AC 2012-4052: DESIGNING FOR IMPROVED SUCCESS IN FIRST-YEAR MATHEMATICS

Prof. Helen M. Doerr, Syracuse University

Professor of mathematics and mathematics education

Dr. Andria Costello Staniec, Syracuse University

Andria Costello Staniec is an Associate Professor in civil and environmental engineering at Syracuse University (SU). Since 2010, she has served as the Associate Dean for Student Affairs in the LC Smith College of Engineering and Computer Science at SU. As Associate Dean, Costello Staniec has focused on student success both through the development of structured retention programs and through one-on-one interventions with students.

Mrs. AnnMarie H. O'Neil

Designing for Improved Success in First Year Mathematics

In responding to the need to improve retention in the first year of engineering, many institutions have developed a range of academic support programs, including learning communities, peer mentoring, summer bridge programs, tutoring and supplemental instructional workshops.[1,2,3,4,5] Other institutional efforts are aimed at curricular changes involving new approaches to teaching and learning, such as student-centered pedagogies and design projects in first-year engineering courses.[6,7] However, students' difficulties with first year mathematics courses remain a widespread and consistent barrier to success in engineering for many students.[8,9] In this paper, we will present the results of three programs designed to address student success in first year mathematics courses. We will also describe the design principles that guided the development and implementation of these programs.

Background

While no single factor or simple explanation accounts for student success and retention in the first year of their engineering program, there is widespread agreement that success in their first mathematics course is strongly linked to student retention.[10,11,12] Much has been learned over the past two decades of mathematics and engineering education research about new approaches to teaching and learning, such as student-centered pedagogies, engineering learning communities, and design projects in first-year engineering courses.[13,14] Such approaches have been shown to lead to improved student outcomes: gains in students' understandings of engineering concepts and gains in student retention, particularly among women and underrepresented minorities. Achieving widespread and systemic implementation of these innovations, however, has been a stubborn and persistent problem.[15,16,17] To address student success in their first year mathematics course, the department of mathematics, in collaboration with the college of engineering, designed three changes in the mathematics program for pre-freshmen and first year students. In the next section we will describe these three programmatic changes. We then describe the design principles that guided the development and implementation of these programs, followed by a discussion of the lessons learned.

Programmatic Changes in Mathematics

We developed and implemented three changes in the mathematics program for engineering students: (1) a revision of the summer bridge program mathematics course; (2) the creation of a modified pre-calculus course for students who would otherwise have been placed in a college algebra course; and (3) the implementation of a Calculus I course for students who had not taken calculus in high school. In each of the sections that follow, we describe the engineering students for whom the change was targeted and our specific goals for the changes we implemented.

The Summer Bridge Program

The summer bridge program is a six-week residential program that provides pre-freshmen engineers with an opportunity to become familiar with the university's academic, social, and cultural life. Students recruited for this program include all first year engineers with an SAT math score less than or equal to 590 and all underrepresented minority women. Over the last five years, the summer program has comprised an average of 28% women and 77% underrepresented minorities. In previous years, students took a six-week version of freshmen college algebra or a precalculus course. This six-week summer course was substantially the same version of the course that was taught over 14 weeks during the regular academic semester. As such, it was essentially an accelerated version of the course content; much of this course content is identical to that taught in high school mathematics courses. Upon successful completion of this mathematics course, students would then be placed into either pre-calculus or Calculus I during the fall semester. We found that students taking a mathematics course as part of this program performed almost a full letter grade lower in their first semester math course than their peers (matched by gender, ethnicity, and SAT math score) who did not participate in the summer program.

To address this gap, we completely re-designed the summer mathematics course to be organized around a deep understanding of the average rate of change, a foundational concept for the later study of calculus. Our primary goal was to prepare students for success in their first mathematics course, whether that placement was pre-calculus or calculus. We explicitly designed the course so as not to be duplicative of content taught in pre-calculus, since some students would likely be placed in that course in the fall. Rather, we wanted to engage students in the study of a topic that brings together core concepts from both engineering and mathematics, namely, quantifying and interpreting change. To do this, we designed tasks that engaged the students in creating and interpreting models of physical phenomena that change. We hoped to achieve a deep understanding of average rate of change through the hands-on investigation of the behavior of linear and non-linear phenomena and how these phenomena change with respect to time.

