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Development and Pilot Test of the Rate and Accumulation Concept Inventory

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Abstract

Many of the rate of change and accumulation processes that we commonly encounter reflect the behaviors of complex systems. Solving complex system problems within engineering requires the ability to interpret the meaning of a function that is modeling a dynamic situation. This ability is essential for engineers. Thus, knowing how students think and learn about rate and accumulation processes in complex systems can help educators better prepare students for their engineering careers.

Engineering students often have various robust misconceptions that can persist throughout their education and may hinder their ability to learn new material. Concept inventories can identify some of these misconceptions. Existing discipline-specific inventories include assessments of conceptual frameworks related to rate and accumulation processes. However, these inventories measure context-specific understandings of rate and accumulation processes and do not delve into the mathematical and scientific conceptual frameworks that may underlie contextual misconceptions.

This paper builds on previous evidence that many engineering students possess robust misconceptions about rate and accumulation processes. These findings led to the development of the Rate and Accumulation Concept Inventory (RACI), which is intended to assess students' conceptual understanding of rate and accumulation processes. Three categories of conceptual understanding are included in the RACI: (1) first order calculus, (2) mass flow, in particular water flow, and (3) heat transfer.

Pilot testing of the RACI took place in a sophomore civil and environmental engineering course. Results from pilot testing indicated the presence of persistent misconceptions among the students in all three categories of understanding. Student performance on the RACI went from 56% to 59% after instruction. Internal consistency reliability was assessed using Cronbach's Alpha; values were 0.77 for the entire instrument and ranged from 0.64 to 0.76 for the three concept categories of the RACI.

Introduction

Mass and energy balances are fundamental process models adopted by engineers in many disciplines and contexts, such as force and momentum balances used by civil and mechanical engineers, heat-work relationships used by chemical engineers, and fate and transport modeling used by environmental engineers. These processes are sometimes referred to as "stocks and flows problems." Each of these processes is related to a particular flow of mass or energy that may accumulate within a given boundary over time. To model such a flow, one must invoke the fundamental theorem of calculus, which relates differential calculus (i.e., the rate of change of a function) with integral calculus (i.e., the accumulation of quantities within a particular boundary). The conceptualization of rate and change processes can be understood as a pivotal aspect of early engineering education.

Engineering students must be able to interpret the complexities of rate and accumulation processes within real systems. For instance, sustainable development requires a "systems thinking" approach to the design of engineered systems, and stocks and flows problems are central to the dynamics of complex systems. Thus, for students to improve their ability to learn about and manage complex systems, they must have a strong conceptual understanding of calculus fundamentals, and then be able to interpret how these fundamentals are associated with real world phenomena in various contexts. Unfortunately, research shows that most people's intuitive understanding of stocks and flows is poor[1,2], and engineering student misconceptions related to rate and accumulation processes has been known for some time[3]. Students may form misconceptions of rate and accumulation processes for numerous reasons. For example, certain focusing phenomena used in the classroom have been linked to students incorrectly generalizing slopes as differences in quantities rather than ratios[4].

Assessing engineering students' conceptual understanding of fundamental processes before a course begins can provide instructors with valuable feedback. Concept inventories are assessment instruments that have been used in several math, science and engineering disciplines as a way to provide reliable and valid assessment of students' misconceptions. While some of these inventories include questions to assess student understanding of particular rate and accumulation processes, they tend to be context-specific. The Rate and Accumulation Concept Inventory (RACI) was designed in part to address the need for an assessment tool which would be able to measure the degree to which a student's misconceptions of rate and accumulation processes is related to mathematical understanding vs. the contextual understanding of a particular process.

This paper describes the development of the RACI. We begin with a brief summary of past exploratory work, which demonstrated the need for the inventory. This is followed by a discussion of the development of the categories and questions included in the RACI. Results from a pilot test were used to assess (1) the level of improvement for question sets and concept categories after course instruction, (2) student confidence in answering question sets, (3) relationships between performance on the RACI and course performance measures, and (4) internal consistency reliability measures of the instrument and categories. The paper ends with a discussion of plans for ongoing and future work.

Summary of Exploratory Work

The primary objective of the exploratory study was to identify and categorize student misconceptions that may impede student learning of engineering concepts related to water flow processes[5]. The context of the study was an urban hydrology unit that is part of a sophomore civil and environmental engineering course. Several existing concept inventories were considered for their suitability as assessment instruments[6–9]; however, they were found to be too context-specific for the purposes of the exploratory study. Original survey instruments were developed to assess student understanding of two fundamental engineering conceptual frameworks: first order calculus and water flow. Multiple types of student understanding were considered in the survey questions, including equation based, graphical, mental models and descriptive. Additional research methods included video and audio recordings of student groups completing activities designed to assess understanding of two particular flow processes, namely groundwater flow and water flows on a green roof. Results from these efforts suggested the existence of persistent misconceptions among the students, specifically misconceptions of rate and accumulation processes.

