2006-1141: DEVELOPING MODELS AND AN UNDERSTANDING OF THEIR LIMITATIONS

Robert Throne, Rose-Hulman Institute of Technology
Robert D. Throne is currently an Associate Professor in the ECE department at Rose-Hulman Institute of Technology. He was in the EE department at the University of Nebraska-Lincoln for ten years prior to moving to Rose-Hulman. His interests are in control systems and the solution of inverse problems.

Developing Models and Understanding Their Limitations

Abstract

Students often do not appreciate the difference between the model of a system and a real system. Typical homework problems often have students apply control strategies on systems (plants) that have been given to them in the problem statement. Utilizing our Education Control Products (ECP) systems, as well as Matlab's Simulink, we developed a sequence of laboratories to emphasize the difference between the model of a system and a real system in our undergraduate controls classes. In the initial labs the students determine models for the different plants they will be utilizing. These models are created using specialized Simulink blocks we developed that drive the ECP systems. The goal of these early labs is actually to develop a model of the plant to replace the Simulink hardware drivers. Subsequently, controllers and/or observers are designed to meet design specifications utilizing these system models. Then the students replace their model of the plant with the hardware driver (Simulink block) for the ECP system. These Simulink systems, with the student designed controllers/observers, are used to run the real ECP systems. Finally, the students compare the predicted response (utilizing their model) with the actual response of the real system. Our assessments of the students indicate that after the course the students have a better understanding that the response of the model will only approximate that of the real system.

Introduction

Students often do not appreciate the difference between the model of a system and a real system. Typical homework problems often have students apply control strategies on systems (plants) that have been given to them in the problem statement. However, this often leaves the students with the belief that the system models in these assignments are exact, and easily obtainable. Utilizing our Education Control Products (ECP) systems, as well as Matlab's Simulink, we developed a sequence of laboratories to emphasize the difference between the model of a system and a real system in our undergraduate controls classes.

All of the electrical engineering students at Rose-Hulman are required to take ECE-320: Linear Control Systems. This is a junior level class offered in the first and third quarter each year. The current prerequisites for the class are ES-205: Analysis and Design of Engineering Systems and ECE-200 Circuits and Systems. There are usually 25-30 students in the class each quarter. The students work in groups of two in the labs. We have fifteen lab stations, and have eleven ECP model 210 rectilinear systems and eleven ECP model 205 torsional systems (some stations have both systems). Each of these ECP systems is connected to a computer through a breakout box, and the ECP systems are controlled via Simulink on the computer. The Simulink programs utilize Matlab's Real-Time Workshop and Real-Time Window Target, and Microsoft's C++

compiler. Most of this software is transparent to the user. In addition, these systems are utilized in electives ECE-520 Discrete-Time Control Systems and ECE-521 Modern Control Systems, which usually have about ten students per year. The majority of this paper discusses the laboratories for the introductory controls course. All of the labs for ECE-320, ECE-520, and ECE-521 can be found using the link http://www.rose-hulman.edu/~throne/CCLI.html. These labs are continually undergoing revision as we gain more experience.

Laboratory Approach

In the initial labs the students determine models for the different plants they will be utilizing. Figure 1 shows one of the "carts" for one of our ECP rectilinear systems. The position encoder is shown toward the back of the system. The carts are moved via a motor with a rack and pinion mechanism. We have developed specialized Simulink blocks that drive the ECP systems, as shown in Figure 2. These blocks, which control the ECP systems, represent the "real" system. Our goal in the early labs is actually to develop mathematical models of the plants to replace these specialized blocks that represent the real system.
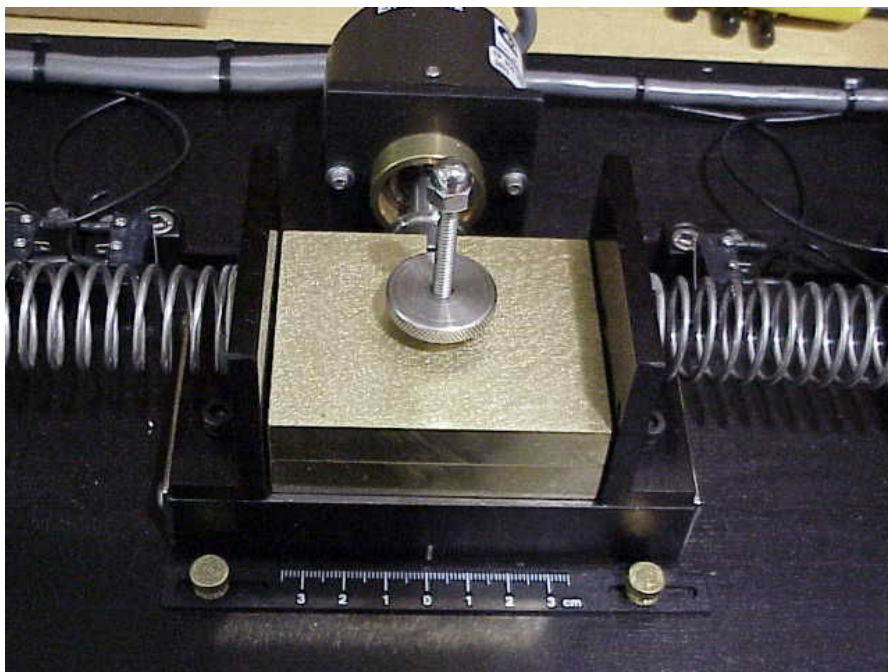