We organized the course around four modeling tasks: (1) working with motion detectors to analyze linear and quadratic motion and their respective rates of change; (2) working with computer simulations to interpret velocity and position graphs; (3) using light sensors to model the intensity of light with respect to the distance from the light source; and (4) building a simple RC circuit to charge a capacitor and then creating a mathematical model that can be used to analyze the change in voltage across the capacitor as it discharges. In light intensity task, students were asked to create a model of the intensity of light with respect to the distance from the light source and to analyze the average rates of change of the intensity at varying distances from the light source. The students measured, described and interpreted the non-constant rate at which the intensity changed as the distance from the light source increased. In the fourth modeling task, students investigated the rate at which a fully charged capacitor in a simple RC circuit discharged with respect to time, a phenomena with an exponential rate of change. The students built the circuits, charged a capacitor, and measured the voltage drop across the capacitor as it discharged. Students were given a set of resistors and capacitors and were asked to develop a model they could use to answer these three questions: (1) How does increasing the resistance affect the rate at which a capacitor discharges? (2) Compare the rates at which the capacitor is discharging at the beginning, middle and end of the total time interval. How does the average rate of change of the function change as time increases? (3) How does increasing the capacitance affect the rate at which a capacitor discharges? Taken together, these modeling tasks focused the students' attention simultaneously on the quantity that was being measured and on how that quantity was changing with respect to some other quantity (i.e., distance or time). A coordinated understanding of these two quantities is at the crux of representing and reasoning about changing phenomena.[18]

Since a primary goal of this course was to prepare students for subsequent success in their first year mathematics courses, we also wanted to develop students' algebra skills. However, we did this within the context of problem situations that were relevant to engineering. We explicitly focused on three topics that are well-known sources of student difficulties and are essential in pre-calculus and calculus:

- rational expressions and complex fractions
- exponential expressions and equations
- logarithmic expressions and equations

These three topics were directly related to the mathematical content in the four tasks described above, thus providing students with the opportunity to use their algebra skills in a meaningful context. In addition, we provided skills practice through the use of an on-line homework system. Together, these changes contributed to closing the previous grade gap between summer and non-summer students in their first semester mathematics course.

An Alternative to a College Algebra Course

While many incoming students enroll in a calculus course, some place into a pre-calculus course and some into a college algebra course. Very few of those students who placed into college algebra in their first semester were retained in engineering. Among other things, a placement into college algebra precluded taking physics in their spring semester, a key pre-requisite for other engineering courses. To address the needs of those students who place into college algebra and prepare them for Calculus I in one semester rather than two, we introduced a new version of pre-calculus, which met five times per week, rather than four. We describe this course as an "algebra infused" pre-calculus course. Our standard pre-calculus course, which has three lectures and a recitation section, is intended to develop students' understandings of functions, essential knowledge for the study of calculus. As such, the course is organized around three main ideas: families of functions and their properties, the average rate of change of a function, and transformations of functions. A faculty member teaches the standard pre-calculus course in a large lecture to engineering freshmen, with the recitation sections taught by teaching assistants. Students who succeed in this course with an A or a B are generally successful in our subsequent course in Calculus I.

While algebra skills are addressed in the standard pre-calculus course, the development of those skills is not a primary focus of the course. Students need a pre-requisite level of fluency in algebra to be successful with the pre-calculus content on functions. Students who placed into college algebra did not meet that pre-requisite. To address this deficiency, we designed an "algebra infused" pre-calculus course. The algebra infused precalculus followed the same syllabus as the standard course, with one essential difference. In this course, additional time was spent on algebra topics that were taught as needed for the pre-calculus topics throughout the semester. Thus, the instructor would spend the additional time pre-teaching algebraic skills needed for the precalculus content, such as finding the equation of line, developing laws of exponents and logarithms, and simplifying rational expressions. Otherwise, the level of difficulty of homework, quizzes, and exams of the algebra infused course was identical to the standard course, which served as a control group for comparison purposes.

The algebra infused course was taught by a teaching assistant, who met with the students five days per week. We selected the teaching assistant from among mathematics education graduate students in the department, in order to have an instructor who was experienced in pedagogies that were oriented toward collaborative work, assessing students' prior knowledge, and addressing student misconceptions. Both courses had the same homework assignments, with some additional algebra homework in the algebra infused course, and the same final exam. The algebra infused precalculus course included only those students who would have otherwise placed into a college algebra course; the standard precalculus course was taken only by those students who had placed into precalculus and thus were judged to be prepared for the course material. We compared the final course grades for these two groups of students.