Development of the Rate and Accumulation Concept Inventory

Work began on the development of a new assessment tool that would more accurately assess both the mathematical and scientific conceptual frameworks that underlie students' understanding of rate and accumulation processes. The work of Hestenes et al. on the Force Concept Inventory[10] established many of the protocols for concept inventory development, which have since been further established by many authors[11–15]. The following steps suggested by Richardson (2005)[11] provided a basis for the development stages of the RACI:

1. Determine the concepts to be included in the inventory.
2. Study and articulate the student learning process for those concepts.
3. Construct a beta version of the inventory with several open-ended questions for each concept. Design multiple-choice answers based on common student misconceptions.
4. Administer the beta version of the inventory to as many students as possible and perform statistical analyses to establish validity, reliability and fairness.
5. Revise the inventory to improve readability, validity, reliability, and fairness.

Steps 1-4 are reflected in the results reported in this paper, while iterations of Steps 3-5 are planned for future stages of this research. The concepts to be included in the RACI were identified using the exploratory study results and observations of student learning. Three categories of concepts were included in the inventory: (1) first order calculus, (2) mass flow, in particular water flow, and (3) heat transfer. Ten sets of questions related to unique prompts were included with thirty individual questions in total. Two calculus concept questions were based on problems from an introductory textbook[16]. These questions were developed to assess students' ability to interpret a phenomenon and its associated graphical representation. A third calculus question was based on research that investigates students' covariational reasoning abilities[17]. The format of this problem was left as an open-ended question since our previous efforts had not investigated concepts related to covariational reasoning. The mass flow category included original inventory items developed in this study over a number of iterations with several engineering instructors and graduate students. These questions stem from the exploratory work that demonstrated student difficulty in distinguishing between factors that affect the rate at which water flows through a system and the total amount of water that flowed over a period of time. The heat transfer inventory items were taken directly from a rate and accumulation processes subsection of the Heat and Energy Concept Inventory (HECI), developed by Prince et al. with the author's permission[18]. One of these question sets (Q10) was designed to be a mass transfer analog to the heat transfer questions. For our analysis purposes, this question set remained a part of the third category of questions, i.e., the heat transfer or "HECI" questions.

The formats for questions were either multiple choice or open-ended. The open-ended questions allowed for the collection of a range of student reasoning responses for each question. Incorrect responses were initially categorized by multiple graders according to the type of misconception suggested in the students' work. These categories were then combined into a single rubric for the grading of each question. In the subsequent version of the RACI, these categories of misconceptions will be developed into multiple-choice responses known as *distractors* to capture patterns of incorrect conceptual reasoning.

The version of the RACI used in this study is included in Appendix A. Table 1 summarizes the point values of each question set. At the end of each set for calculus and mass flow categories, and after every question in the HECI category, students were asked to assess their level of confidence in answering the questions. This allowed for a greater understanding as to how students were interpreting the inventory questions. The confidence rating scores are considered separately from the point values for each question set.

Table 1. Question Set Point Score Values

Category	Question Set	Prompt Situation	Total point value
Calculus	**Q1**	Reservoir fill	3[a]
	Q2	Walking paths	6
	Q3[b]	Bottle fill	2
Mass Flow	**Q4**	Hydrostatic equilibrium	2
	Q5	Bathtubs draining	2
	Q6	Planter boxes	6
	Q7	Graduated cylinders	2
HECI	**Q8**[c]	Ice blocks	4
	Q9[c]	Cool tea	2
	Q10[c]	Sponge dye	2

a. Question 1a scored with a 2 point value (1 point for each numerical value and units)
b. Question modified from Carlson et al. (2002)[17]
c. Questions taken with permission from the HECI[18]

Administration

The instrument was administered in the Spring 2014 semester in a sophomore civil and environmental engineering class of 78 students (57 civil engineers, 15 environmental engineers, 4 other). The average GPA at the beginning of course instruction was 3.05. The RACI was administered during the first week of classes (referred to as "Survey 1" in the Results section) and again during the final week of classes (referred to as "Survey 2" in the Results section) to assess the students' conceptual understanding at the beginning and end of a course that included extensive content related to mass transfer and energy transfer principles. The instrument was administered during normal class periods to all students enrolled in the course. Full assessment results were collected for 75 students.

Several specific pieces of student learning were tracked throughout the semester to evaluate the progression of student learning. These included questions on bi-weekly quizzes which were designed to assess student understanding of rate and accumulation processes within the context of new course material, such as population growth models and resource extraction models. Interviews were also conducted after the initial administration of the RACI to further assess student responses, in particular the open ended responses. The interviews were semi-structured, 20 minute interviews held within a week of the students' completion of the inventory. The option to participate as an interview subject was open to all students in the course. Twelve interviews were completed, each of which provided valuable feedback in the development of the rubrics used to code open ended responses.

Results

1. Pre- and Post-Instruction RACI Scores

Table 2 summarizes results for each concept category developed for the RACI and the total score for the instrument. The combined results for the calculus and mass flow categories were also analyzed, as these questions had not been tested for their reliability or validity. Student performance on the RACI increased modestly from 56% to 59% after instruction. Statistically significant improvements are found in Survey 2 for the entire instrument as well as in each category other than the HECI category.