Figure 1: One cart (with adjustable masses) connected to two (adjustable) springs of the ECP-210a system. The position encoder is shown in the background.
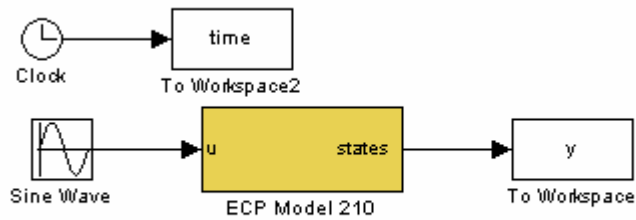
Figure 2: Simulink file used to generate the model for the ECP Model 210 system. The yellow block is the Simulink driver that runs the actual hardware. It has been constructed so that if the input units are in cm, then the output units are in cm.

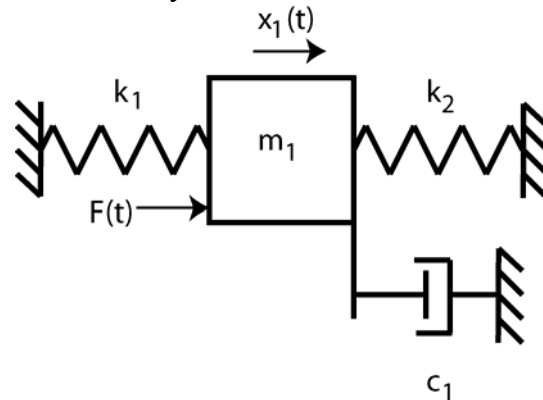This rectilinear one degree of freedom system can be modeled as shown below in Figure 3.

Figure 3. Model of a one degree of freedom system. Only one cart is free to move, and there is at least one spring attached.

The transfer function for this model can easily be shown to be

$$\frac{X_1(s)}{F(s)} = \frac{K}{\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1}$$

where K is the static gain, ω_n is the natural frequency, and ζ is the damping ratio. In the time domain, both ζ and ω_n can be estimated using the log decrement method, and the static gain can be estimated by determining the ratio of the steady state value of the cart position $x_{1,ss}$ and the amplitude of the step response input. Next the system is excited at various frequencies, and a Bode plot of experimental data is constructed. System identification utilizing frequency domain response is a fairly common method of determining the transfer function of an unknown system.[1-4] This experimental frequency response data is compared with the frequency response

of the transfer function to determine the parameters (K , ω_n , and ζ) again. A typical result is displayed in Figure 4, which shows the measured frequency response as discrete diamonds and the fit of the transfer function to the measured frequency response as a solid line. For this configuration the initial and final estimates of the transfer function parameters are shown in Table 1. These results are fairly typical in that the largest difference between the time-domain and frequency domain estimates are the damping ratio estimates. It is emphasized to the students that they need to compare their initial estimates with the final estimates as a check on their work, and this comparison must be included in their lab report.
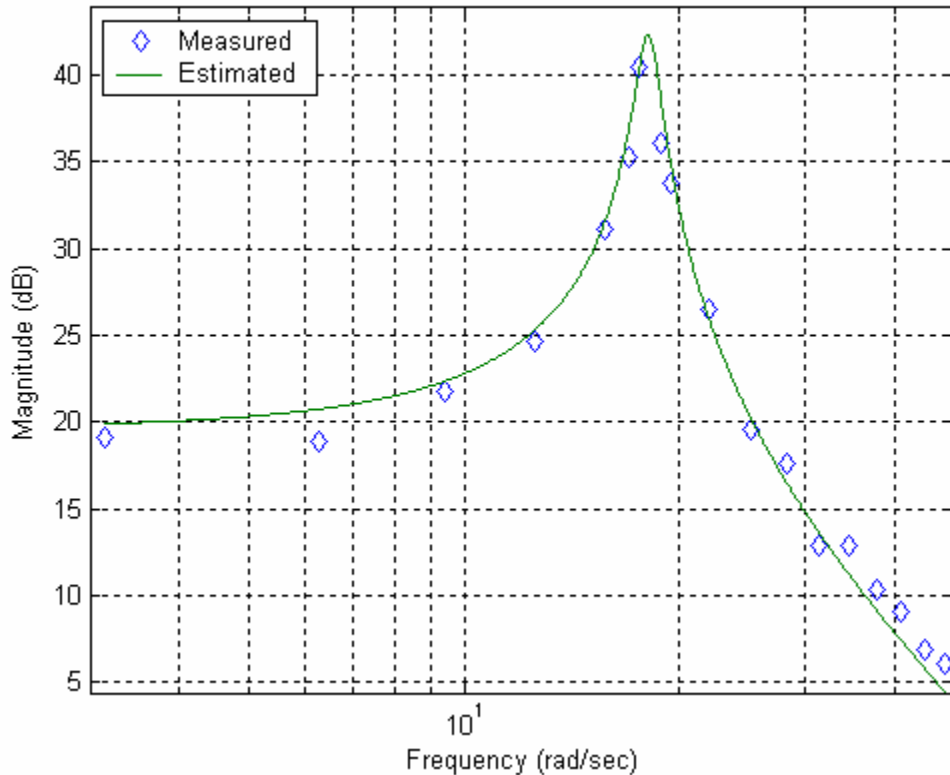

Figure 4: Fit of measured frequency response and estimated transfer function for the one degree of freedom system.

