Developing Excel Macros for Solving Heat Diffusion Problems

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Abstract

This paper describes the use of Microsoft Excel Spreadsheet and Macro in solving diffusion problems. A one dimensional heat diffusion equation was transformed into a finite difference solution for a vertical grain storage bin. Crank-Nicholson method was added in the time dimension for a stable solution. The Excel spreadsheet has numerous tools that can solve differential equation transformed into finite difference form for both steady and unsteady boundary conditions. Its iterative scheme is easy in solving a problem with steady boundary condition. This paper describes a method to solve heat diffusion problem with unsteady boundary conditions using Excel based macros. Results obtained from the solution agreed well with the measured product temperature distribution for a period of two and half years.

Introduction

Commercial software or custom programs are usually used in studying complex problems. A commercial program may have limitations to certain conditions of a problem. A custom program requires adequate programming skills of the researchers. Microsoft Excel spreadsheet together with its macro capabilities, on the other hand, is a handy tool that can easily be programmed to suit the needs of solving diversified problems. Ketkar and Reddy [1] used Microsoft Excel for numerical solution of an unsteady state heat equation. In this study a heat diffusion problem was solved using the Excel spreadsheet. The time varying boundary condition of the problem did not follow any pattern. This dynamically changing behavior of the boundary condition was taken care of by interfacing some Excel macros with values in the spreadsheet.

Bala et al [2] wrote an elaborate computer program in BASIC language to simulate temperatures of wheat stored for two and a half years in a concrete bin of 778.7-m3 capacity with a diameter of 5.5m and a height of 33.5m located in Cheney, Kansas. This paper used their data but solved the same problem using Microsoft Excel macros interfaced with Excel spreadsheet.

Finite Difference Model for Heat Flow

The following partial differential equation describes transient heat transfer in the radial direction.

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial^2 T}{\partial r} \right) \tag{1}$$

where

T = temperature, °C
t = storage time, hour
r = radial distance from the center of bin, m
α = thermal diffusivities of grain, bin wall materials, and air, m²/hour

Analytical solution of this equation is difficult due to the fact that the diffusivity of grain, bin wall materials and ambient air are different and the boundary condition also varied irregularly. A finite difference method was developed using the following assumptions:
- The heat flow pattern was assumed to be symmetric around the vertical axis of the bin
- There was no heat flow in the vertical direction
- The physical and thermal properties of grain was uniform throughout the bin
- There was no heat generation within the grain mass.

The finite difference method determines temperatures at discrete grid (node) points in the bin. Temperature at each grid or node is based on previous temperature at the node and current and previous temperature at it neighboring nodes. The method, embedding the Crank-Nicholson's [3] implicit technique, yielded equations (2), (3), (4) and (5).

Considering n number of nodes and designating the central node as node number 0 and hence the bin outside wall node number to be (n-1), the equation for the central node is:

$$\left(1 + \frac{2\alpha\Delta t}{\Delta r^2}\right)T_{0,j+1} - \frac{2\alpha\Delta t}{\Delta r^2}T_{1,j+1} = \left(1 - \frac{2\alpha\Delta t}{\Delta r^2}\right)T_{0,j} + \frac{2\alpha\Delta t}{\Delta r^2}T_{1,j} \tag{2}$$

where

α = diffusivity of grain, m²/h
Δr = nodal distance, m
$T_{0,j}$ = Temperature at node 0 at time j, °K
Δt = time difference in between two calculations, h

Equation at any node excluding nodes at the center and within the bin materials is

$$\frac{\alpha\Delta t}{2\Delta r^2}\left(-1 + \frac{1}{2i}\right)T_{i-1,j+1} + \left(1 + \frac{\alpha\Delta t}{\Delta r^2}\right)T_{i,j+1} - \frac{\alpha\Delta t}{2\Delta r^2}\left(1 + \frac{1}{2i}\right)T_{i+1,j+1}$$

$$= \frac{\alpha\Delta t}{2\Delta r^2}\left(1 - \frac{1}{2i}\right)T_{i-1,j} + \left(1 - \frac{\alpha\Delta t}{\Delta r^2}\right)T_{i,j} + \frac{\alpha\Delta t}{2\Delta r^2}\left(1 + \frac{1}{2i}\right)T_{i+1,j} \tag{3}$$