We measured the student success by the percentage of students who achieved an A or a B in the course; our historical data has shown that such students are well prepared for subsequent success in Calculus I. In the first offering of both courses, 62% of the students enrolled ($n = 76$) in the standard course achieved a final grade of an A or a B and 63% of the students enrolled ($n = 32$) in the algebra infused course achieved an A or a B. Aggregating the data over the four years we have taught both courses, we found that 58% of the students ($n = 274$) in the standard course achieved a final grade of an A or a B and that 56% of the students ($n = 161$) in the algebra infused course achieved a final grade of A or B. We take this to be a positive outcome. A significant number of freshmen engineering students who otherwise would have taken two semesters to prepare for the study of Calculus were successfully prepared in one semester.

A First Course in Calculus

Many first year students have had a year of calculus in high school, but for a variety of reasons either do not take or do not succeed on an examination (such as the AP Calculus exam) so as to earn college credit for Calculus I. In general, we have found that approximately 70% to 80% of the students who place into Calculus I have had a high school course in calculus, and 20% to 30% of those who place into Calculus I have no previous calculus background. Not surprisingly, this difference in background led to a full letter grade difference in students' performance in the course. The median course grade for those who had had calculus in high school was a B and for those who had not had calculus in high school the median grade was a C. However, we suspected that the reasons for this difference were not simply attributable to better mathematics abilities on the part of those who had had a course in calculus in high school. Indeed, students who have been prepared for and succeeded in a high school calculus course will have taken and succeeded on the AP Calculus exam (or its equivalent) and placed into Calculus II. Anecdotally, we found that some students (without prior knowledge of calculus from high school) were discouraged in a large lecture course where seemingly everyone else was familiar with what to them was new and challenging material. These "new to calculus" students often felt that others were learning the material more quickly and with greater ease, not realizing that most of their "peers" were, in fact, re-learning the course material. Again, largely anecdotally, we found that some students were frustrated in recitation sections, where the teaching assistant would often move at pace suitable for those who were learning the material for the second time. In such a setting, some of these "new to calculus" students were reluctant to ask for help or clarification on material that was not immediately obvious to them. We suspected that these factors might have contributed to the gap in performance between these two groups of students.

To address the letter grade performance, we divided the Calculus I course into two sections, with one section of the course exclusively for those students who have not had calculus in high school. The other section was comprised of students who were re-taking calculus. Both sections were taught by a faculty member in a large lecture format, meeting three days per week with a recitation section taught by a teaching assistant. The syllabus for the two sections of the course, the homework assignments and the final examination were identical. The median grade for both sections of the course was a B- in the first year of implementation; in the second year of implementation, the median course grade for those who had had calculus was a B- and for those who had not had calculus the median grade was a B. We closed the full letter grade gap that had existed between students who had taken calculus in high school and those who had not. This suggests that many of the students who were new to calculus were, in fact, well prepared from their high school background for the study of calculus and were better served by being in a course where they were learning with peers who were also learning the material for the first time.

Design Principles

The changes we have described were the outcome of collaborative efforts between faculty in the college of engineering and in the department of mathematics. In this section, we wish to describe what we see as the underlying design principles that guided the development and implementation of these changes. As we noted earlier, widespread changes in practice for preparing engineers have been difficult to achieve, despite the repeated calls for change and despite much quality research on the nature of effective changes. Hence, we wish to argue that what is needed in the field of engineering education are not just exemplars of improved practice. We also need principles that would guide the implementation of improvements in practice on a widespread basis. To that end, we put forward four principles that have in large part guided the changes we have described:

- First, the changes have been *data driven*.
- Second, the changes have been designed as *incremental*, rather than a large restructuring of our programs.
- Third, the changes have been designed to be *sustainable*. Many innovations falter when the enthusiasms of early adopters are not shared by those who follow.
- Fourth, the changes have been designed to be *generative* of further changes.

By beginning with *data driven* changes, we have been able to identify, with some level of specificity, a particular problem for which we need to design a solution. Our understanding of the data also allows us to become clearer about what constitutes a satisfactory solution or improvement. Thus, for example, by identifying the need for students to successfully complete pre-calculus and its pre-requisite algebra in one semester, instead of two, we had a specific problem to address. At the same, we were able to be clearer about what would constitute success in our particular setting. Our goal was to design a solution that would enable the same or nearly the same percentage of students in a revised course to achieve a final grade of an A or a B as did the students in the standard course. Other universities, with other settings and other constraints, might well have different criteria for success. By having clear criteria for success, we were able to closely monitor our progress toward this goal during the first implementation. This resulted in some changes and modifications as we first taught the course. But, more importantly, it allowed us, in this case, to judge the success of the re-designed course in meeting our design specifications.