Table 2. Mean Scores for Concept Categories and Overall Instrument

Category	Survey 1 (N=75)	Survey 2 (N=75)
Calculus	64%	68%[a]
Mass Flow	68%	73%[a]
HECI	29%	30%
Calculus & Mass Flow	66%	71%[b]
Overall Instrument	56%	59%[b]

a. Indicates one tailed t-test showed significant improvement at the $p<0.05$ level

b. Indicates one tailed t-test showed significant improvement at the $p<0.01$ level

Scores for the calculus question sets are summarized in Table 3. While Survey 2 scores improved for each question set, no improvements were statistically significant other than Q1 at the $p<0.1$ level. Similar results are found for the mass flow question sets and are summarized in Table 4. Significant improvement was seen in the hydrostatic equilibrium question (Q4) despite the fact that this concept was not directly covered in any course material. All other questions in this category have modest improvements, including Q6 which is statistically significant at the 0.1 level.

Table 3. Mean Scores for Calculus Question Sets

Question	**Survey 1** (N=75)	**Survey 2** (N=75)
Q1	57%	63%
Q2	72%	75%
Q3	50%	57%[a]

a. Indicates one tailed t-test showed significant improvement at the $p<0.1$ level

Table 4. Mean Scores for Mass Flow Question Sets

Question	**Survey 1** (N=75)	**Survey 2** (N=75)
Q4	79%	88%[b]
Q5	34%	33%
Q6	81%	85%[a]
Q7	55%	59%

a. Indicates one tailed t-test showed significant improvement at the $p<0.1$ level
b. Indicates one tailed t-test showed significant improvement at the $p<0.01$ level

Within the mass flow category, question sets Q5 and Q7 were designed to be analogous questions to assess student understanding of the physical factors affecting water flow. Tables 5 and 6 include matrices that depict how students answered both questions, which highlight whether or not common misconceptions can be assessed in these questions. Only 28% of students correctly answered both questions on Survey 1, and even less (23%) on Survey 2 (highlighted in yellow in Tables 5 and 6).

Two interesting patterns of responses emerged from the results: (1) students choosing "A" for both questions, i.e., the flow rate in one set up is greater than the other (incorrect for

Q5a); and (2) students choosing "C" for both questions, i.e., that the flow rates are equal in both the bathtub and graduated cylinder systems (incorrect for Q7a). Surprisingly, both of these categories of incomplete conceptual understanding increased on Survey 2. Students who answered "A" for both questions may believe that total water volume drives water flow rates rather than pressure due to the height of the water. Of the students in this response category (highlighted in orange in Tables 5 and 6), 9 out of 11 in Survey 1 (82%) and 14 out of 21 in Survey 2 (67%) had answers to 5b that suggested this misconception (e.g., "depends on the area" or "depends on the volume of water"). Likewise, students who answered "C" for both may believe that water flow rate in the given systems is determined only by a singular physical constraint, such as the size of outlet drains. Of the students in this category (highlighted in green in Tables 5 and 6), 10 out of 12 in Survey 1 (83%) and 11 out of 18 in Survey 2 (61%) had answers to 7a that suggested this misconception (e.g., "the structures are identical").

Table 5. Survey 1 Responses for Q5a and Q7a

		Q5a			
		A	**B**	**C** (correct)	**D**
Q7a	**A** (correct)	**11 (15%)**	9	**21 (28%)**	3
	B	0	4	2	0
	C	4	6	**12 (16%)**	0
	D	1	0	2	0

Table 6. Survey 2 Responses for Q5a and Q7a

		Q5a			
		A	**B**	**C** (correct)	**D**
Q7a	**A** (correct)	**21 (28%)**	9	**17 (23%)**	1
	B	0	3	0	0
	C	2	4	**18 (24%)**	0
	D	0	0	0	0

Table 7 summarizes results for the rate and accumulation questions taken from the HECI. There are minor improvements for the mean score of each question set, none of which were found to be statistically significant. Although course instruction included a brief discussion of energy transfer principles, these results suggest that the instruction did not sufficiently address the students' misconceptions related to heat transfer.

Table 7. Mean Scores for HECI Question Sets

Question	Survey 1 (N=75)	Survey 2 (N=75)
Q8	22%	23%
Q9	35%	39%
Q10	33%	34%

2. Confidence Scores

Confidence scores were included in the inventory for several reasons. If any question received a large number of "Total guess" ratings, it could be considered too difficult or confusing for students to answer. In both Survey 1 and Survey 2 results, the majority of questions received either one or zero guesses among all student responses. The questions with the most guesses included Q5a (7 guesses on Survey 1 and 3 guesses on Survey 2) and Q7a (5 guesses on Survey 1 and 5 on Survey 2). This level of guessing was not deemed to be so large as to skew the analysis of the results or to consider the removal of a question from the analysis.

While assessing the student confidence ratings, it was noted that there were some distinctions between the ratings among the female and male students. Table 8 includes a breakdown of the confidence ratings for female and male students for each concept category, as well as their mean category score. Most ratings for all questions on both Survey 1 and Survey 2 fall in the "low" to "moderate" range. There are several instances of statistically significant gains in confidence ratings and improvements in mean category scores, particularly among the male students. For all categories in both Survey 1 and Survey 2, male students had higher mean scores and higher confidence ratings than the female students.