For the node just outside the bin wall (node, n-1), the equation is

$$\frac{\Delta t}{2\Delta r^2}\left(-\alpha_2 + \frac{\alpha_3}{2(n-1)}\right)T_{n-2,j+1} + \left\{1 + \frac{\Delta t}{2\Delta r^2}(\alpha_1 + \alpha_2)\right\}T_{n-1,j+1} - \frac{\Delta t}{2\Delta r^2}\left(\alpha_1 + \frac{\alpha_3}{2(n-1)}\right)T_{n,j+1}$$

$$= \frac{\Delta t}{2\Delta r^2}\left(\alpha_2 - \frac{\alpha_3}{2(n-1)}\right)T_{n-2,j} + \left\{1 - \frac{\Delta t}{2\Delta r^2}(\alpha_1 + \alpha_2)\right\}T_{n-1,j} + \frac{\Delta t}{2\Delta r^2}\left(\alpha_1 + \frac{\alpha_3}{2(n-1)}\right)T_{n,j} \tag{4}$$

where α_1 = diffusivity for the materials in between the node (n-1) and node n
$\alpha_2 = \alpha$ = diffusivity of grain
α_3 = diffusivity for materials in between the node (n-2) and node n

Equation for node n, just outside the bin wall, is:

$$
-\frac{\Delta t}{2\Delta r^2}(\alpha_{11}+\alpha_{22})T_{n-1,j+1} + \left\{1 + \frac{\Delta t}{2\Delta r^2}\left(\alpha_1\left(\frac{2\Delta r h_c}{k}+1\right)+\alpha_{22}+\frac{\alpha_{33}}{2n}\frac{2\Delta r h_c}{k}\right)\right\}T_{n,j+1}
$$

$$
= \frac{\Delta t}{2\Delta r^2}(\alpha_{11}+\alpha_{22})T_{n-1,j} + \left\{1 + \frac{\Delta t}{2\Delta r^2}\left(-\alpha_1\left(\frac{2\Delta r h_c}{k}+1\right)-\alpha_{22}-\frac{\alpha_{33}}{2n}\frac{2\Delta r h_c}{k}\right)\right\}T_{n,j} \qquad (5)
$$

$$
+ \frac{\Delta t}{\Delta r}\frac{h_c}{k}\left(\alpha_{11}+\frac{\alpha_{33}}{2n}\right)(T_{a,j}+T_{a,j+1})
$$

where α_{11} = diffusivity of air
α_{22} = diffusivity of materials in between node (n-1) and node n
α_{33} = diffusivity of materials in between node (n-1) and a node in air away from node n

Finite Difference Nodes
In this study the radial dimension of the bin was discretized into 9 equal sections with 10 nodes. These nodes, as mentioned in equations (2), (3), (4) and (5), are shown in Figure 1.
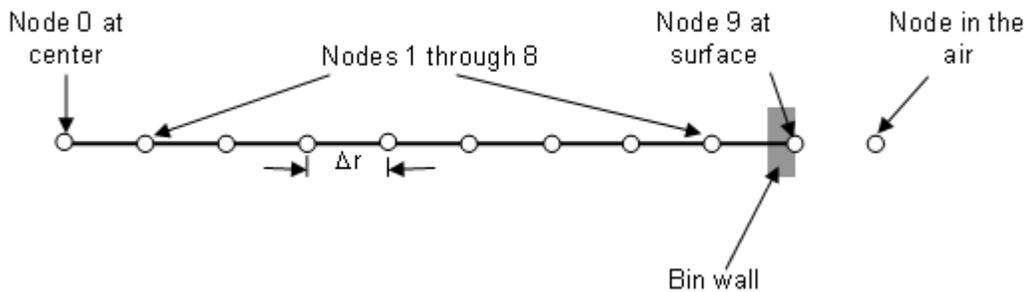
Figure 1. Discrete nodes along radial direction of the bin

According to the principle of the finite difference method, the temperature at the next step time (at time j+1) at node i will be estimated based on the current temperatures at nodes (i-1), i, and at (i+1).

Tri-diagonal Matrix Equation
Equations (2), (3), (4) and (5) form a tri-diagonal matrix equation with constant matrix, time varying force vector and unknown nodal temperatures. For 10 nodes, the finite difference method yields 10 equations find 10 unknown temperatures at time t + Δt. The known nodal temperatures and outside air temperature at time t are used to find temperatures at time t + Δt.

This matrix equation can be produced in the following format:

$$AT = H$$

where

        A = tri-diagonal constant matrix as function of the thermal diffusion properties of grain, bin wall materials and outside air,

        T = vector of unknown nodal temperatures at time, $t + \Delta t$ (i.e. time, j+1)

        H = known vector as function of the thermal diffusion of grain, bin wall materials and outside air temperatures and known nodal temperatures at time, t (i.e. time j)

Use of Microsoft Excel Spreadsheet

Individual elements of matrix A and vector H are calculated from the grain, bin materials, nodal positions and distances and air temperatures. These properties are entered in the spreadsheet. As shown in Figure 2, the elements in the matrix are represented by a, b, and c. The elements of vector H are represented by h. Each value of h is calculated from the values of e, f, and g.