Parameter	Initial Estimate	Final Estimate
K	8.4	9.6
ω_n	18.1	18.1
ζ	0.025	0.037

Table 1. Initial and final estimates of the parameters for the one degree of freedom model. The final estimates are those obtained by matching the assumed form of the transfer function with the measured frequency response data.

In order to determine the state model, we define $q_1 = x_1$, and $q_2 = \dot{x}_1$, and obtain the following state equations

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K\omega_n^2 \end{bmatrix} F$$

Clearly all of the parameters in the state variable model can be obtained from the transfer function for this system. The mathematical models are actually stored as state variable models and converted to transfer functions when needed. In the initial two or three labs the students must construct models for at least two different systems. These models will be used throughout the course. The students are also given loose guidelines in terms of how to physically build each system. The result of this is that there is a great deal of variety in the systems used in labs, and each group of students must modify the control schemes to work with their particular system.

Once the students have developed their mathematical models of the systems, they construct a simple closed loop model of the system in Simulink, as shown in Figure 5. In this Simulink model, the mathematical model of the plant is shown in a different color to emphasize that it is just a model of the real system, *it is not the real system!* This configuration allows us to utilize the model of the plant with a variety of classical control methods, such as model matching, utilizing Diophantine equations, and various forms of PID controllers. In addition, we can investigate either constant or dynamic prefilters using this configuration.

This Simulink model utilizing the mathematical model of the plant is actually driven (controlled) by a Matlab script file, which reads in the appropriate mathematical model of the plant and allows for easy modification of the control scheme. When the Matlab file is run, it puts all of the necessary variables in the workspace. These variables can then be accessed by the Simulink files as necessary. A typical output of this Matlab file is shown in Figure 6. This plot indicates the predicted response of the system (based on the model) and the required control effort. Note that there are two solid lines on the control effort graph, which indicate the saturation level and force the students to deal with the fact that a real system has real limits on the control effort. Utilizing these mathematical models of the plant allows the students to become comfortable with each control scheme before implementing them on a real system, and develop an understanding of how their system is likely to react to various controllers. Before each lab, as part of their weekly homework, the students are required to implement various control schemes on their models, and verify that their models meet the design constraints without saturating the motor or breaking anything.

In addition to a Simulink model which utilizes their mathematical model of the real system, during labs the students also utilize a similar Simulink model file which controls the real system. This Simulink model file is shown in Figure 7. Comparing Figures 5 and 7, we see the only distinction is that the mathematical model of the system in Figure 5 is replaced by the yellow ECP block (the real system) in Figure 7. The names of the controller transfer function, the prefilter transfer function, the input, and the time to run the simulation are the same as when utilizing the Simulink file with the mathematical model of the plant.
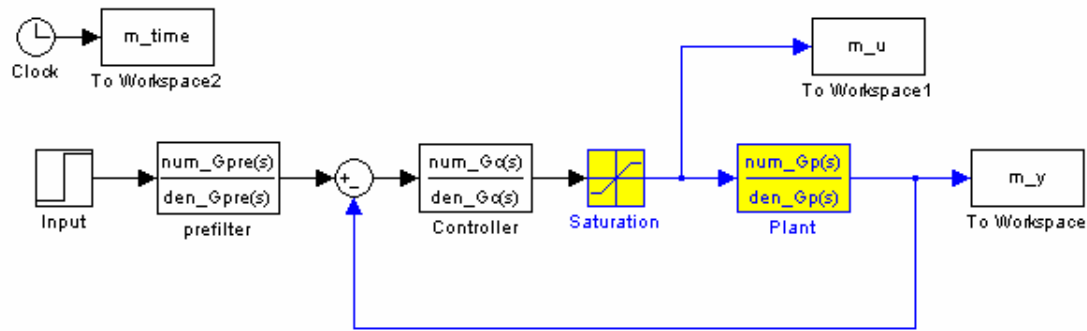
Figure 5. Standard Simulink model utilizing the mathematical model of the plant for use with a variety of classical control techniques.
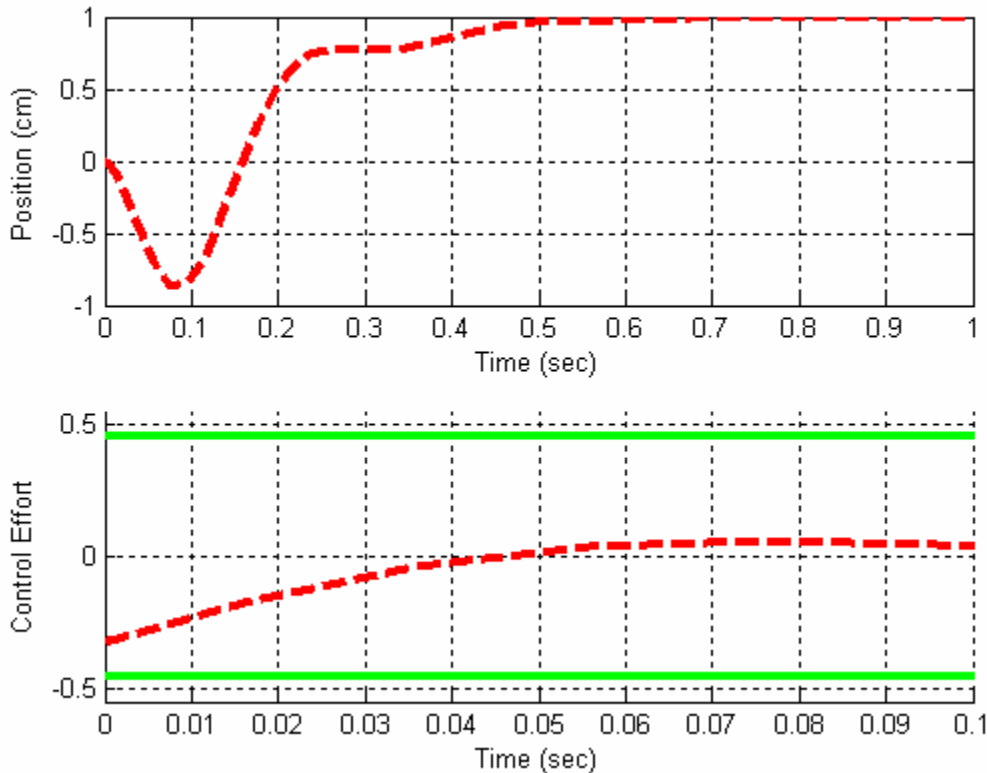