Correlations for individual question sets and confidence ratings were calculated, though no significant correlations were discovered. Correlative analysis was also performed on category scores and average category confidence ratings, as well as the on the total inventory score and average confidence score for all questions (based on a 0-3 point scale). Results from these findings are summarized in Table 9. While most correlations are weak, some patterns do emerge. For instance, most results for female students, particularly on Survey 2, indicate a positive relationship between confidence levels and mean scores, whereas most results indicate negative, albeit weak, relationships between confidence levels and mean scores for male students.

Table 8. Female and Male Student Confidence Ratings and Mean Scores

Category	Value Type	Female Students (N=23)		Male Students (N=52)	
		Survey 1	*Survey 2*	*Survey 1*	*Survey 2*
Calculus	*Average Confidence*	2.06	2.10	2.40	2.56[c]
	Mean Score	62%	68%[a]	65%	68%[a]
Mass Flow	*Average Confidence*	1.81	2.07	2.24	2.36[a]
	Mean Score	65%	68%	70%	75%[b]
HECI	*Average Confidence**	1.66	1.67	1.86	1.97[b]
	Mean Score	17%	21%	33%	33%
Average Instrument Confidence		1.83	1.96[a]	2.17	2.30[c]
Overall Instrument Score		52%	56%[b]	59%	62%[c]

a. Indicates one tailed t-test showed significant improvement at the $p<0.10$ level.

b. Indicates one tailed t-test showed significant improvement at the $p<0.05$ level.

c. Indicates one tailed t-test showed significant improvement at the $p<0.01$ level.

Notes: 0-3 point scale for confidence (0= Total guess, 1 = Low, 2= Moderate, 3= High); * denotes 0-5 point scale rating values (0= Total guess, 1= Low, 2= Low-Moderate, 3= Moderate, 4= Moderate-High, 5= High) that were converted to a 0-3 point scale

Table 9. Correlations for Female and Male Student Confidence Ratings and Mean Scores

Category	Female Students (N=23)		Male Students (N=52)	
	Survey 1	*Survey 2*	*Survey 1*	*Survey 2*
Calculus	0.14	0.29	-0.23	-0.24
Mass Flow	0.24	0.26	-0.07	0.02
HECI	-0.07	0.52	0.15	-0.09
Overall Instrument	0.12	0.46	-0.09	-0.15

3. Comparison of RACI Scores and Course Performance

Possible uses of the RACI would be to administer it as either a formative assessment tool, which would provide feedback to an instructor on how to best design course instruction, or as a summative assessment tool, which would serve as an evaluation of student learning following course instruction. Several factors were considered to analyze its suitability for these purposes, including students' initial GPAs and final course grades. Table 10 summarizes the relationships between students' GPAs (before course instruction), their final course grade, the mean category and total inventory scores for Survey 1 and Survey 2, and a total score for student learning items tracked throughout the course (this includes rate and accumulation questions from six quizzes and two recitation activities). No negative relationships were discovered between any of the variables. The strongest relationship is found between the total mean scores for Survey 1 and Survey 2, which suggests that individual students' pre- and post-test scores are consistent among the overall population trends.

Initial GPA is most strongly related to the final course grade, and it is moderately linked to performance on Survey 1 and Survey 2. This suggests the RACI may not be suitable as a reliable formative assessment tool for course performance in the course used in this study. An increase in the strength of the relationship between final course grade and total inventory score between Survey 1 and Survey 2 suggests that there is improvement in the level of understanding on Survey 2 among students that performed well in the class. However, this level of increase does not suggest its suitability as a summative assessment tool for course performance. The moderate relationship between course grade and total Survey 2 score also hints at the possibility that high student performance in this course is not associated with an increased conceptual understanding of rate and accumulation processes.

Table 10. Correlations for Inventory Scores and Course Performance Metrics

		GPA	Course Grade	Survey 1				Quizzes and Activities	Survey 2			
				1-3	4-7	8-10	Total		1-3	4-7	8-10	Total
GPA		1.00										
Course Grade		0.74	1.00									
Survey 1	1-3	0.38	0.46	1.00								
	4-7	0.23	0.31	0.50	1.00							
	8-10	0.21	0.27	0.26	0.31	1.00						
	Total	0.37	0.47	0.82	0.80	0.64	1.00					
Quizzes and Activities		0.41	0.46	0.46	0.44	0.13	0.47	1.00				
Survey 2	1-3	0.47	0.56	0.77	0.55	0.23	0.72	0.57	1.00			
	4-7	0.34	0.39	0.49	0.58	0.20	0.57	0.33	0.43	1.00		
	8-10	0.32	0.31	0.25	0.29	0.51	0.44	0.24	0.19	0.09	1.00	
	Total	0.54	0.60	0.73	0.68	0.44	0.83	0.55	0.80	0.71	0.59	1.00

4. Reliability Measures

Internal consistency reliability was determined for the entire instrument as well as various categories using the Cronbach's Alpha on post-course assessment. Reliability results are summarized in Table 11. The reliability for the entire instrument is 0.77, with category scores ranging from 0.64 to 0.76.

