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Developing an Interactive Computer Program to Enhance Student Learning of Dynamical Systems

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Abstract

Today's students are quite accustomed to availing themselves of the latest in computer innovations and technology to aid in learning and the attainment of student outcomes. For example, use of tablets and cellphones in the classroom to take notes, collaborate on projects and to search the web for information is commonplace. Likewise, advancements in computer software and tools afford in-depth simulations of both mechanical and thermal systems. MATHEMATICA, with its symbolic and visual capabilities, is one such tool that, despite its robustness, has seen little utilization in the classroom environment, yet it is viewed as a tool for those who pursue research in every discipline from economics to engineering. In this paper, the capabilities of MATHEMATICA are explored as a tool to model and visualize the forced mechanical response of viscoelastically-damped, multiple degree of freedoms systems obtained through a Newtonian approach. Proportional damping permits the resulting Eigen-value problem to be diagonalized using a Cholesky Decomposition method. In addition, multiple harmonics can be included as part of the forcing function. Displacement results for each mass permit the generation of graphical results and also provide the needed inputs for animated motions of all included masses. While the ultimate goal is to solve the dynamic response of general n^{th} degree of freedom systems, explicit results are presented for the second, fourth, and tenth order degree of freedom system to demonstrate the efficiency of the software. All results are demonstrated in an interactive, user friendly program developed explicitly for this purpose.

Introduction

The development of low cost, high power computers continues to revolutionize how engineers perform their work in industry, academia and research oriented positions. Through enhanced graphical capabilities, these systems support modelling and analysis of complex systems that bring the solution of real world problems to the desktop. Universities maintain the latest of these systems, recognizing the direct benefit towards the attainment of student outcomes, especially in the engineering disciplines which need to comply with EAC-ABET criteria. Johannesen suggests that "When understood, more interesting and complicated situations can be explored with the help of computational tools"[1].Tajvidi et al note that "Particularly in engineering dynamics, Computer Simulation and Animation [CSA] modules can demonstrate motion of particles and rigid bodies through computer animations, helping students picture the concepts taught in the course"[2].Computers have their greatest impact not by displacing the entire course, but by providing an additional tool for aiding conceptual understanding, particularly by providing visualizations of complicated systems which are otherwise difficult to obtain and employ[2].Therefore, the continuing focus on developing effective tools to exploit both the power and potential of computers for the benefit of student understanding offers an excellent opportunity.

Continuing to access the power of computers requires the development of software tools accessible to students. One platform for these tools is Wolfram Research's MATHEMATICA, a computational software program widely used for its versatility and robustness. Importantly, MATHEMATICA operates on personal computers and possesses a large existing user base and documentation of its features. This paper presents the work of one student's research project, which is focused on using the visual and computational tools in MATHEMATICA to develop a program to study dynamical systems with the goal of enhancing the attainment of student learning outcomes. Using a forced spring-damper-mass system as the test case, a generalized code is developed that features input menus for system parameters, interactive graphics to output mechanical responses, and user tool that controls animations.

Background

This project originates from work performed by Midshipman 1/C Angela Roush, USN, as part of the Trident Scholar Program in *Dynamic Analysis of an Optical Laser Platform*[4]. In that work, Roush used a Lagrangian approaches to develop and solve models for the vibrations of a rigid plate mounted on viscoelastic supports under a variety of loads and initial conditions. While that project features the use of MATHEMATICA to solve large systems of equations, it does not develop the process into a generalized model, instead only focusing on the problem at hand. As an initial goal of this work, the dynamics of a single degree of freedom spring mass damper system is modelled in using Newtonian mechanics and extended to a general model applicable to a multiple degree of freedom system. . This model possesses enormous value for the student since it allows for the exploration of more complicated dynamic systems.

The derivation of the governing equations used in this work follows a similar approach of that presented Roush.[4] In short, the governing equations are developed, here a Newtonian approach is used instead of a Lagrangian approach, ; the Eigenvalue problem is formed incorporating proportional damping; and finally, Cholesky decomposition is used to determine the system's mechanical response using MATHEMATICA.

Problem Formulation

In this section, the equilibrium formulation yielding the governing equations and preferred method to determine the mechanical response for a two degree of freedom system is presented.

Figure 1 presents the two degree of freedom system containing dampers, masses and springs under consideration.