Figure 6. Predicted output of the Simulink model utilizing the mathematical model of the plant. The thick lines indicate the levels at which the control effort will saturate the system. Since for this particular kind of controller the control effort is largest near the beginning of the step, the control effort axis has been modified to only show the initial control effort.
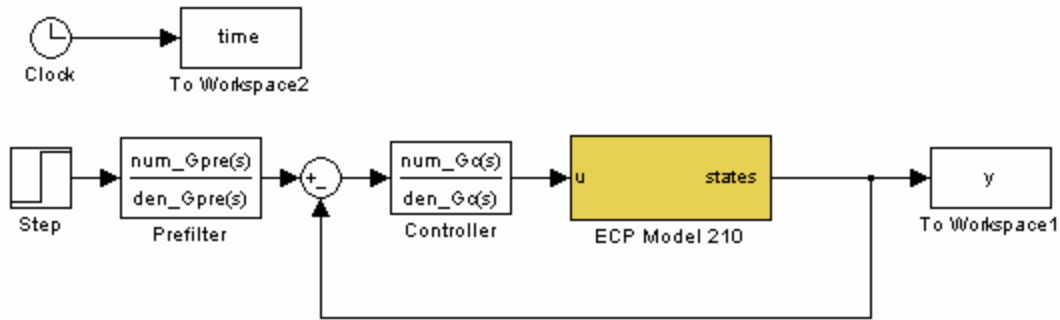
Figure 7. Simulink model utilizing the real plant for use with classical control techniques.

Finally, the students have to implement their control strategies on the real systems (Figure 7) and see how well both their control schemes worked and how well their mathematical models predicted the actual response of the system. As part of their lab they must include a plot similar to Figure 8, which compares the predicted response of the mathematical model and the response of the real system.
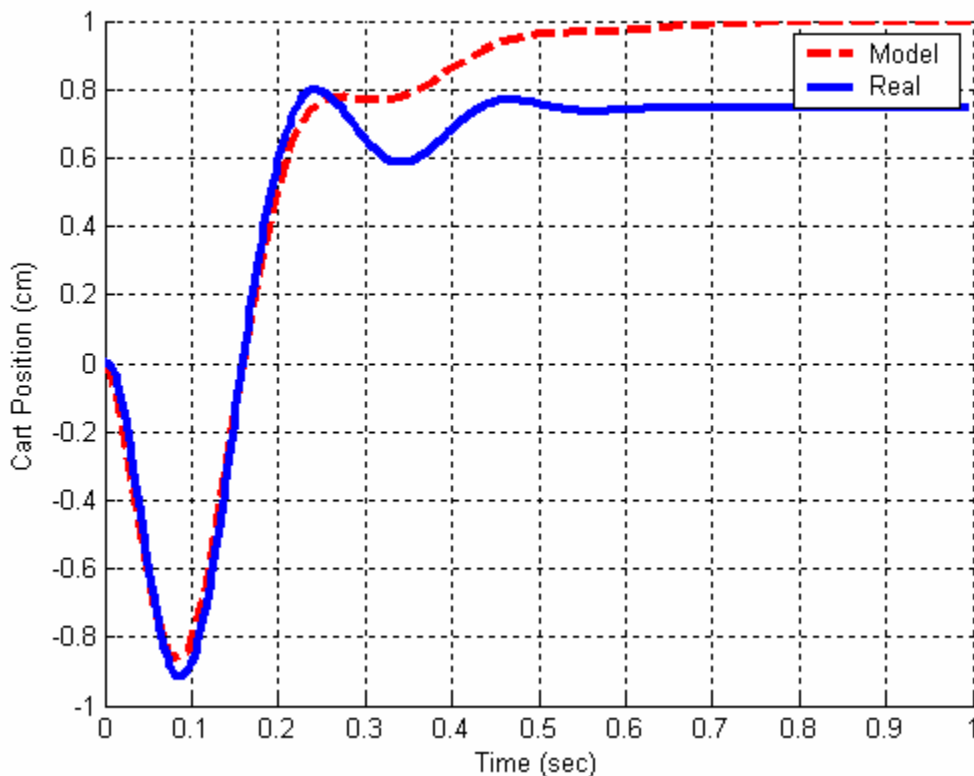

Figure 8. Predicted response of the mathematical model and the response of real system. The input to the system was a one cm step. A graph like this is required for each control scheme analyzed in lab.

