

# **Designing a Multi-Disciplinary Hybrid Vehicle Systems Course Curriculum Suitable for Multiple Departments**

**Vincent Winstead**

**Department of Electrical and Computer Engineering and Technology  
Minnesota State University, Mankato**

## **Abstract**

In recent years an increasing emphasis is being placed on the inclusion of multi-disciplinary programs or in courses having multi-disciplinary content. Including this content can be challenging especially among the various engineering disciplines. This is exacerbated by the challenges associated with making this type of course content accessible to a wide range of students with varying levels of background in mathematics and simultaneously ensure its relevancy and technology advancement contemporaneousness. Students from different traditional engineering disciplines, such as electrical, computer and mechanical engineering, and also from non-traditional technology-based disciplines have different course requirements and depth in core mathematics, static systems, dynamic systems, systems modeling, power systems and electronics. Developing a course curriculum which crosses over multiple programs and disciplines and yet is relevant to a broad class of students is difficult. This paper describes some results from a concerted effort to accomplish the inclusion of a multi-disciplinary content in a new multi-department course series. The hybrid vehicle systems course series (2 semesters) was designed to accommodate the needs of traditional electrical and mechanical engineering curriculums while including the necessary content to be relevant and welcomed as part of the electrical and automotive engineering technology programs at Minnesota State University, Mankato. This paper discusses the differences in curriculum between these different departments as it pertains to core subject preparation and competency and the modifications made in developing the hybrid vehicle course sequence to enable broad applicability to the students in different departments. The paper concludes with the specific curriculum topics and the teaching methods used to convey similar analysis results and conclusions using both calculus and algebraic concepts.

## **Introduction**

Modern automotive vehicles are filled with new and complex technologies. This is reflected most notably in more recent entries in the automobile market, namely hybrid-electric vehicles. These developments include combining a torque producing chemical energy engine with torque producing electrical energy motor/generators, regenerative braking, advanced engine displays and advanced high speed communication networks among others. This has led the traditional engineering disciplines of electrical engineering, computer engineering, mechanical engineering and automotive engineering to broaden their scope and consider areas of overlap. It has, in part, blurred the lines delineating the subject matter of these traditional programs. Particularly in the case of hybrid-electric vehicles, the automotive industry has a definite need for engineers who recognize these overlaps and can operate comfortably in multiple camps. This is the essence of multi-disciplinary engineering. In responding to this need, we have considered the development of a hybrid vehicle systems two-course sequence in which a multi-disciplinary teaching approach is taken. This methodology is not intended to be restricted to the topic of hybrid vehicles although the technologies associated with this topic seemed to be a good match for the teaching methodologies discussed herein. The goal being to effectively teach the topics to an engineering "diverse" audience.

The paper is organized as follows. First, a background of the three predominant disciplines expected to show interest in the topics of a course sequence in hybrid vehicle systems is described in the Engineering Disciplines Background section. Then, in the Teaching Methodologies section, a summary of the teaching methodologies proposed for the first course in the two-course sequence is described along with derivation examples which provide a bridge between the capabilities of the audience to understand various topics included in the course sequence and in learning an optimization problem solving approach. Finally, the paper concludes with some closing remarks about the application of the techniques discussed to other topics such as modeling and simulation, which are effectively used teaching tools at many academic institutions.

## Engineering Disciplines Background

The Minnesota State University, Mankato college of Science, Engineering and Technology has a number of engineering programs leading to bachelor and masters degrees including three which are considered to have a particular connection with the subject matter associated with a course sequence in hybrid vehicle systems. These are electrical engineering, mechanical engineering and automotive engineering technology.

The Automotive and Manufacturing Engineering Technology department includes the Automotive Engineering Technology program. It incorporates a broad curriculum in automotive design and diagnostics courses. Its courses in the areas of mathematics and physics mirror those of the electrical and mechanical engineering disciplines with the exception of calculus. This department does not require advancement in the area of calculus beyond methods of integration, typically covered as a subset of topics in the second semester calculus course. Further, methods of partial and ordinary differential equations are not a part of the required curriculum. The Automotive Engineering Technology curriculum does include courses in direct current (DC) circuits and statics, dynamics and mechanics of materials as well as basic concepts from thermodynamics.

The Electrical and Computer Engineering and Technology department includes the Electrical Engineering program. It incorporates a broad curriculum focused in electronics (digital and analog), circuit theory, systems theory (including communication and control), electromagnetics and microprocessor systems. Its courses in the areas of mathematics and physics include requirements for two semesters of physics and four semesters of calculus through concepts of partial and ordinary differential equations. Additional required courses are in the subject areas of statics and thermal systems engineering.