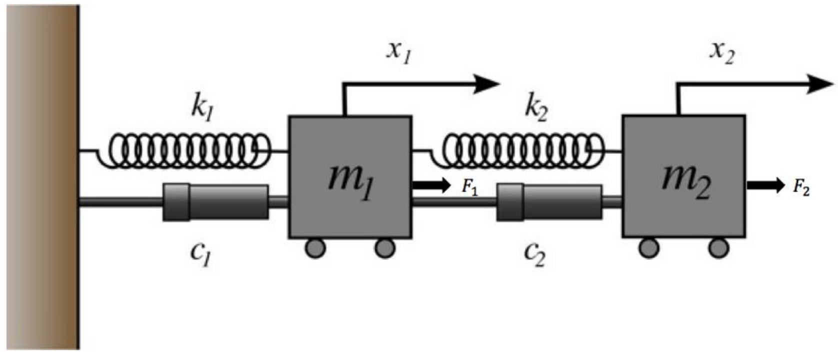

Figure 1: Spring-Mass-Damper System

While there are several approaches to determine the governing equations of a dynamical system, a Newtonian approach was used for this problem. By holding mass m_1 fixed and summing forces on mass m_2, the governing equation becomes

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = F_2(t) \qquad \text{Eqn.(1)}$$

Repeating this process for the mass m_2 provides the following equation

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = F_1(t) \qquad \text{Eqn.(2)}$$

Combining Eqn (1) and Eqn (2) into a matrix system of equations provides

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix} \qquad \text{Eqn.(3)}$$

Equation (3) is a system of equations coupled both dynamically and statically. Damping complicates the determining a solution in several ways. First, it is challenging to experimentally measure, and secondly, with damping present, there were no assurances that the matrix of eigenvectors would diagonalize the damping matrix, thus decoupling the system. In fact, Caugley and O'Kelly [13] have shown exactly that

$$[k][m]^{-1}[c] = [c][m]^{-1}[k] \qquad \text{Eqn.(4)}$$
$$[m][k]^{-1}[c] = [c][k]^{-1}[m] \qquad \text{Eqn.(5)}$$
$$[m][c]^{-1}[k] = [k][c]^{-1}[m] \qquad \text{Eqn.(6)}$$

must be satisfied in order for the system to be diagonalized.

Whenever Eqn.(4), Eqn.(5), and Eqn.(6) do not hold, then a proportional damping model is recommended in which the damping matrix [c] is expressed as a linear combination of the mass and stiffness matrix given by

$$[c] = \alpha[m] + \beta[k] \qquad \text{Eqn.(7)}$$

Constants α and β were selected to produce a desired damping ratio, based upon experimental results or design considerations.

Method of Solution

In light of the complicated structure of the governing equations, a modal solution is the preferred method of solution. An overview of this approach is presented here as it is not the focus of the paper but offers the means to solving the mechanical system response. This approach incorporates several steps including

1. A coordinate transformation using Cholesky Decomposition
2. A modal transformation
3. Solution in the problem in modal space
4. Transformation back to original coordinate space.

The code that was developed incorporates these steps quite efficiently and can solve most systems quickly.

Application of MATHEMATICA

The significant contribution provided by MATHEMATICA to a student rests on fact that regardless of a student's ability or inclination to solve a complicated, coupled system of equations by hand, with a single function call any student can rapidly create and analyze as complicated a system as he or she wishes. Naturally, an understanding of the mechanics performed by the program in the background is essential to the holistic understanding of the subject, but the program here functions similarly to an experiment: armed with theoretical knowledge, the student can rapidly explore numerous variations of the problem unencumbered by the need to perform repetitious calculations. Equipping the student with this code enables a more efficient exploration of the dynamics at play in these systems.

The functions containing the code, which mirrors the process of solving these types of problems manually, are encapsulated into a software package that any user can load into their copy of MATHEMATICA. Once loaded into MATHEMATICA, users can then call the appropriate function for their necessary operation. The function will then prompt the user to input system parameters including the number of degrees of freedom. Having stated the requisite system size, the user then inputs the required system parameters – masses, stiffnesses, forcing functions, and initial conditions. In short, the user merely has to enter the physical data governing the system, any initial conditions, and forcing functions, whereupon the program automatically generates the matrix system of governing equations and applies the methods mentioned above to produce a system of solutions. Students therefore do not have to actually perform calculations on the system, merely enter the system parameters, to analyze a system.