For the particular system whose results are displayed in Figure 8, the mathematical model matches the real system quite well initially, but they reach different steady state values. In our labs this difference in steady state values is a very common distinction between the mathematical models of the system and the response of the real system. At this point the students begin to understand why having a type one system (a system with integral control action) is beneficial – the integral control action will produce zero steady state error and will eliminate this problem.

Figure 9 shows both the predicted response based on the mathematical model of the system and the true response of the system, for a control scheme that forces the system to be a type one system. As this figure shows, the steady state values of the predicted and actual responses now match (the input to the system was a one cm step and hence the steady state error is zero), and the real system is a bit slower than the real system.
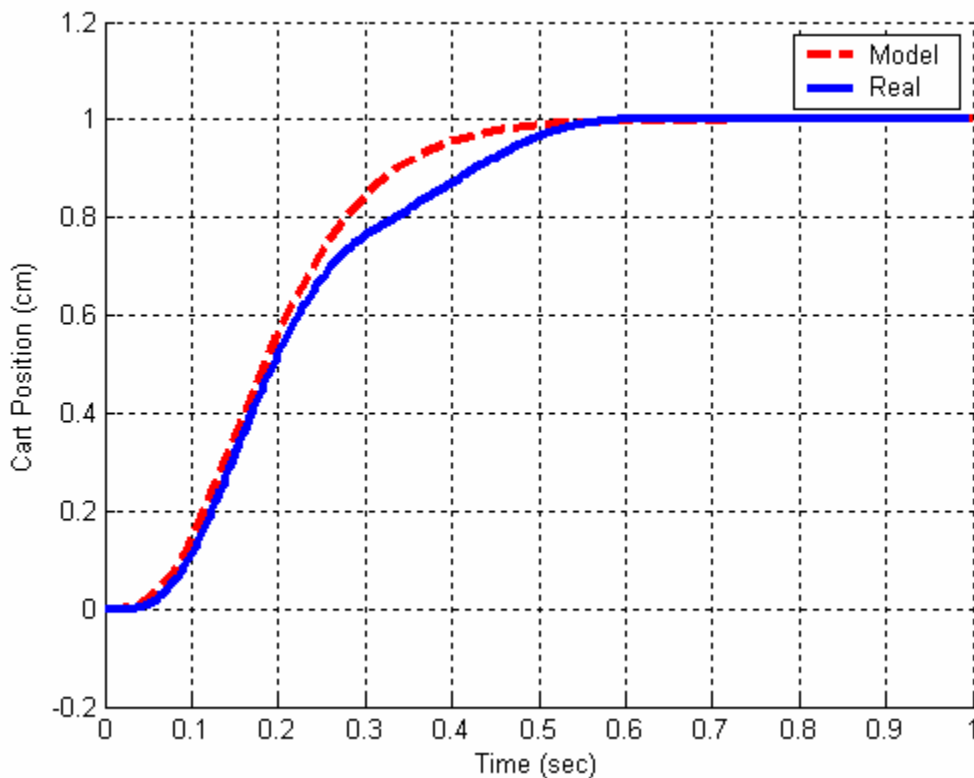

Figure 9. Predicted response of the mathematical model and the response of real system for a control scheme that forces the system to be a type one system. The input to the system was a one cm step.

In addition to transfer function models, we also utilize the same basic approach with state variable models. Figures 10 and 11 show the Simulink models for a one degree of freedom system utilizing the mathematical model of the system and the real system, respectively. Again, the students develop their control strategies utilizing their mathematical model of the plant, being sure they do not saturate the motor. During lab they again compare their predicted response with the response of the real system. Figure 12 shows a typical response. Here the position of the cart

is fairly well predicted by the model, but again there is a difference in the steady state values. The prediction of the velocity of the cart shows a very typical result – the model predicts a velocity that is quite smooth, while the measured velocity of the real system is quite noisy.
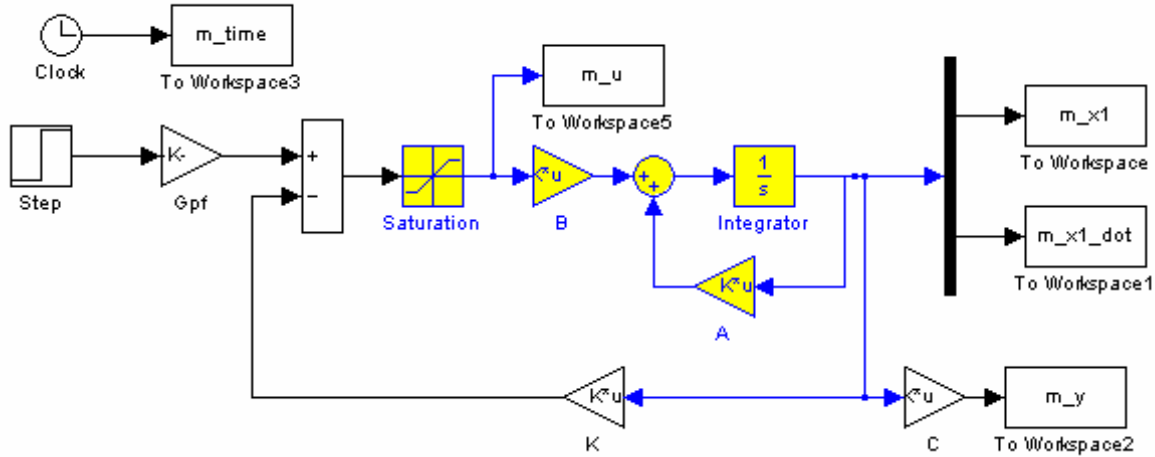

