. Table 1 lists the topics covered in this course.

Table 1: Topics covered in ECE8803

Chapters	Topics
1. Probability Reviews	Probability models
	The axioms of probability
	Conditional probability and independence
	Sequential experiment
2. Random Variables	Probability distribution and density
	Expectation
	Conditional expectation and its properties
	Functions of random variables
	Inequalities
	Convergence
	Laws of large numbers
	Central limit theorem

3. Random Processes	Random processes specifications
	Stationary random processes
	Markov processes
	Continuity of random processes
	Ergodicity
	Karhunen-Loeve expansion
4. Analysis and Processing of Random Signals	Power spectral density
	Response of linear systems to random signals
	Optimum linear systems

Many graduate students do not like this course, because the content is difficult and the classes are boring. This has been reflected by historically low student evaluation. Students usually avoid taking this course, unless their academic advisors force them to take it.

2.2 Improvement Approach

The approach to improving the teaching and learning effectiveness of ECE8803 is to design application-oriented computer projects with four objectives: to help students deeply understand fundamental concepts; to provide students practice in solving relatively complex tasks encountered by engineering professionals; to offer students hands-on experience with important software package; and to make this mathematical class more interesting for higher motivation.

All projects are designed to utilize `MATLAB`. `MATLAB` is a registered trademark of the `MathWorks, Inc.`, which is a technical software environment. It has been extensively used in practice throughout the engineering community. It is easy to learn and use, because no compiler is needed and a large number of built-in functions can be called. Furthermore, `MATLAB` has nice plotting capabilities that can greatly assist data visualization. `MATLAB` has a Statistics Toolbox that can be used. All students taking ECE8803 are required to have `MATLAB` coding skill.

There are dozens of books published on probability and statistics, and among them Refs.1-9 are frequently used for EE students. Although some of them have computer exercise/projects, the limited available resource in current texts mainly teaches students how to use built-in functions for a certain purpose. For instance, it is taught how to use the function `normrand` to generate Gaussian random variables. Students can easily complete an exercise/project without knowing what are the fundamental techniques for the generation of random variables. If they meet a practical problem requiring the generation of random variables with a specific distribution that is not directly supported by any software package, they may be unable to handle. Another problem is that most computer problems still do not have a clear connection with practical application. For instance, how to generate random samples with a specific probability density function can be a small computer project, but its importance in graduate research is rarely mentioned.

The projects designed for ECE8803 are very different. They require critical thinking instead of simple software function/command manipulation. Concept understanding and software utilization will be effectively integrated. As a result, students will be familiar with and know how to take advantage of software package but not dependent on it. They will understand deeply the technique behind a built-in function. And students will be trained to have a problem-solving capability. More interestingly, all these projects are designed based on the applications of probability and random processes in Digital Image Processing (DIP) and Digital Speech Processing (DSP). The reasons for this choice are: DIP and DSP are two major areas for the application of the knowledge of probability and random processes; image and speech signals have very vivid presentation, which can motivate students to learn; image and speech signals can be easily manipulated in MATLAB, the widely used engineering software, so students are able to do these projects without much prior background in DIP and DSP.

Since communications is another area where probability and random processes theories are widely used, computer projects in this area will be designed as well. Most of textbooks have examples in digital communications. These examples will be translated into interesting computer projects to deepen the understanding. Moreover, this may also be helpful to encourage the undergraduate students to take elective courses in communications in their senior year, such as ECE 4813/6813 Communications Theory, ECE 4823/6823 Digital Communications.

An Example Project in Digital Image Processing:

A simple DIP-related computer project is designed for the calculation of mean and variance of a random sequence. The original example in a textbook looks like this:

Let y_1, y_2, Λ, y_N *be a sequence of random variables, and* $y_i = x + \eta_i$, *where x is a deterministic value and* η_i *are independent identically distributed (iid) random variables with zero mean. Let* $\bar{y}$ *be the sample average:* $\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ *. What are the mean and variance of* y_i *and* $\bar{y}$ *?*

If only this original example is used, students may know how to do the necessary derivation but have no clue about its practical importance. But if this example is reformulated by using a simple but very effective application in image enhancement, which is referred to as *image averaging*, as shown below, the impact will be significantly different. After repackaging, the following project can not only show how to calculate the mean and variance of the average of iid random samples, but also tells students how useful this technique is. In particular, the computer simulation using a real image will impress students and make somewhat boring mathematical materials much more enjoyable. Students will start to appreciate the magic of mathematics, and become more willing to learn.