All three of the programmatic changes we have made have been *incremental* changes to our overall programs. Thus, we did a re-design of the mathematics course in the summer bridge program to integrate engineering and mathematics topics while addressing a core concept (average rate of change) for subsequent success in mathematics courses. This was a single change within the overall program, as opposed to a complete restructuring of the summer bridge program. The advantage of this approach is twofold. First, this gives us a manageable problem to work on. We chose the mathematics course because of its key role (based on the literature) in the first year success of engineering students. As an incremental and manageable problem, we can use largely existing resources to design solutions to the problem. Second, as an incremental change, we are more confident in our abilities to measure the success of the change. As with any educational innovation (as in any complex system), there are numerous interacting factors, multiple feedback loops, and indirect as well as direct effects to account for. By making an incremental change, we have an opportunity to better understand the particulars of a local change that is happening within a larger, complex system. There are of course limitations to an incremental approach. A potential limitation is that the changes made will not make a significant difference because other factors that account for more of the variance have not been addressed. However, we have attempted to address this limitation by our focus on problems for which we have data that help us define criteria for measuring success. This approach is fundamentally based on designing solutions to local problems, but with a global view in mind, and in this way is similar to the approach to educational improvement in Japan known as lesson study and to design-based research approaches in mathematics and science education.[19, 20]

In designing these three programmatic changes, we paid careful attention to developing and implementing changes that would be *sustainable* largely within existing resources. As noted earlier, many innovations falter when the enthusiasms and energies of early adopters are not shared by those who follow. By having a sustainable and local view, we take as given the constraints in our particular setting and we design solutions that will work within those constraints over time. Thus, for example, in dividing our first year engineering students into those who have had calculus before and those who have not, we wanted both sections of the course to be taught by any faculty member with the usual teaching assistants. The same syllabus continues to be used in both sections, with a common final exam. The mathematics department was not able to easily identify those students who have had calculus in high school. To do so, we enlisted the help of the student records staff in the college of engineering who then registered students in the appropriate sections. In the second year, we realized that we needed to design a mechanism to prevent students from changing sections after they had been assigned to the appropriate section. Our results to date have sustained closing the gap between who had had calculus in high school and those who had not. Similarly, different instructors and different teaching assistants have taught the standard and the algebra infused pre-calculus course with good results over a period of four years. While there has been variation between instructors and between years, the aggregated results have sustained the achievement specified by our initial goals.

Finally, in designing these changes, we have attempted to make those changes *generative* of further change. By this we mean that we intend that the local changes, which are data driven, incremental and sustainable, will in turn lead to other changes in our programmatic efforts to improve student success in first year mathematics and ultimately in the retention of students in engineering. First, we would point out that this set of changes was in and of itself generative in the following sense. The number of students placing in college algebra instead of pre-calculus had been identified early on as a problematic area. As we made progress in addressing that issue, we began to examine the placement of summer bridge program students into their first course. We found that this program was not having the success that we wished. At the same time, we identified students who, despite adequate high school preparation but lacking calculus, were not succeeding as well as their peers. So collectively, these incremental (and local) changes contributed to what became a larger set of initiatives. However, at the same time, as part of our on-going efforts to improve success and retention, we continue to examine programmatic data so as to identify specific problems for which we can design new solutions.

Lessons Learned

Our goal in this paper has been twofold: first, to describe changes in the first year mathematics courses for engineers that we have developed and successfully implemented and, second, to propose principles that can be used to guide the further design of effective changes. One of the lessons that we have learned in this work is the need to attend to both the actual changes that are being made and to the processes of change. In so doing, we have generated principles that can guide our continuing work. A second lesson learned is the centrality of taking a long term view of the changes being made. This long-term view means that while we are acting locally to design changes that result in current improvements, we do so with an eye to the global issues at hand. In addressing changes in any complex human system, such as education, there is much variation present, most of which cannot be controlled in any meaningful sense. Hence, we have taken a design-based approach that can yield improvements that can be measured locally and aggregated over time, while at the same time giving us insight into how to be effective in implementing change.