Table 11. Internal Reliability of Post-Tests

Category	Cronbach's Alpha
Calculus	0.70
Mass Flow	0.64
HECI	0.75
Calculus & Mass Flow	0.76
Overall Instrument	**0.77**

Deleting individual questions within question sets had little significance in changing the overall value of the total Cronbach's Alpha, regardless of the question. The greatest increase from an individual question was 0.02, which was noted in questions within Q1a, Q4b, Q5a, and Q7b. This small difference was considered insignificant in the overall impact of the reliability of the instrument. Table 12 summarizes the Cronbach's Alpha for the entire instrument when entire question sets are removed. Removing set Q2, Q6 or Q8 each increased Cronbach's alpha to 0.80 or above. Q6 had the greatest degree on increase, to 0.84. This large increase suggests that Q6 could be removed or altered in future iterations of the RACI to produce a more reliable instrument.

Summary and Ongoing Work

This paper discusses efforts that have been made to assess student misconceptions that may impede learning of applied engineering concepts related to rate and accumulation processes. The Rate and Accumulation Concept Inventory (RACI) was designed to assess conceptual understanding of fundamental concepts related to these processes. Results from a pilot test suggest that the overall instrument and subcategories within the RACI provide reasonably reliable measures for sophomore engineering students' conceptual understanding of rate and accumulation processes.

Table 12. Internal Reliability of Post-Tests with Question Sets or Categories Removed

Question Set or Category Removed	Cronbach's Alpha
Q1	0.78
Q2	**0.82**
Q3	0.76
Calculus	0.69
Q4	0.78
Q5	0.79
Q6	**0.84**
Q7	0.79
Mass Flow	0.74
Calculus & Mass Flow	0.75
Q8	**0.80**
Q9	0.78
Q10	0.78
HECI	0.76

Development of this instrument is an on-going and iterative process that will continue to go through several additional versions to further establish reliability and validity. The results of this study have led to the refinement of several questions as well as the development of multiple choice questions to take the place of open ended questions. Supplementary questions have also been added to certain subcategories. For instance, an updated version of the RACI includes questions from Precalculus Concept Assessment instrument that are classified as rate and accumulation questions[19]. These questions will provide further insight into students' conceptual understanding of calculus principles. Current research methods also include a measure of students' *procedural knowledge,* i.e., their ability to recall problem solving procedures and strategies[20]. A first order calculus assessment tool was developed to provide a measure of students' procedural knowledge, which will be compared with their conceptual knowledge as measured by the calculus subcategory of the RACI. It is hypothesized that both knowledge types will affect students' *knowledge transfer*, or their ability to extend an understanding of calculus principles beyond the contextual setting of calculus coursework[21].

The conclusions drawn from this study have certain limitations that should be acknowledged. The sample of students is from a single class in a single institution. Thus, many of these findings may be unique to this particular population of students. As rate and accumulation processes represent a fundamental conceptual framework that spans many engineering disciplines, the RACI is not inherently a discipline specific concept inventory. Future iterations of this study will seek to include larger samples of engineering students in various institutions and disciplines. Additionally, there are several other factors that shape a student's conceptual understanding that were not captured within this study. These results of this research have revealed additional questions and variables to consider in future work.

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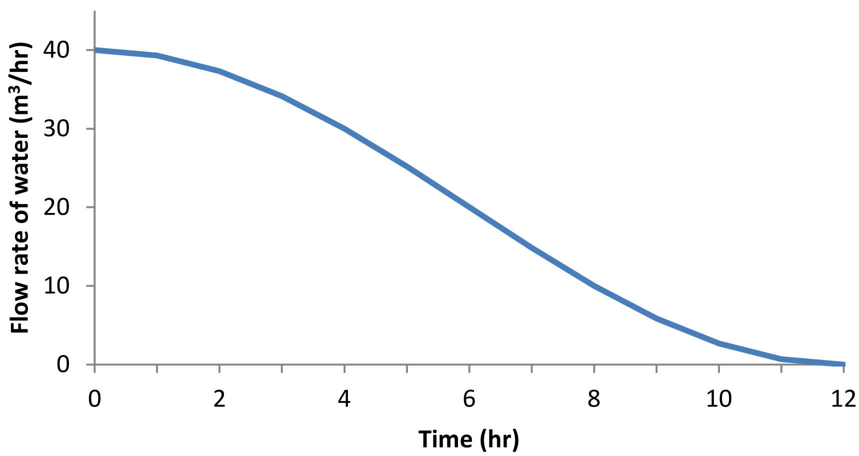
References