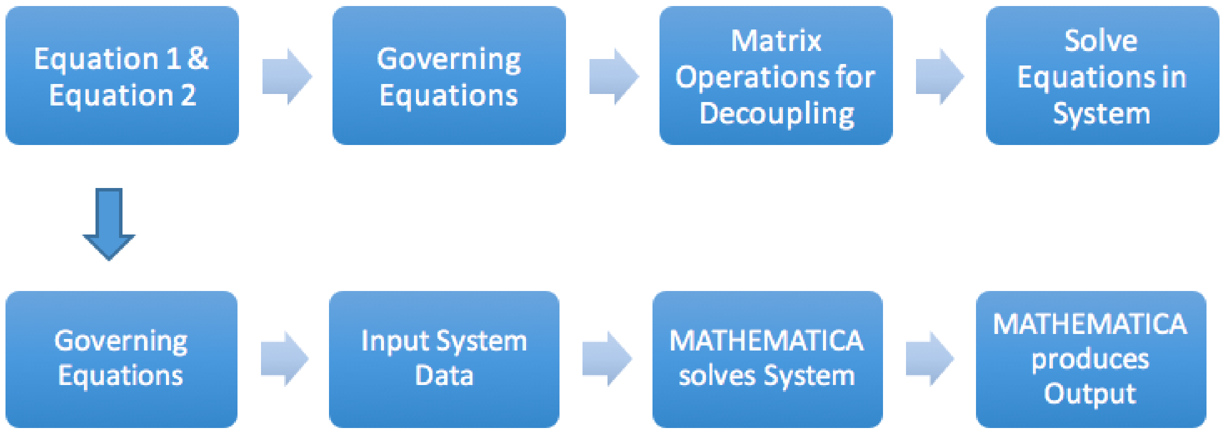

Figure 2: Upper Row details Solution Process, Lower Row Details MATHEMATICA Interaction

A test example is generated using data from Inman[3]. Here, the system parameters are

$$m_1 = 9\ kg \qquad k_1 = 24\frac{N}{m}$$

$$m_2 = 1\ kg \qquad k_2 = 3\frac{N}{m}$$

With initial velocities of each mass taken as zero, the initial displacements for mass is

$$x_1 = 1\ m \qquad x_2 = 0\ m$$

In addition, no forcing functions are applied and the damping coefficients are quite small:

$$\alpha = 0.00125\frac{1}{kg * s} \qquad \beta = 0.00125\frac{s}{kg}$$

Upon entering the information into the program, the function called massSpringSystem[] runs the code required to simulate the system. Upon completion, it outputs (Figure 1):

- system information input by the user to achieve this result is displayed;
- the solution for the mechanical response of each mass;
- the governing system of equations is presented in matrix form,
- a displacement versus time graph.

Figure 3 Program Output for 2 DOF free, moderately damped system

Notably, this graph has an adjustable time axis, allowing the user to dynamically adjust the length of time displayed between 0-100 seconds, including both the start and end times, to functionally zoom in on different parts of the graph as desired. Each of these sections also includes the ability to hide that piece of the output as desired by using the small arrow next to each section header. This feature provides significant value for very large systems where hiding the governing equations and their solutions can efficiently control the size of the output while preserving all information for future use. The program manages information this way to allow the user to prioritize which pieces of the information are most relevant without destroying the rest, emphasizing utility over unnecessary information.

The most useful feature, however, is the animation section. For any size, nth degree system, a simple representative animation is produced where the masses move according to their respective solutions. The animation features provide an excellent visualization feature.

This base case displays the capabilities of the MATHEMATICA driven software. Operations that, on paper, require tedious computation have successfully undergone automation such that, with available data, any user can analyze a system. Furthermore, the software here can perform these operations reliably time and time again. These two properties unlock a world of potential for a student. While a sufficient knowledge of the mathematics required to the produce the solutions remains essential for proper understanding, a student with this software can experimentally explore these systems relieved of the cost of constructing a physical experiment and unburdened by the need for tedious and repetitive matrix calculations. Unlocking the potential for students to explore large systems such as these allows them to garner an improved understanding they might not otherwise garner from traditional methods of instruction.