Sample Project 1 (simplified): Get a gray scale image (download from the Internet or take it using your own digital camera). Denote it as the original image $f(x, y)$*, where* (x, y) *is a pair of coordinates of a pixel. Zero-mean Gaussian random noise* $\eta(x, y)$ *is generated using* `MATLAB` *function* `normrand` *and added to* $f(x, y)$*, which may be induced in an image acquisition system. The resultant noisy image is denoted as* $g(x, y)$*, that is,* $g(x, y) = f(x, y) + \eta(x, y)$*. Generate a set of N noisy images,* $\{g_i(x, y)\}$*. An averaged image* $\bar{g}(x, y)$ *is formed by averaging N different noisy images, i.e.,* $\bar{g}(x, y) = \frac{1}{N}\sum_{i=1}^{N} g_i(x, y)$*. Plot the averaged image when 3, 10, 20,* *and 100 noisy images are averaged. What phenomena do you find? Provide some theoretical analysis. (Hint: determine the mean and variance of* $g(x, y)$ *and* $\overline{g}(x, y)$*)*

Solution to Project 1 (partial): For $g(x, y)$, $E\{g(x, y)\} = f(x, y)$ and $Var\{g(x, y)\} = \sigma^2_{\eta(x,y)}$.
Since the noise is iid, $g(x, y)$ is iid. Then the mean of $\overline{g}(x, y)$ is

$$E\{\overline{g}(x, y)\} = \frac{1}{N}\sum_{i=1}^{N} E\{g_i(x, y)\} = \frac{1}{N}\sum_{i=1}^{N} f(x, y) = f(x, y),$$

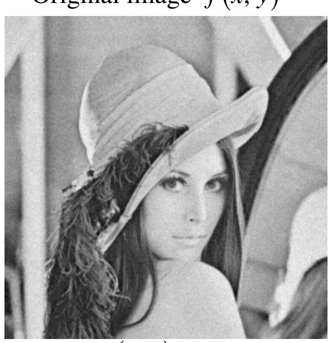
and the variance is $Var\{\overline{g}(x, y)\} = E\left\{(\overline{g}(x, y) - f(x, y))^2\right\} = E\left\{\left(\frac{1}{N}\sum_{i=1}^{N}\eta(x, y)\right)^2\right\} = \frac{1}{N}\sigma^2_{\eta(x,y)}$.

This indicates the variability of pixel values in the sum image decrease as *N* increases. Because the mean of $\overline{g}(x, y)$ is the original image $f(x, y)$, $\overline{g}(x, y)$ approaches $f(x, y)$ as the number of noisy images used in the averaging process increases.

This simple averaging technique is very useful in image enhancement. As shown in Figure 1, an original image $f(x, y)$ is corrupted by uncorrelated Gaussian noise with zero mean and variance being 10. Ten noisy images are simulated. If averaging using three noisy images, part of noise effect can be removed. If all the ten images are used for averaging, the resultant image $\overline{g}(x, y)$ is very close to the original image.


Original image $f(x, y)$


One of noisy images $g(x, y)$


Averaged image $\overline{g}(x, y)$ using 3 noisy images


Averaged image $\overline{g}(x, y)$ using 10 noisy images

Figure 1: Demonstration about Image Averaging

<u>An Example in Digital Speech Processing:</u>

A simple DSP-related computer project is designed for the optimal linear system functioning as a predictor. The original example in a textbook looks like this:

Let X_1, X_2, Λ *be a random sequence. Suppose that a second-order prediction system is to be designed such that a sample is predicted by the previous two samples. Find the system parameters a and b that yield the minimum prediction error.*

If only this original example is used, students may practice on system design using the formula provided in the textbook without knowing its practical importance. Actually, optimal linear predictor has very important applications in DSP (as well as DIP). An application-oriented computer projector can be designed based on this sample as below.

Sample Project 2 (simplified): Record your voice into MATLAB. You will get a speech waveform. Design the optimal first-order, second-order, and third-order linear predictors. Determine the prediction errors. Play the reconstructed waveforms. Can you tell the difference from the original one? Compute the entropies of the original waveform and residue waveform, and estimate the theoretical compression ratios.

This project gives students an opportunity to manipulate the speech signals with a deep understanding of the concept and application of optimal linear predictor. To students who have taken Information Theory and/or Data Compression, this project provides the correlation with what they have learned in these courses. To students who did not take these related courses, the concept of source coding can be easily introduced. This may encourage those interested students to take these courses in the future.