1. Sweeney, L. B. & Sterman, J. D. Bathtub dynamics: initial results of a systems thinking inventory. *Syst. Dyn. Rev.* **16,** 249–286 (2000).
2. Sweeney, L. B. & Sterman, J. D. Thinking about systems: student and teacher conceptions of natural and social systems. *Syst. Dyn. Rev.* **23,** 285–311 (2007).
3. Thompson, P. W. in *Learning Mathematics* 125–170 (Springer, 1994).
4. Lobato, J., Ellis, A. B. & Munoz, R. How 'Focusing Phenomena' in the Instructional Environment Support Individual Students' Generalizations. *Math. Think. Learn.* **5,** 1–36 (2003).
5. Flynn, C. D., Davidson, C. I. & Dotger, S. Engineering Student Misconceptions about Rate and Accumulation Processes. *ASEE Zone I Conference Proceedings* (2014). at <http://asee-ne.org/proceedings/2014/Student%20Papers/150.pdf>
6. Gray, G. L. *et al.* The dynamics concept inventory assessment test: A progress report and some results. *American Society for Engineering Education Annual Conference & Exposition* (2005).
7. Martin, J., Mitchell, J. & Newell, T. Development of a concept inventory for fluid mechanics. in *33rd Annual Frontiers in Education, 2003. FIE 2003* **1,** T3D–23 – T3D–28 Vol.1 (2003).
8. *Fluid Mechanics Concept Inventory*. (2011). at <https://cihub.org/resources/9>
9. Shallcross, D. C. A concept inventory for material and energy balances. *Educ. Chem. Eng.* **5,** e1–e12 (2010).
10. Hestenes, D., Wells, M. & Swackhamer, G. Force concept inventory. *Phys. Teach.* **30,** 141–158 (1992).
11. Richardson, J. Concept inventories: Tools for uncovering STEM students' misconceptions. *Invent. Impact Build. Excell. Undergrad. Sci. Technol. Eng. Math. STEM Educ.* 19–25 (2005).
12. Jarrett, L., Ferry, B. & Takacs, G. Development and validation of a concept inventory for introductory-level climate change science. *Fac. Eng. Inf. Sci. - Pap.* 25–41 (2012).
13. Knight, J. Biology concept assessment tools: design and use. *Microbiol. Aust.* **31,** 5–8 (2010).
14. Libarkin, J. Concept inventories in higher education science. *BOSE Conf* (2008).
15. Adams, W. K. & Wieman, C. E. Development and validation of instruments to measure learning of expert-like thinking. *Int. J. Sci. Educ.* **33,** 1289–1312 (2011).
16. W. Briggs & L. Cochran. *Calculus*. (Pearson, 2010).
17. Carlson, M., Jacobs, S., Coe, E., Larsen, S. & Hsu, E. Applying covariational reasoning while modeling dynamic events: A framework and a study. *J. Res. Math. Educ.* 352–378 (2002).
18. Prince, M., Vigeant, M. & Nottis, K. Development of the heat and energy concept inventory: Preliminary results on the prevalence and persistence of engineering students' misconceptions. *J. Eng. Educ.* **101,** 412–438 (2012).
19. Carlson, M., Oehrtman, M. & Engelke, N. The precalculus concept assessment: A tool for assessing students' reasoning abilities and understandings. *Cogn. Instr.* **28,** 113–145 (2010).
20. Hiebert, J. *Conceptual and procedural knowledge: The case of mathematics*. (Routledge, 2013).
21. Bransford, J. D. & Schwartz, D. L. Rethinking transfer: A simple proposal with multiple implications. *Rev. Res. Educ.* 61–100 (1999).

Appendix A: RACI questions (2014 version)

1. **A reservoir is filled with a single inflow pipe. The reservoir is empty when the inflow pipe is opened at *t* = 0. The flow rate of water into the reservoir (in m^3/hr) is shown below.**



   a. **Estimate how much water flows into the reservoir in the first 2 hours.**

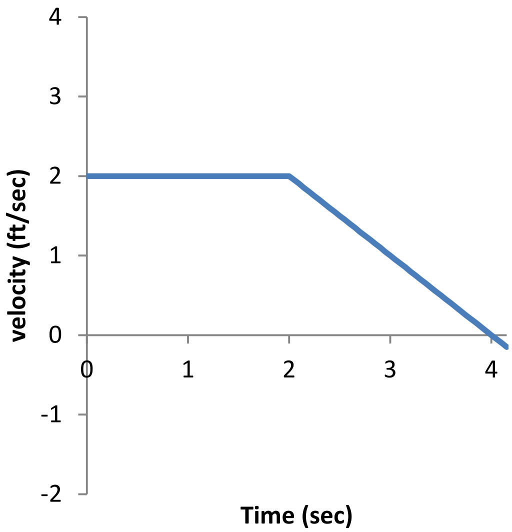
   b. **Please explain your answer below.**

Indicate how confident you are in your responses for this question.