The Mechanical and Civil Engineering department includes the Mechanical Engineering program. It incorporates a broad curriculum covering many topics including thermodynamics, mechanics (solid materials and fluids), computer aided design and machine design. Its courses in the areas of mathematics and physics include requirements for two semesters of physics and four semesters of calculus through concepts of partial and ordinary differential equations. Additional required courses are in the subject areas of systems theory (linear systems and control), statics, dynamics and DC circuits.

The overlap between the Electrical Engineering and the Mechanical Engineering curricula is strong in the areas of mathematics and physics as well as in systems theory and thermodynamics. The overlap among the aforementioned and the Automotive Engineering Technology curricula is less broad since the level of mathematics in the Automotive Engineering Technology required courses is limited to two calculus semesters and there are no required courses in systems theoretic concepts. See Table 1 for a summary of course topics which are part of multiple curricula.

<i>Course Topics</i>	<i>Electrical</i>	<i>Mechanical</i>	<i>Automotive</i>
Digital Systems	X	X	X
Electronics	X	X	X
Signals and Systems	X	X	
Control Systems	X	X	
Electromechanics	X		
Thermal Systems	X	X	X
Calculus (Limits, Continuity, Derivatives, ...)	X	X	X
Calculus (Transcendentals, Integration, ...)	X	X	X
Calculus (Vector Calculus, Partial Diff., ...)	X	X	
Ordinary Differential Equations	X	X	
Physics	X	X	X
Mechanics of Materials		X	X
Statics	X	X	X
Dynamics		X	X

Table 1: Table of course topics and program coverage.

A hybrid vehicle systems course covers the broad topics of automotive system analysis and design with an emphasis in the areas of electric propulsion, mechanical and electrical power conversion, energy conversion, transfer and storage, control system concepts and applications of optimization techniques. These areas appear to create a broad umbrella which envelops portions of the Automotive Engineering Technology, Electrical Engineering and Mechanical Engineering programs. This is illustrated in Table 2.

<i>Hybrid Vehicle Systems Course Enablers</i>		
<i>Automotive</i>	<i>Electrical</i>	<i>Mechanical</i>
DC Circuits	DC Circuits	Circuits
Physics	Physics	Physics
Differentiation	Differentiation	Differentiation
Integration	Integration	Integration
Statics	Vector Calculus	Vector Calculus
Dynamics	Ordinary Diff. Eq.	Ordinary Diff. Eq.
Thermodynamics	Statics	Statics
Automotive Tech. & Systems	Dynamics	Thermal Systems
	Thermodynamics	Electronics
	Control Systems	Control Systems
	Linear Systems	Signals and Systems

Table 2: Table of course enablers.

From Table 1, one can see the overlap of common course topics which form the fundamental concepts providing prerequisites for the topics associated with a hybrid vehicle systems course curriculum. Tables 1 and 2 show course topics which are common for all three disciplines. In an effort to provide consistent learning within the community of students from the separate departments, specific areas of focus for

course development were followed. Namely, a focus on alternatives to the use of differential equations to describe hybrid vehicle component dynamics, a focus on the use of interchangeable (or equivalent) mechanical/electrical modeling concepts, and a focus on optimization techniques which can be described using non-differential equations in an effort to provide a digestible course format considering the backgrounds of the Automotive Engineering Technology, Electrical Engineering and Mechanical Engineering program students. The two-course sequence includes one course in theory and another course dedicated to applied project activities similar to a senior design project course such as hybridizing a conventional drive train vehicle, designing an efficient series power train solar vehicle having a small ICE, etc. This allows students from different departments to collaborate and bring their particular backgrounds into a multi-disciplinary design project.

### Teaching Methodologies

Hybrid vehicle systems can be characterized as having multiple power sources whose output can be measured in Watts of power. Mechanical and electrical systems alike can be described this way and so in describing a best case or *optimal* distribution of power,  $P_{S_i}$ , from the individual sources ( $S_i$ ), the high level driving function is a linear sum-of-components equation with normalized components summing to one,

$$\sum_i \frac{P_{S_i}}{N} = 1. \quad (1)$$

This concept of ordered sums is covered as part of the second course in calculus which is part of the curricula for the students of all three disciplines. From here it remains to work both in the reverse to derive equations or functional relationships between the individual components,  $S_i$ , and the energy source of the component, e.g. with respect to an internal combustion engine (ICE), the relationship between the fuel burned and the output power of the ICE and to work forward to determine a functional relationship between the output power,  $N$ , and the velocity and acceleration of the vehicle. Consider first a parallel hybrid system, such as the Honda Civic Hybrid, in which the ICE and electric motor provide torque to the ICE drive shaft. In this case, both power sources apply power at the same rotational speed and therefore the sum of normalized power components can be written as a sum of normalized torque components using a rotational speed scaling term. For conventional hybrid vehicles, output power is applied as torque to some mechanical drive line at a given angular velocity,  $P = T \cdot \omega$ , which is in turn connected to a final drive gear set and the vehicle drive wheels, working forward also allows linear computations for gear ratio and scaling constants for wheel radius. Modeling of differential slip and other nonlinear aspects of conventional drive lines can be considered once the steady state relationships are established. Acceleration and velocity of a vehicle can then be computed by knowing the torque at the vehicle wheels and through a model of the vehicle loads, friction and mass. An example of this type of model with a fixed gear ratio from [5] is shown,