To see how this software allows the user to experience how changing variables effects behavior, let us consider the previous example, but with both damping coefficients raised to 0.75. Entering this data produces:

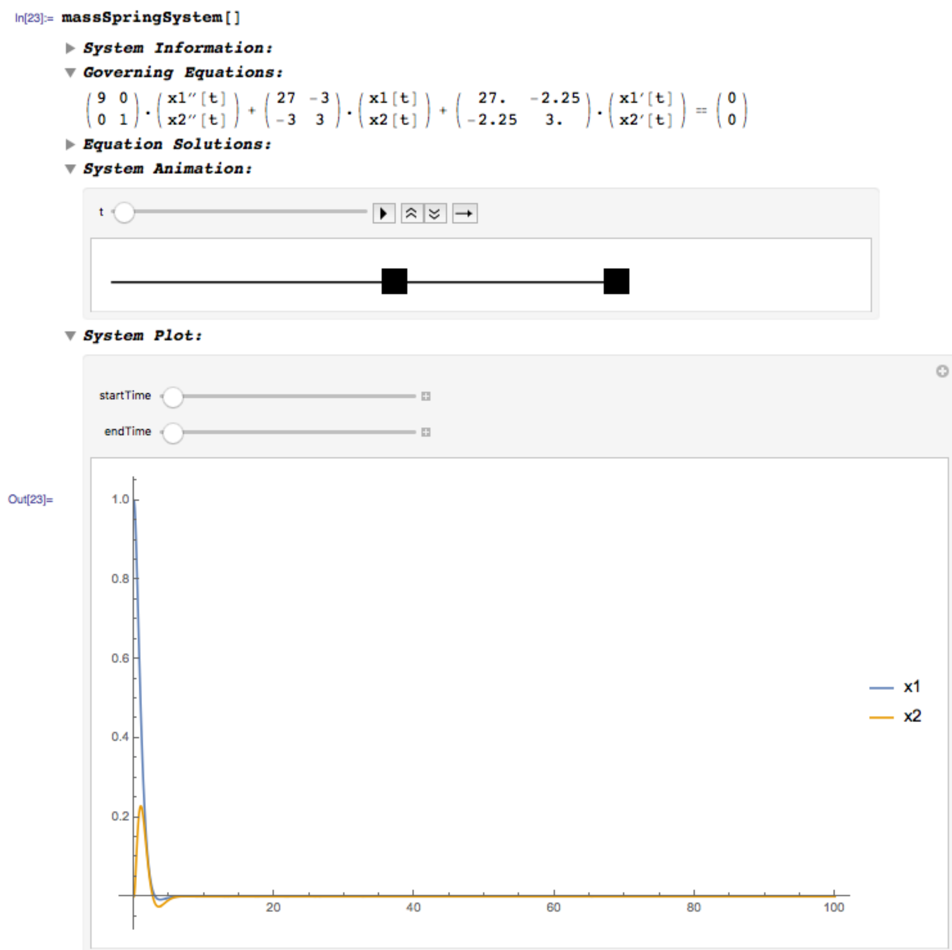

Figure 4 Program Output for 2 DOF free, heavily damped system

While this is a rather extreme shift in performance, it does amply demonstrate how the software operates. The change of only two variables dramatically altered the behavior of the system, information which the program provides readily to the student to explore the behavior of these systems.

For an additional exploration of these abilities, let us examine a system featuring the same mass and spring terms, and with high proportional damping coefficients of 0.75 but now add a multiple harmonic forcing function to further demonstrate the program's functionality.