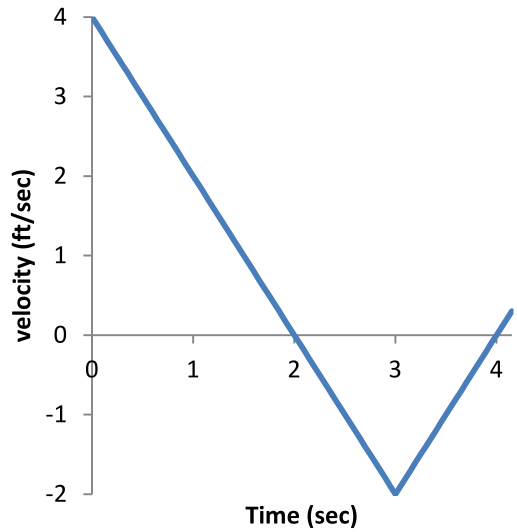
_ Total guess
_ Low
_ Moderate
_ High

2. **The figures below show velocity functions for two people walking along two straight paths.**

Person A:



Person B:



a. **Which person is further from their starting position at t=4?**

_ Person A
_ Person B
_ Both are the same distance from their respective starting point
_ Unknown (not enough information to select one of these three answers)

b. **Please explain your answer below. If the solution that you selected is "Unknown", state what information must be added to answer the question.**

c. Which person travels a greater total distance over the time interval t=0 to t=4?

_ Person A
_ Person B
_ Both travel the same total distance
_ Unknown (not enough information to select one of these three answers)

d. Please explain your answer below. If the solution that you selected is "Unknown", state what information must be added to answer the question.

e. Which person has a greater acceleration at t=4?

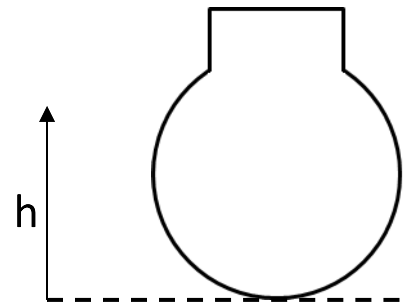
_ Person A
_ Person B
_ Both have the same acceleration at this time
_ Unknown (not enough information to select one of these three answers)

f. Please explain your answer below. If the solution that you selected is "Unknown", state what information must be added to answer the question.

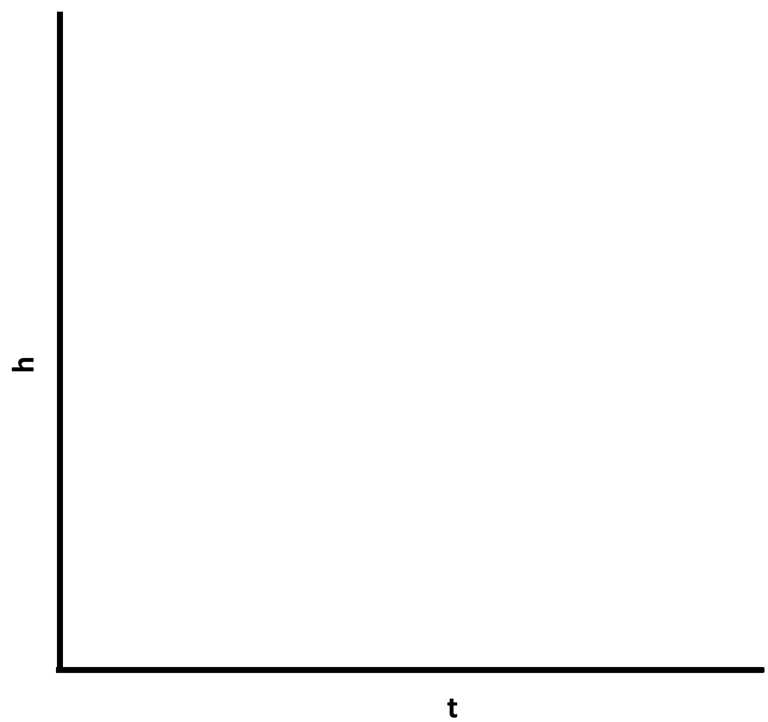
Indicate how confident you are in your responses for this question.

_ Total guess
_ Low
_ Moderate
_ High

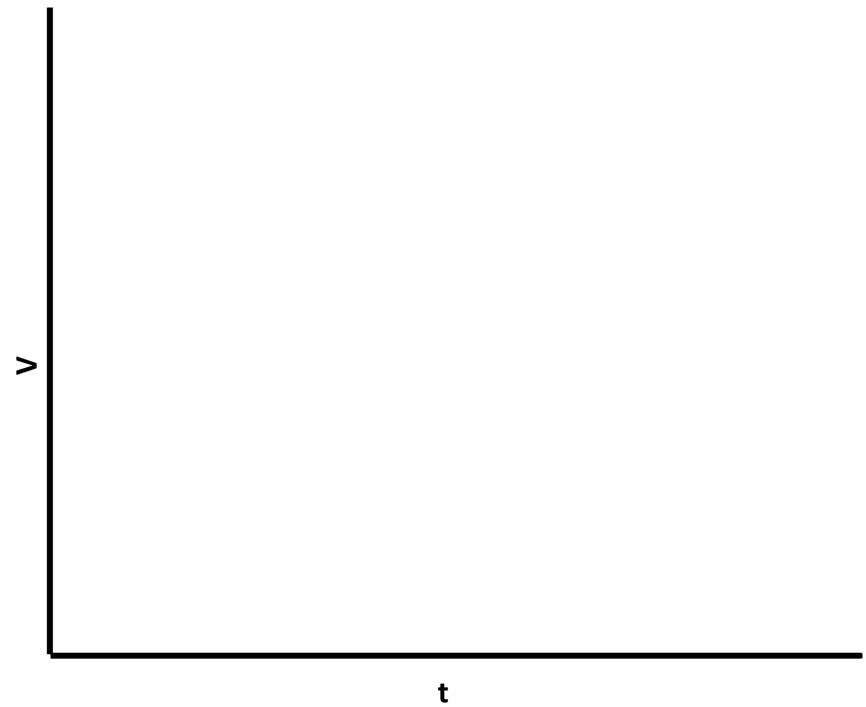
3. **Imagine the bottle shown below is filling up with water at a constant flow rate.**