Figure 10. Simulink model utilizing the mathematical model of the plant using state variable feedback.
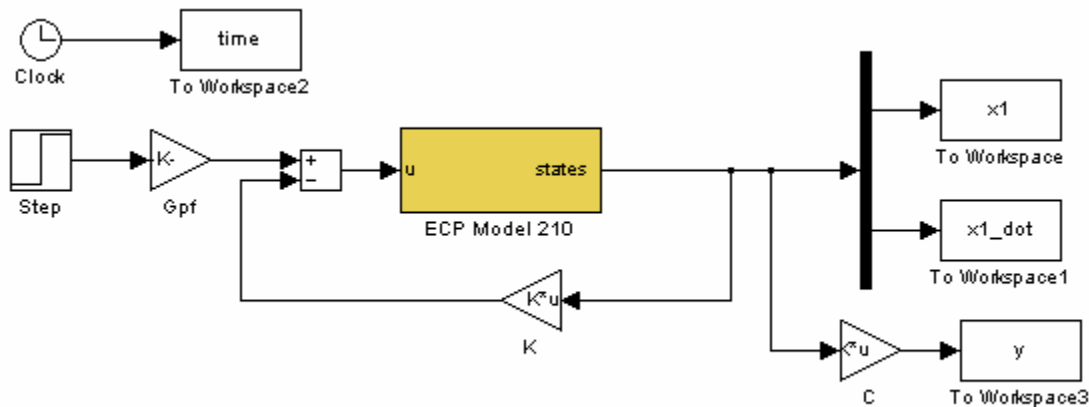

Figure 11. Simulink model utilizing the real plant with state variable feedback.
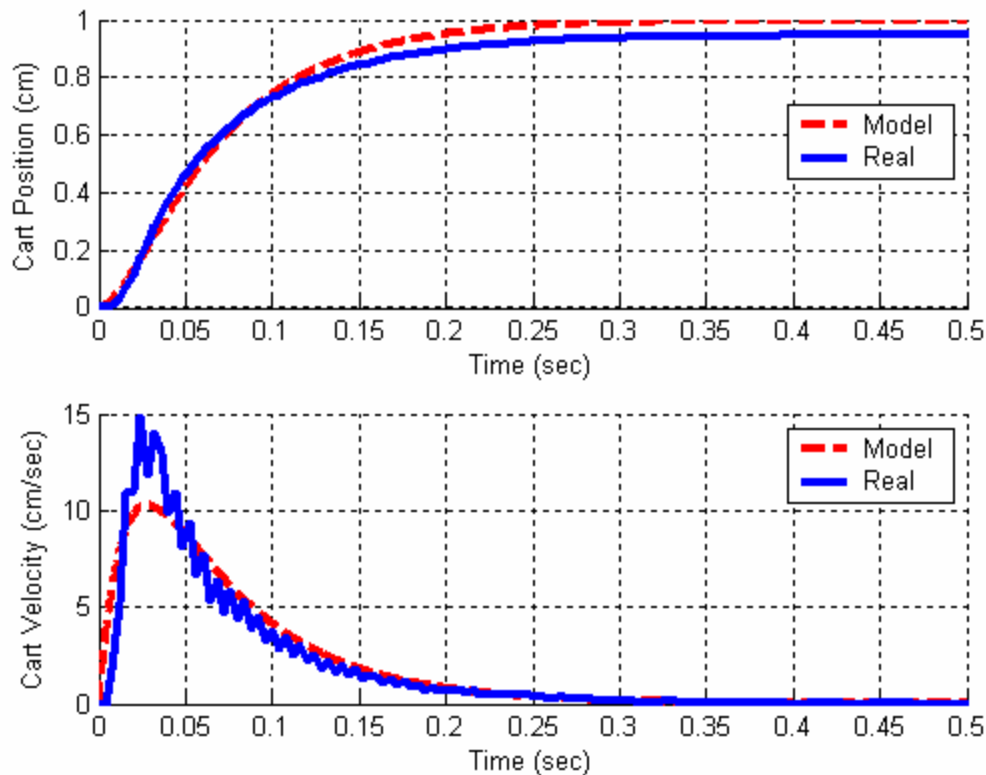
Figure 12. Predicted response using the mathematical model of the plant and the response of the real system for a state variable feedback system. The input to the system was a one cm step. The model predicts no steady state error in the system, but the real system does not quite reach the correct final value. In addition, the measured velocity in the system is much larger and noisier than that predicted by the model.