<u>An Example in Digital Communications:</u>

A typical example in Digital Communications in many textbooks looks like this:

In a binary communication system, the input 0 and 1 are equal likely. The error probability for the output being 1 with input 0 or 0 with input 1 is ε*. Calculate the probabilities of output 0 and 1, and calculate the four posterior probabilities.*

The purpose of this example is to illustrate the use of the conditional probability and Bayes' Rule. The students will be more impressed with the impact of random noise on practical communication channels, if a related computer project is conducted as below.

Sample Project 3 (simplified): In a binary communication system, the input 0 and 1 are equal likely. Because of noise, the output may be 1 when the input is 0, and vise versa. Assume this error probability is 0.2. Simulate this communication channel using MATLAB. Determine the probabilities of output 0 and 1, and the four posterior probabilities (i.e., the probability of an input given the output). Compare them with theoretical values. Sending an input of 0 and 1 with equal probability can be simulated as flipping an unbiased coin. Receiving an output can be simulated by keeping or changing the input, determined by the outcome of flipping a biased coin. (Hint: coin-flipping experiment can be realized using the MATLAB function `rand`*).*

This project will also help the students understand the meaning of posterior probability and its practical importance, which usually is difficult to understand.

2.3 Preliminary Outcome

Many students expressed their appreciation about computer projects. They think computer projects help them learn and think actively. A large population of this class is international students, and some of them have taken a similar course in their home country. They said the major difference in ECE8803 at MSU, compared the one they took before, was that there were interesting computer projects.

In order to get enough feedback and refine the projects in a timely manner, surveys will be designed and used for each project in Fall 2006. An example survey questionnaire is shown in Table 2.

Table 2: An Example Survey Questionnaire in ECE8803.

Evaluation of Project X in ECE8803

SA = Strongly Agree; *A = Agree*; *N = Neutral*; *D = Disagree*; *SD = Strongly Disagree*

1. The project provided me with a better understanding of concepts learned in theory.
   SA A N D SD
2. The project gave me the opportunity to demonstrate individual initiative and creativity.
   SA A N D SD
3. The project was clearly outlined and objectives were well explained.
   SA A N D SD
4. I believe that this project is valuable to my research.
   SA A N D SD
5. Handouts and reading assignments were useful and informative.
   SA A N D SD
6. I recommend this experience to other students.
   SA A N D SD
7. Things I like about this project.
8. Suggestions for improvement about this project.

III. Extension to MA4533/6533

The MATH Department at MSU is tasked to offer MA4533/6533 Probability and Random Processes to junior-level undergraduate students in the ECE Department. It is a required course, and the prerequisite of ECE8803. This course plays a significant role in fulfilling the Accreditation Board for Engineering and Technology (ABET) program requirement that students have "knowledge of probability and statistics, including applications appropriate to the program name and objectives". But it is considered as the most difficult course by the undergraduates, and received lots of complaints.

Faculty members in the ECE and Math departments work closely to improve this course. The experience from ECE8803 will be extended to MA4533/6533. Students at the junior-level in the ECE department have basic skill in `MATLAB` coding. But the concern is if our undergraduate students can handle projects in DIP and DSP without prior background.

The initial efforts are taken in Spring 2006. Firstly, to mitigate the pressure from projects, only two optional computer projects are assigned, and the students who complete a project get bonus points. Secondly, an ECE faculty member goes to the classes of MA4533/6533, and provides basic concepts of DIP and DSP before project assignment (e.g., a digital image is a 2D matrix, each element is called pixel, and image processing is matrix manipulation, etc). Thirdly, sample `MATLAB` programs for each project are posted at the course website to reduce the workload (e.g., the sample code for figure plotting is provided so that students do not need to take extra time to figure it out, and some built-in functions that can be used for specific purposes are introduced).

The impact needs to be carefully assessed. Based on the assessment, projects will be dynamically modified. The goal is to enhance the teaching and learning effectiveness without creating too much extra workload for undergraduate students.

IV. Conclusion

An approach to improving a graduate-level course, ECE8803 Random Signals & System at MSU, is presented. The basic idea is to design application-oriented computer projects in Digital Image Processing, Digital Speech Processing, and Digital Communications, major areas that employ the knowledge of probability and random processes. These projects provide students obvious correlation between the knowledge in ECE8803 and practical applications. They also make the historically boring classes much more interesting. They may even inspire some interested students get into these research areas. Up to now, the preliminary outcome includes very positive feedback.

The extension of this experience to an undergraduate-level course, MA4533/6533 Probability and Random Processes, is also discussed. Basically, a project in ECE8803 will be cut into several small projects for MA4533/6533. Considering the background level of our undergraduates, more explanation work needs to be done before project assignments. This also requires a smooth collaboration between faculty members in the ECE and MATH departments.

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