$$r_g^2 \dot{\omega} = (T_e - T_{fb} - T_{aero} - J_e \dot{\omega}) \cdot \frac{1}{M} - \frac{gr_g^2}{\cos(\alpha)} \cdot \sin(\beta + \beta_\mu) \quad (2)$$

where  $J_e$  is the ICE lumped inertia,  $M$  is the vehicle mass,  $r_g$  the total gear ratio and  $\{T_x : x \in \{e, fb, aero\}\}$  are the individual ICE, friction and aerodynamic torque components. The parameter,  $\mu$ , is the coefficient of rolling resistance and  $\beta$  the road grade. Aerodynamic loading is dependent on vehicle velocity,

$$T_{aero} = C_q r_g^3 \omega^2.$$

and

$$\beta_\mu = \tan^{-1}(\mu).$$

This is a very general construction relating torque, rotational velocity and vehicle physical parameters. It is interesting to note that equation (2) can be written as a linear equation in  $\theta$  where

$$\theta = \left[ \frac{1}{M}, \sin(\beta + \beta_\mu) \right]^T$$

for the purpose of studying linear effects due to vehicle mass change or change in road grade. By considering a discrete time model with index  $k$ , equation (2) can be written in a difference equation form which approximates the dynamics of the ordinary differential equation form. We have at time  $t$ ,

$$\begin{aligned} \dot{\omega}(t) &= (T_e(t) - T_{fb}(t) - T_{aero}(t) - J_e \dot{\omega}(t)) \cdot \frac{1}{M \cdot r_g^2} - \frac{g \cdot \sin(\beta(t) + \beta_\mu(t))}{\cos(\alpha)}, \\ &= (C_1 P_e(t) - C_2) \cdot \omega(t) - C_3 \omega^2(t) - C_4, \\ &\approx \frac{1}{T} \cdot (\hat{\omega}(k+1) - \hat{\omega}(k)), \end{aligned}$$

where  $T > 0$  is the discrete time interval and  $\omega(k) = \omega(t)$  for  $kT \leq t < (k+1)T$ . In this way one can write

$$\hat{\omega}(k) = \hat{\omega}(k-1) + T \cdot \left( (C_1 \hat{P}_e(k-1) - C_2) \cdot \hat{\omega}(k-1) - C_3 \hat{\omega}^2(k-1) - C_4 \right),$$

where  $\hat{P}_e(k)$  is the discrete version of the ICE power  $P_e$ . Note that now the approximate velocity,  $\hat{\omega}$ , can be solved using a quadratic function of prior (in time) velocity terms. Of course for steady state computations,  $\dot{\omega} = 0$  and  $\omega(t)$  can be found by solving the resulting quadratic equation. Both difference equation concepts and finding the roots of quadratic functions are covered in coursework which is common to all of the engineering disciplines discussed in this paper. Working in reverse from the *power* terms,  $P_{S_i}$ , one can consider the voltage-amperage relationship with electrical power, i.e.  $P = VI$ , and continue to work backwards through simplified circuit equivalent DC or AC motor models. One can also consider physics-based or (non-physics)-based regression derived functions relating ICE output power to input air quantity (or throttle opening), fuel quantity and spark timing. Once these key input parameters are identified and their functional relationship to the output power of devices  $S_i$  is established, various concepts and methodologies for optimization of these parameters can be considered. Schemes for the minimization or maximization of quadratic objective functions can be introduced using derivatives and vector calculus. These types of functional relationships can be understood and manipulated by the engineering students given their mathematical background. See Figure 3 for a summary of this process.