In our higher-level controls courses we continue this use of a mathematical model of the plant in Simulink for developing or implementing a control scheme, and then replacing the mathematical model of the plant with the real plant and comparing the predicted response with the response of the real system. Specifically, this technique is utilized in implementing state variable feedback for two and three degree of freedom systems, incorporating full order and reduced order observers with state variable feedback, and including integral control in conjunction with state variable feedback (with and without observers). As an example, Figure 13 shows a Simulink model which includes a discrete-time model of a one degree of freedom plant. In this implementation state variable feedback is being utilized as well as integral control. Figure 14 shows the same basic structure with the mathematical model of the plant replaced with the real plant. Finally, Figure 15 is a comparison between the predicted response of the system using the mathematical model of the plant and the measured response when the actual plant is used. Again this figure shows the real system takes longer to reach steady state than the model and the derivative in the model is much cleaner than that for the real system.
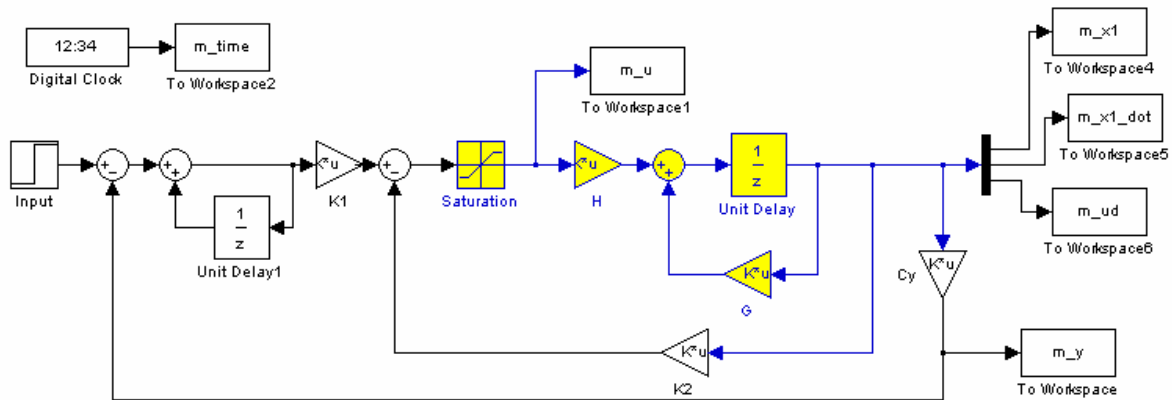
Figure 13. Simulink model for a discrete-time control system utilizing the mathematical model of the plant with both integral control and state variable feedback.
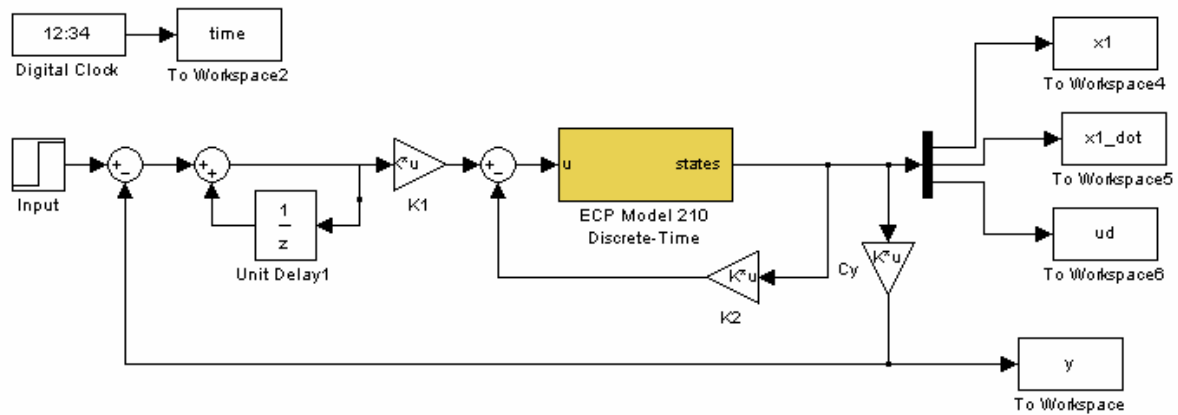

Figure 14. Simulink model for discrete-time control utilizing the real plant with both integral control and state variable feedback.
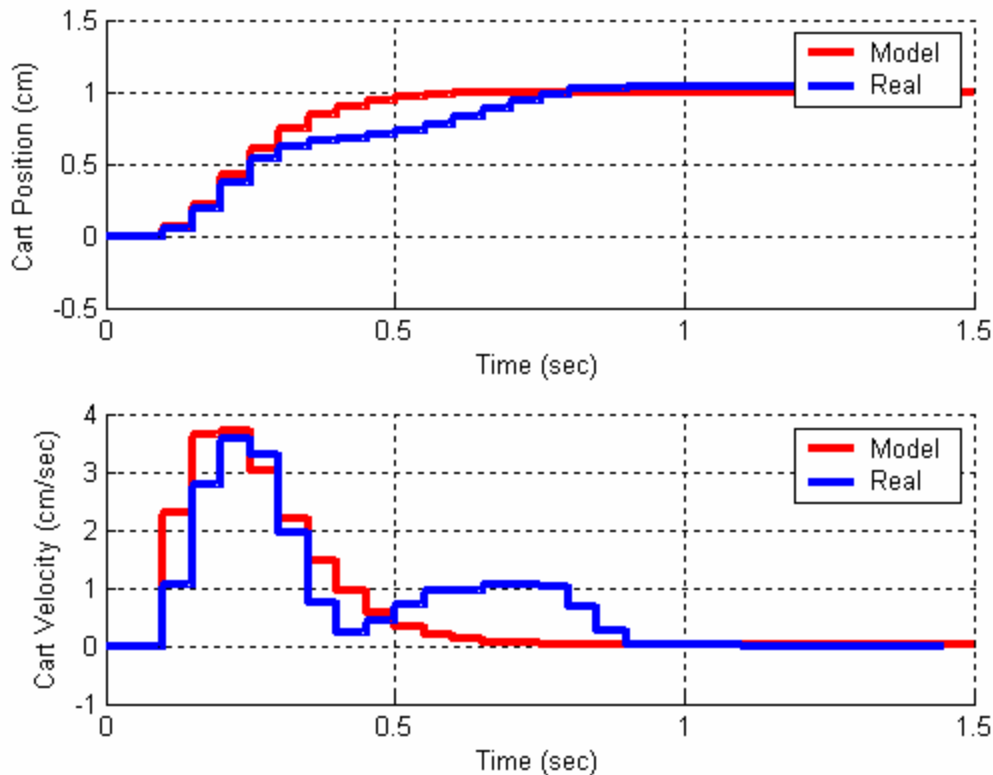
Figure 15. Predicted response using the mathematical model of the plant and the response of the real system for a discrete-time model using integral control and state variable feedback. The input to the system was a one cm step.