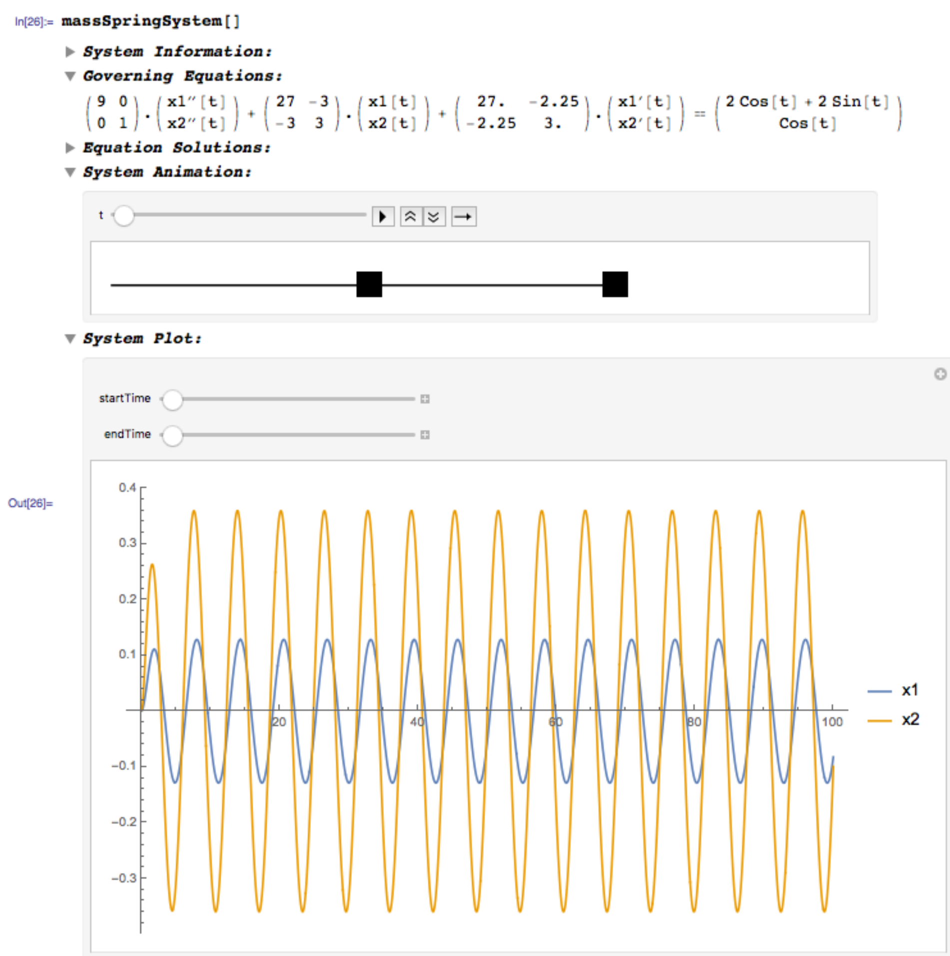

Figure 5 Program Output for 2 DOF forced, heavily damped system

Continuing the exploration of this dynamic system, the addition of the forcing functions overwhelms even the strong damping coefficients due to their own large amplitude. When shown in the actual program, each of these three related systems would appear next to each other, allowing for the behavior of the systems to be compared quickly. A student, therefore, can explore the changes in behavior caused by adjusting the values of variables and go back and see the prior behavior on demand.

For systems containing many degrees of freedom, analyze, Inman provides [3] the following data:

$$m_1 = 4000\ kg \qquad k_1 = 5000\ N/m \qquad x_1(0) = 0.025\ m$$
$$m_2 = 4000\ kg \qquad k_2 = 5000\ N/m \qquad x_2(0) = 0.020\ m$$
$$m_3 = 4000\ kg \qquad k_3 = 5000\ N/m \qquad x_3(0) = 0.010\ m$$
$$m_4 = 4000\ kg \qquad k_4 = 5000\ N/m \qquad x_4(0) = 0.001\ m$$

Again here the initial mass velocities are all zero. Inputting this information to the program, produces the following output:

Figure 6 Multiple DOF system, undamped, freely vibrating

Although solving this system does not pose any challenge to a computer, it nonetheless illustrates the capability to solve more large systems and thus prove more useful as an educational tool. This example simulates the motion of a building.

To demonstrate the real usefulness of computers in this analysis, the mass and spring terms do not equal each other, the damping constants are non-symmetric, and the initial displacements vary. The only simplification permitted is no forcing functions.