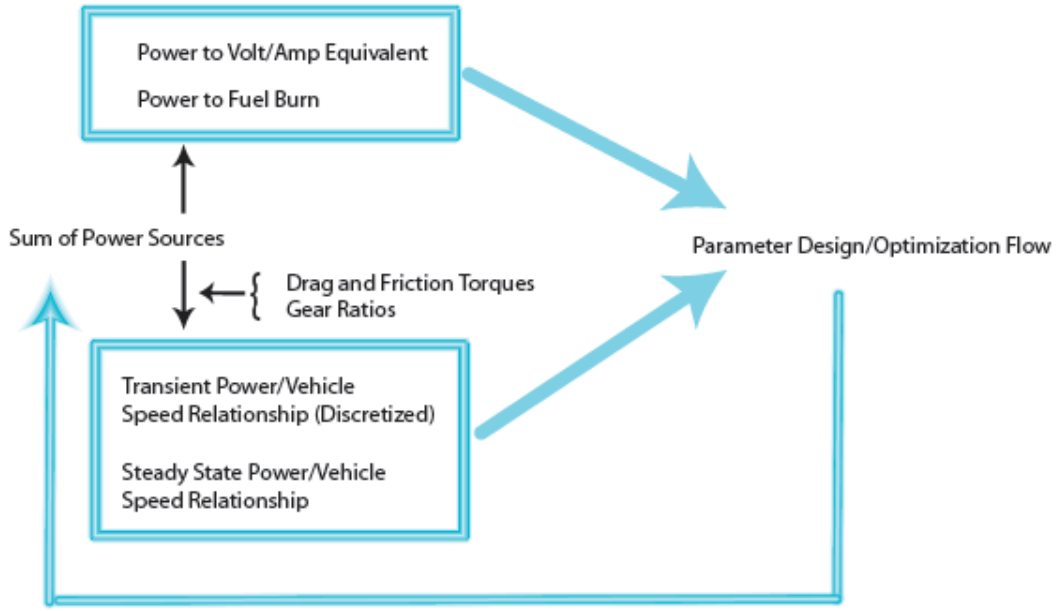


Figure 3: Derivation flow from the sum in equation (1) to the free parameters.

In a series/parallel system such as is found in the Toyota Prius and the Ford Hybrid Escape, a planetary gear set provides the mechanical link connecting the ICE, the electric drive motor and an electric motor/generator which are independently controlled to allow power flow to and from all three components, typically called *Power Split*. Each power source component in equation (1), in this case, is divided by its independent rotational velocity term to yield component torque,

$$T_i = \frac{P_{S_i}}{\omega_i}.$$

The planetary gear set, having a *sun* gear, *planet* or *carrier* gears and the *ring* gear, provides a linear relationship between the component rotational velocities through the following [6],

$$\omega_s + k\omega_r - (k + 1)\omega_c = 0,$$

where  $\omega_s$  is the rotational velocity of the *sun* gear,  $\omega_r$  the rotational velocity of the *ring* gear,  $\omega_c$  the rotational velocity of the *carrier* gears and  $k$  the ratio of the *ring* and *sun* gear radii. This configuration allows one to retain the linear and quadratic functional relationships between the power sources and the free parameters shown in Figure 3. The individual torque levels of the planetary gears can be described using ordinary differential equations and can either be included in the curriculum using the difference equation approximation methodology described earlier or revisited in more advanced coursework. This simplified derivation process is utilized in the list of curriculum topics to provide a broad overview the hybrid vehicle systems and associated technologies. The curriculum list is provided below.

- Basic Vehicle Dynamics and Dynamic Modeling
- Road Load and Friction
- Fuel Consumption and ICE modeling (basic)
- Hybrid Vehicle Components (Mild vs. Full)
- Series Hybrid
- Parallel Hybrid
- Power Split Hybrid
- Planetary Gear set and CVTs
- Electric Drives
- Brakes and Energy Recovery
- Battery Storage and Reliability
- Power plant sizing
- HV Electronics
- Sensors and Communication
- Optimization
- Linear Control Primer (discrete-time)
- Efficiencies and Trade-Offs

#### Concluding Remarks and Other Opportunities

This paper has shown a methodology to introduce hybrid vehicle systems topics to a broad engineering audience having varying levels of background in mathematics and electronics. This methodology emphasizes the use of linear and quadratic equations in an effort to incorporate optimization concepts at the undergraduate level and to provide a common ground for the students. Other opportunities to follow a similar functional flow process as is shown in Figure 3 are possible such as in the development of mathematical models for simulation. Similar functional relationships can be exercised in simulation by testing ranges of values for the free parameters quickly through Monte Carlo or similar techniques.

## Bibliography

- [1] C. Chen (1984), *Linear System Theory and Design*, Holt, Rinehart and Winston, Inc., New York.
- [2] G. Strang (1988), *Linear Algebra and Its Applications*, Harcourt Brace Jovanovich, Inc.
- [3] C. Desoer and E. Kuh (1969), *Basic Circuit Theory*, McGraw-Hill, New York.
- [4] C. Goffman (1965), *Calculus of Several Variables*, Harper and Row Publishers, New York.
- [5] A. Vahidi et. al. (2003), “Experiments for Online Estimation of Heavy Vehicle’s Mass and Time-Varying Road Grade”, IMECE 2003.
- [6] J. Miller (2004), *Propulsion Systems for Hybrid Vehicles*, Institution for Electrical Engineers, London, UK.