Assessment

At the beginning of the introductory controls course, and again at the end of the course, the students taking the class were asked to complete questionnaires designed by Rose-Hulman's Office of Institutional Research, Planning, and Assessment. These questionnaires had two types of questions: survey questions and questions about the level of knowledge and confidence in various course concepts. The results of the relevant survey questions are displayed in Table 2, which indicates that most of the students felt that the course helped them with the use and limitations of models in control systems. Table 3 presents the results for the relevant average knowledge and average confidence part of the questionnaire. The knowledge score was based on the following scale:

Pre Course
1 = No clue, this concept is new to me
2 = Low, I had only heard about the concept
3 = Moderate, I knew the concept but had not applied it
4 = High, I knew the concept and had applied it

Post Course
1 = No Clue, I do not know the concept
2 = Low, I have only heard about the concept, but do not know it well enough to apply it.
3 = Moderate, I know the concept but do not know it well enough to apply it
4 = High, I know the concept and have applied it in this course.

The Confidence score was based on the following scale:

Pre Course
1 = No Clue, I have no idea if I can apply the concept
2 = Low, I had heard of the concept but had little confidence that I could apply it
3 = Moderate, I was somewhat confident that I understood the concept and was fairly sure I could apply it to a new problem
4 = High, I was confident that I understood and apply it to a new problem

Post Course
1 = No Clue, I am not confident that I can apply the concept
2 = Low, I have heard of the concept but am not sure if I can apply it
3 = Moderate, I am somewhat confident that I understand the concept and I can apply it to a new problem
4 = High, I am confident that I understand and can apply the concept to problems.

As the average knowledge/confidence levels indicate, the students felt their knowledge level had increased during the course, and their confidence had also increased. The continual use of mathematical models to develop and implement control schemes, and then the comparison between the response predicted by the mathematical models and the response of the real system seems to be having an effect on the students' understanding of the use of models. The largest difference between the pre-course and post-course responses concerned the knowledge that the size of the control signal in a real system is not unlimited. This is a concept that becomes very clear to the students during the labs since, if they exceed the control signals limiter, they often have to shut off their computers and reboot them.

Question	Strongly Agree	Agree	Disagree	Strongly Disagree	I Don't Know
As a result of this class, I understand the uses of models in designing control systems that will help me in future classes.	60 %	40 %	0 %	0 %	0 %
As a result of this class, I understand the limitations of models in designing control systems that will help me in future classes.	60 %	40 %	0 %	0%	0%

Table 2: Results of student questionaires.

Concept	Pre Course Knowledge	Pre Course Confidence	Post Course Knowledge	Post Course Confidence
Various approaches to modeling dynamical systems.	2.78	2.39	3.67	3.42
Distinctions between a model and a real dynamical system.	3.33	2.78	4.00	3.75
Comparison between the predicted response of a mathematical model and the response of a physical system.	3.17	2.67	3.92	3.75
Size limitations on control signals of real systems.	1.67	1.61	3.42	3.58

Table 3: Pre- and post-course knowledge and confidence (average score).

Conclusions

Students often do not appreciate the difference between the model of a system and a real system. We have presented our approach to helping students understand the distinction between a real system and a mathematical model of a system in our control systems classes. Our initial labs focus on developing mathematical models of real systems. These models are then incorporated into various control strategies in a Simulink file. Once the control strategy works acceptably with the model of the system, the mathematical model of the plant is replaced with the real plant and the control strategy is tried on the real system. Finally, the students must plot and compare the response of the system predicted by their model with the response of the real system. Observing the difference between the predicted response and the response of the real system generally helps the students understand that the models tend to have less friction than the real systems, that the models have smoother derivatives than real systems, and that we want integral control action in our control schemes to overcome modeling errors. Our assessment indicates that our students feel they understand that there is a difference between a model of a system and a real system.

Bibliography

1. R. Throne. Frequency domain system identification of one, two, and three degree of freedom systems in an introductory controls class. Proc. 2005 ASEE Annual Conference and Exposition (Portland, 2005)
2. G. Plett and D. Schmidt. Multidisciplinary lab-based controls curriculum. Proc. 2001 ASEE Annual Conference & Exposition (Albuquerque, 2001)

3. G. Plett and D. Schmidt. A multidisciplinary digital-control-system laboratory. Proc. 2002 ASEE Annual Conference & Exposition (Montreal, 2002).
4. B. Armstrong. A controls laboratory program with an accent on system identification. Proc. 1997 ASEE Annual Conference & Exposition (Milwaukee, 1997)