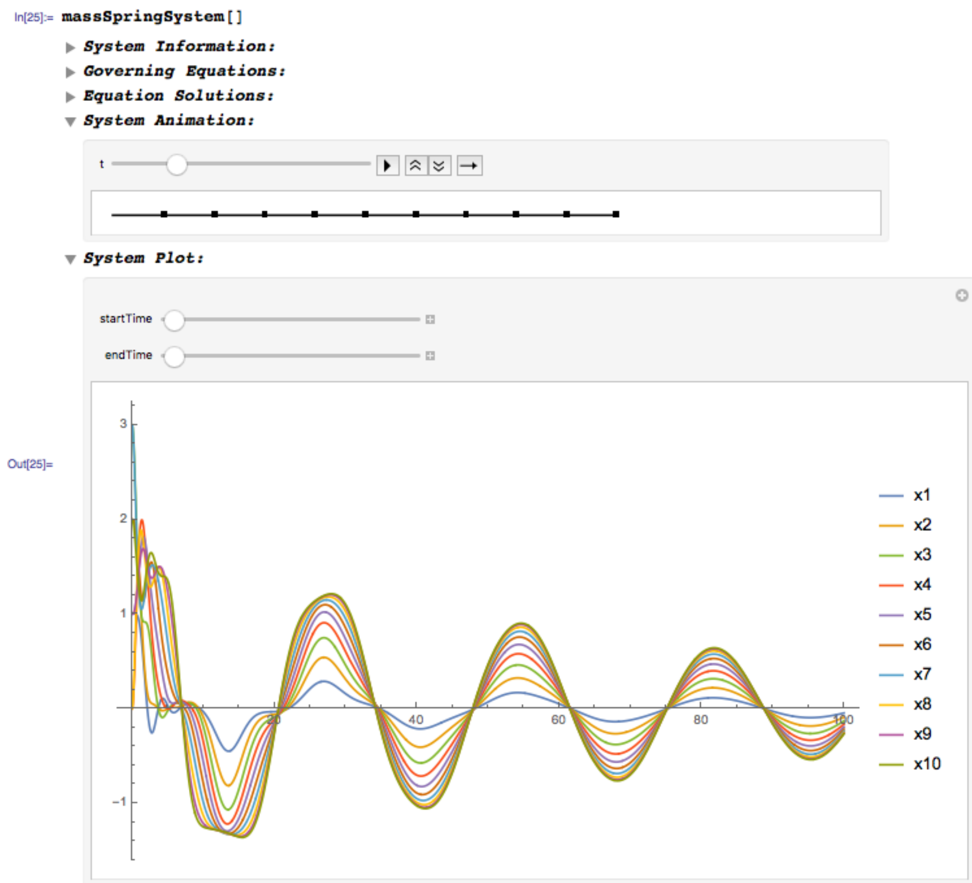

Figure 7Multiple DOF system with varying system parameters.

Despite the increased size, the pattern continues of consistently scaling the size of the program output to the size of the system. Importantly, due to the size of this system, the governing equations and solutions have been minimized to demonstrate the output control features of the program. As explored in the previous examples, the program essentially captures the properties of the mathematics and combines them with the efficiency and power of a computer to produce software that simplifies the work required of the user, particularly a student not yet well-versed in these systems.

Several features included in the software improve the flexibility of the simulation. First, the user can control the display of the solutions. Thus, for a relatively simple system as user may want to see the equations for immediate analysis, while for a system with many degrees of freedom the user may wish to hide the output in the interests of space until required. As mentioned before, the solutions are contained within a dropdown section which can be opened or closed at any time. Second, as touched on before, each system plot contains a summary of the the system's data. This makes comparing two systems straightforward, since all the relevant data needed to describe it appears with the corresponding graph. Third, the solution of these problems on a computer often gives rise to functions with extremely small coefficients, once witnessed with an order of magnitude of 10^{-29}. Functions with coefficients this small contribute nothing significant to the system, and as a form of trimming unnecessary excess, the program excises any function with a coefficient of 10^{-10} or smaller. This allows for the impactful functions to appear at the fore, uncluttered by miniscule numbers with equally miniscule physical significance. These features aim to improve the ability of a user to actually *use* the program without losing track of the objective in tiny details that detract from the main goal of the model. Especially from a student perspective, cutting the waste generated from using the software greatly enhances the potential for deriving understanding the system without a glut of useless data.

Implementation

While the code development is in its early stages, components are sufficient well-developed for use in the department's elective vibration course ME 499. During it offering, feedback from students will be captured after demonstrating features of the code during class and used to modify the code. Extension to distributed systems – rods, plates and shells – is possible with continue support for the OSCAR's undergraduate research support program.

Conclusion

Engineering students must accumulate and assimilate a truly staggering quantity of information throughout the course of their studies. While traditional teaching methods remain effective in imparting the necessary knowledge, technology increasingly offers new techniques which can add to the methods already employed. Specifically, research grade instruments such as MATHEMATICA can add substantially to the learning process for students by enabling them to explore areas of knowledge in ways unavailable through a whiteboard. As a representative case, software capable of modelling dynamic spring-mass systems of various sizes has been developed and presented, software which allows the student to run numerous experiments in the pursuit of understanding. By enabling the student to run these virtual experiments without the restrictions of cost and time requisite for building similar physical systems, the bounds of his or her understanding can be steadily increased by leveraging technology in this manner as a useful addition to teaching methods already widely deployed.

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