The three coefficients at time j are shown in three columns right before the column for vector H. The spreadsheet multiplies these coefficients with corresponding known temperatures at nodes, including the air temperatures for the surface equation, to determine the individual entries of vector H.

Equation (2) for the node at the center of the bin has two coefficients each at time j and time j+1. Therefore, the first row of matrix A has two entries b1 and c1. Likewise vector H has two entries f1 and g1. As for examples, the following coefficients are extracted from equation (2).

$$b1 = 1 + \frac{2\alpha\Delta t}{\Delta r^2}, \qquad c1 = -\frac{2\alpha\Delta t}{\Delta r^2}, \qquad f1 = 1 - \frac{2\alpha\Delta t}{\Delta r^2}, \qquad g1 = \frac{2\alpha\Delta t}{\Delta r^2}$$

For each of the intermediate nodes as seen in Equation (3), there are three entries, a_k, b_k and c_k for matrix A and three entries, e_k, f_k, and g_k for vector H. The Equation (5) for node n has two coefficients at time j+1. Therefore, the last row of matrix A has two entries. However, the vector H has three entries because the third one comes from the coefficients of ambient air temperatures at time j and at time j+1.

Based on appropriate equations, the spreadsheet calculates all elements (represented by a, b, c, e, f, g and h) from the basic parameters of grain, bin materials, and nodal values. The last line includes the air parameters, air temperature and surface conductivity.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1																	
2–6		Constant coefficients at time j+1											Coefficients at time j				Vector H
		Center - Node 0	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7	Node 8	Node 9		i-1	i	i+1		
8		b1	c1												f1	g1		h1
9		a2	b2	c2									e2	f2	g2		h2
10			a3	b3	c3								e3	f3	g3		h3
11				a4	b4	c4							e4	f4	g4		h4
12					a5	b5	c5						e5	f5	g5		h5
13						a6	b6	c6					e6	f6	g6		h6
14							a7	b7	c7				e7	f7	g7		h7
15								a8	b8	c8			e8	f8	g8		h8
16									a9	b9	c9		e9	f9	g9		h9
17										a10	b10		e10	f10	g10		h10

Figure 2. The constant tri-diagonal matrix and the known vector H derived from the thermal diffusion properties of gain, bin wall materials and air

Visual Basic Macros of Microsoft Excel

To determine grain temperatures as a function of the outside temperature, an iterative scheme is required. The Excel spreadsheet can perform such iterations for static boundary condition. However, this scheme could not be used because of the dynamic nature of the outside air temperature which varied irregularly over the period of storage duration. This irregular boundary condition was accommodated using the macro capability of the Microsoft Excel. Macro was used to interpolate missing air temperatures and as well as to solve the tri-diagonal matrix and to populate spreadsheet cells with nodal grain temperatures at each time step. The tri-diagonal matrix was solved using the Thomas method as described by Sastry [4].

The air temperatures outside the bin were measured at an interval of 15 days (360 hours) for a period of two and a half years. Any temperature in between two measured temperatures was determined using straight-line interpolation method using Excel macro.

Unknown grain temperatures at time j+1 were calculated based on known grain temperatures at time j and known air temperatures at times j and j+1. Macro was written such that grain temperatures can be predicted at any time interval (Δt) of 1, 12, 24, 36, 72 or 360 hours. However, the predicted temperatures were written back to the spreadsheet cells at an interval of 360 hours for all time intervals. As an example, with $\Delta t = 1$ h, 360 intermediate iterations are made to write only one set of nodal grain temperatures on the spreadsheet cells.