References

[1] Gattis, C., Hill, B., & Lachowsky, A. (2007). A successful engineering peer mentoring program. In *American Society for Engineering Education Annual Conference and Exposition, Conference Proceedings.*

[2] Jones, S., Rusch, K., Waggenspack, W., Seals, R., & Henderson, V. (2010). S-STEM: Eng^2 scholars for success engineering engagement. In *American Society for Engineering Education Annual Conference and Exposition, Conference Proceedings.*

[3] Kukreti, A., Simonson, K., Johnson, K., & Evans, L. (2009). A NSF-supported S-STEM program for recruitment and retention of underrepresented ethnic and women students in engineering. In *American Society for Engineering Education Annual Conference and Exposition, Conference Proceedings.*

[4] Massi, L., Georgiopoulos, M., Young, C., Ducharme, A., Ford, C., Small, K., Lancey, P., & Bhatia, D. (2010). YES: A NSF S-STEM scholarship program experience at the University of Central Florida. In *American Society for Engineering Education Annual Conference and Exposition, Conference Proceedings.*

[5] Russomanno, D., Best, R., Ivey, S., Haddock, J., Franceschetti, D., & Hairston, R. (2010). MemphiStep: A STEM talent expansion program at the University of Memphis. *Journal of STEM Education, 11*(1 & 2), 69-81.

[6] Hoit, M., & Ohland, M. (1998). Impact of a discipline-based introduction to engineering course on improving retention. *Journal of Engineering Education, 87*(1), 79-85.

[7] Kilgore, D., Atman, C. J., Yasuhara, K., Barker, T. J., & Morozov, A. (2007). Considering context: A study of first-year engineering students. *Journal of Engineering Education, 96*(4), 321-334.

[8] Bullock, D., Callahan, J., Ban, Y., Ahlgren, A., & Schrader, C. (2009). The implementation of an online mathematics placement exam and its effects on student success in precalculus and calculus. In *American Society for Engineering Education Annual Conference and Exposition, Conference Proceedings*.

[9] Alkhasawneh, R., & Hobson, R. (2010). Pre-college mathematics preparation: Does it work? In *American Society for Engineering Education Annual Conference and Exposition, Conference Proceedings.*

[10] Hampikian, J., Gardner, J., Moll, A., Pyke, P., & Schrader, C. (2006). Integrated pre-freshman engineering and precalculus mathematics. In *American Society for Engineering Education Annual Conference and Exposition, Conference Proceedings.*

[11] Li, Q., Swaminathan, H., & Tang, J. (2009). Development of a classification system for engineering student characteristics affecting college enrollment and retention. *Journal of Engineering Education, 98*(4), 361-376.

[12] Weatherton, Y., Kruzic, A., Isbell, B., Peterson, L., Tiernan, J., & Pham, V. (2011). Mathematics performance and first year retention of students in engineering learning communities. In *American Society for Engineering Education Annual Conference and Exposition, Conference Proceedings.*

[13] Hoit, M., & Ohland, M. (1998). Impact of a discipline-based introduction to engineering course on improving retention. *Journal of Engineering Education, 87*(1), 79-85.

[14] Kilgore, D., Atman, C. J., Yasuhara, K., Barker, T. J., & Morozov, A. (2007). Considering context: A study of first-year engineering students. *Journal of Engineering Education, 96*(4), 321-334.

[15] Borrego, M., Froyd, J. E., & Hall, T. S. (2010). Diffusion of engineering education innovations: A survey of awareness and adoption rates in U. S. engineering departments. *Journal of Engineering Education, 99*(3), 185-207.

[16] Felder, R. M., Brent, R., & Prince, M. J. (2011). Engineering instructional development: Programs, best practices, and recommendations. *Journal of Engineering Education, 100*(1), 89-122.

[17] National Science Foundation. (2008). *Innovations in Engineering Education, Curriculum, and Infrastructure.* Retrieved on 4/20/2011 from http://www.nsf.gov/pubs/2008/nsf08542/nsf08542.htm

[18] Oehrtman, M., Carlson, M., & Thompson, P. (2008). Foundational reasoning abilities that promote coherence in students' function understanding. In M. Carlson & C. Rasmussen (Eds.), *Making the connection:*

Research and teaching in undergraduate mathematics education (pp. 27-41). Washington, DC: Mathematical Association of America.

[19] Lewis, C. (2002). *Lesson study: A handbook of teacher-led instructional change*. Philadelphia, PA: Research for Better Schools.

[20] Kelly, A., Lesh, R. A., & Baek, J. Y. (2008). *Handbook of design research methods in education: Innovations in science, technology, engineering and mathematics learning and teaching.* NY: Routledge.