Figure 3 shows the flowchart for the development of the macro as described above.
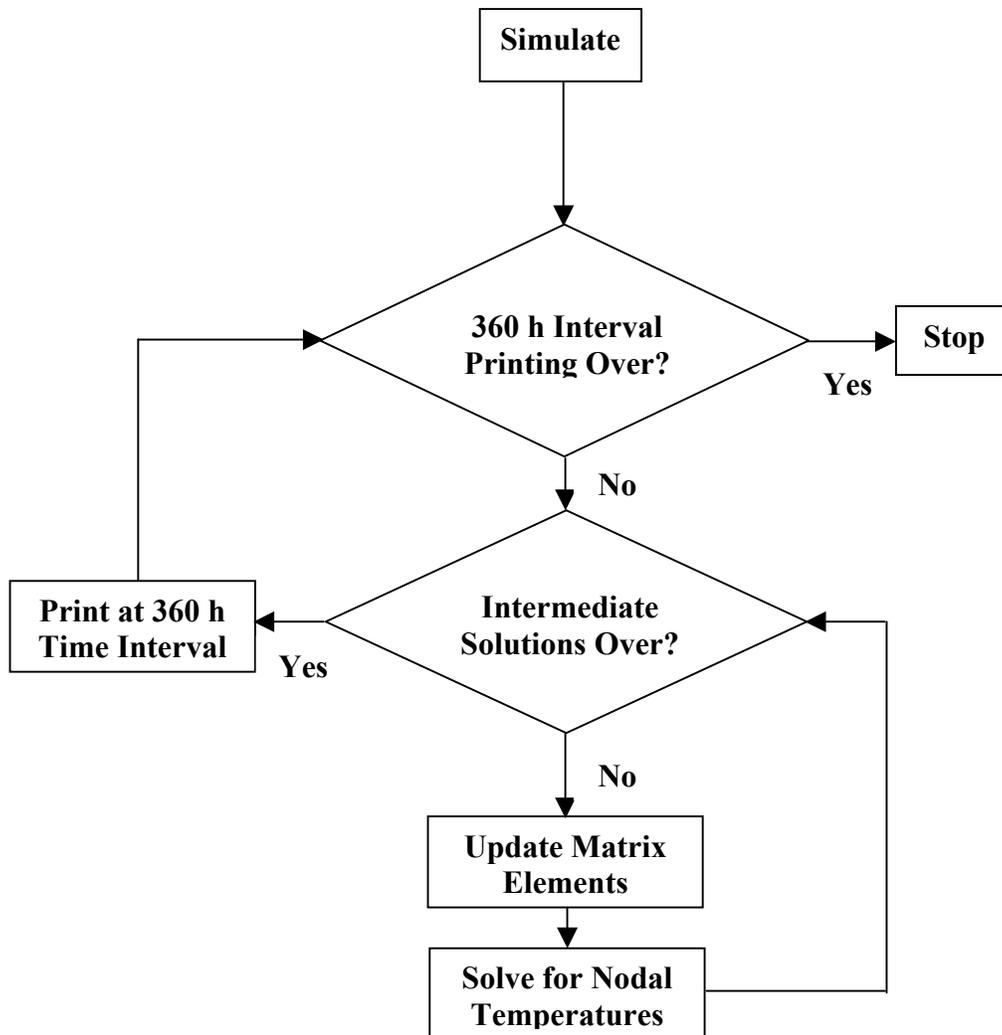
Figure 3. Flowchart for macro development

Results and Discussion

Figure 4 shows a partial view of simulated temperatures at 10 nodal points. Grain was assumed to store at a uniform temperature of 6.67 $^{\circ}$C. As the button labeled, "Simulate" is pressed, the simulation starts and the designated cells are populated with the simulated temperatures of the grain at nodal points for the duration of storage.

The spreadsheet was designed to simulate grain temperatures with reasonable accuracy for all time steps of 1, 12, 24, 36, 72 and 360 hours for the duration of storage of two and a half years (780 days). Figure 5 shows good agreement between the measured and the simulated grain temperatures at a distance of 1.39 m from bin center using 360 h as the time step. Outside air temperature is superimposed to compare it with the grain temperature. This Figure also shows how the grain temperature always lags behind the air temperature.

Days data taken	Center	Intermediate nodes								Surface	
	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7	Node 8	Node 9	Node10	
0	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	Initial grain temps
15	6.67	6.67	6.66	6.65	6.63	6.56	6.36	5.81	4.24	1.3	
30	6.65	6.64	6.62	6.57	6.45	6.19	5.66	4.72	3.59	4.45	
45	6.58	6.56	6.51	6.39	6.18	5.85	5.48	5.42	6.59	8.27	
60	6.46	6.43	6.36	6.23	6.07	5.98	6.17	6.95	8.35	9.61	
75	6.33	6.32	6.27	6.24	6.31	6.61	7.37	8.92	12	16.5	
90	6.32	6.33	6.38	6.54	6.92	7.7	9.17	11.8	16	19	
105	6.51	6.57	6.75	7.15	7.87	9.11	11	13.7	16.4	17.7	
120	6.97	7.07	7.39	8.01	9	10.5	12.5	15	18.4	22.3	
135	7.67	7.81	8.25	9.02	10.2	11.9	14.1	17.3	21.8	25.2	
150	8.57	8.74	9.26	10.2	11.5	13.5	16.2	19.8	24.3	28.1	
165	9.64	9.83	10.4	11.5	13	15.2	18	21.4	25.1	26.6	
180	10.9	11.1	11.7	12.9	14.6	16.8	19.6	23	27.1	31.7	
195	12.2	12.4	13.1	14.4	16.1	18.3	20.9	23.7	25.7	23.4	
210	13.6	13.8	14.5	15.7	17.3	19.3	21.3	23.1	24.1	26.9	
225	15	15.2	15.8	16.9	18.3	19.8	21.2	22.2	22.4	18.9	

Figure 4. Simulated grain temperature at nodal points.
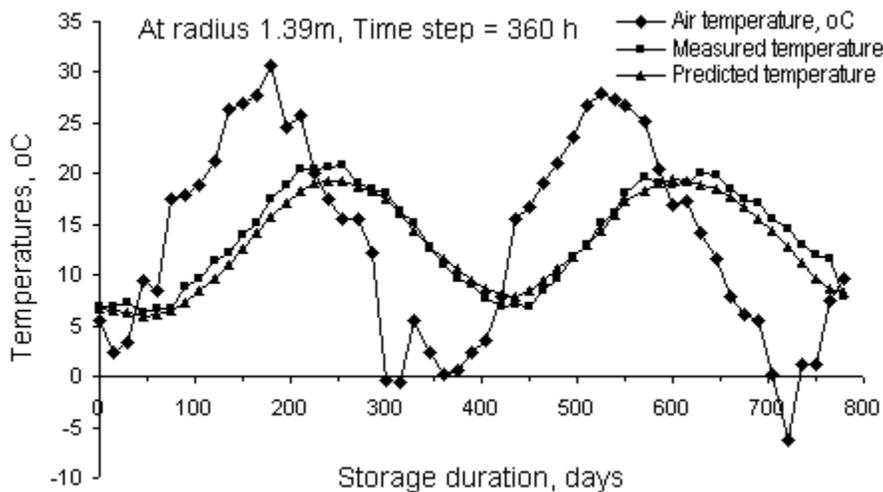The button, Simulate, starts simulation.

Figure 5 Grain temperature response to the outside air temperature variations
at 1.39m from the center of the bin wall.

The use of Δt = 360 hours as time step requires a total of only 53 predictions for the total period of storage. The use of Δt = 1 hour as time step requires 360 intermediate predictions for every 15 days resulting in a total of 18,720 predictions for the same period of storage. This extra fineness (Δt = 1 hour) does not produce any appreciable improvement in the simulation result except the exact air temperature at the surface of the bin. In other words, the time step of Δt = 360 hours is good enough to obtain the desired prediction result. The variation between the predicted and

measured temperatures may be attributed to other factors, such as, assumptions used in this study, possible errors in measurement of temperatures, uncounted external temperature variations between two successive measurement at an interval of 15 days, etc.

Conclusions

Microsoft Excel spreadsheet is a very handy tool in quick calculations of parameters. Macro capability is an added tool to customize its use in accordance with diversified requirements. A command button can be added to a spreadsheet from the Tools menu. A single click on it opens a Visual Basic IDE with subroutine template. Students can easily use the spreadsheet and macro combination as may be required.

The smaller values of time step, Δt, allow finer temperature prediction. Likewise, smaller values of nodal step, Δr, have the potential to improve the similar accuracy. However, any change in Δr, causes a corresponding change in the number of equations to solve. Thus it requires a total redesign of the spreadsheet. An alternative to implement this modification is to write subroutines in the macro. This reduces the size of the spreadsheet at the cost of increasing the programming effort. Thus the more flexibility is added to the Excel solution, the more the spreadsheet loses its share in calculation by becoming just a sheet for keeping records of calculation.

References

1. Ketkar, M. A. and G. B. Reddy. 2003. Microsoft Excel-based numerical solution of linear, homogeneous 1D transient partial differential equations. Proc. of American Society of Engineering Education Annual Conference & Exposition. Nashville, Tennessee. June 22-24, 2004.
2. Bala, B. K., N.N. Sarker, M. A. Basunia and M. M. Alam. 1990. J. Stored Prod. Res. Vol. 26, No. 1, pp. 1-6.
3. Crank J. and Nicholson, P. 1947. A practical method for numerical evaluation of solutions of partial differential equations of the heat conduction types. Proc. Camb. Phil. Soc. Biol. Sci. 43. pp. 50-67.
4. Sastry, S. S. 1984. Introductory Methods of Numerical Analysis. Prentice Hall, India.

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