a. **Consider the height of the water level (h) in the bottle as a function of time (t). Graph height vs. time starting from the bottle being empty until it is full.**



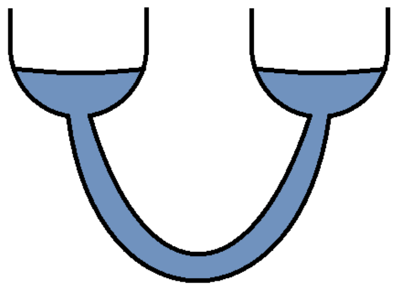
b. Consider volume of water in the bottle (V) in the bottle as a function of time (t). Graph the volume of water vs. time starting from the bottle being empty until it is full.



Indicate how confident you are in your responses for this question.

- _ Total guess
- _ Low
- _ Moderate
- _ High

4. **Two identical sinks are connected with a single pipe as shown. Both sinks are partially filled with water. The sinks are fixed at the same height.**



a. **Additional water is added to Sink 1 by pouring water from a pitcher. As the water is being added to Sink 1, the water level in Sink 2 will be:**

_ Rising
_ Falling
_ Remaining the same
_ Unknown (not enough information to select one of these three answers)

b. **Please explain your answer below. If the solution that you selected is "Unknown", state what information must be added to answer the question.**

Indicate how confident you are in your responses for this question.

_ Total guess
_ Low
_ Moderate
_ High

5. **Two bathtubs are partially filled with water and have small outlet drains which are plugged. The width and height of the bathtubs are equal, but the length of Bathtub 1 is twice that of Bathtub 2. The water level in the bathtubs is equal and no more water is entering either bathtub.**

   a. **If the outlet drains of each bathtub are unplugged at the same time, how will the how will the water flow rates of the outlet drains compare?**

      _ Outlet water flow rate in Bathtub 1 is greater than that of Bathtub 2
      _ Outlet water flow rate in Bathtub 1 is less than that of Bathtub 2
      _ Outlet water flow rate in Bathtubs are equal
      _ Unknown (not enough information to select one of these three answers)

   b. **Please explain your answer below. If solution is unknown, state what information must be added to answer the question.**

Indicate how confident you are in your responses for this question.

_ Total guess
_ Low
_ Moderate
_ High

6. **A gardener has two identical planter boxes that are filled with different mixtures of potting soil. The first box contains soil with 50% porosity (or void space) and the second box contains soil with 40% porosity. Both planters are completely dry, so the gardener uses two hoses with equal constant water flow rates to water both planters simultaneously.**

   a. **If each planter is watered until just before it overflows, which planter has collected more water?**
      _ Planter 1.
      _ Planter 2.
      _ Both planters collect the same amount of water.
      _ Unknown (not enough information to select one of these three answers)

   b. **Please explain your answer below. If the solution that you selected is "Unknown", state what information must be added to answer the question.**

   c. **Which of the planters will reach overflow sooner?**
      _ Planter 1 will reach overflow sooner
      _ Planter 2 will reach overflow sooner
      _ Both systems will reach overflow at the same time.
      _ Unknown (not enough information to select one of these three answers)

   d. **Please explain your answer below. If the solution that you selected is "Unknown", state what information must be added to answer the question.**

e. Which of the planters will collect water at a faster rate?

_ Planter 1 will collect water at a faster rate.
_ Planter 2 will collect water at a faster rate.
_ Both systems will collect water at the same rate.
_ Unknown (not enough information to select one of these three answers)

f. Please explain your answer below. If the solution that you selected is "Unknown", state what information must be added to answer the question.

Indicate how confident you are in your responses for this question.

_ Total guess
_ Low
_ Moderate
_ High

7. Two identical graduated cylinders with spigots at the bottom were partially filled with water. The water level in Graduated Cylinder 1 (GC1) is twice that of Graduated Cylinder 2 (GC2).

a. If the spigots of each graduated cylinder are opened fully at the same time, how will the water flow rates of the spigots compare?

- _ Spigot water flow of GC1 will be greater than that of GC 2
- _ Spigot water flow of GC1 will be less than that of GC 2
- _ Spigot water flows are equal
- _ Unknown (not enough information to select one of these three answers)

b. Please explain your answer below. If the solution that you selected is "Unknown", state what information must be added to answer the question.

Indicate how confident you are in your responses for this question.

- _ Total guess
- _ Low
- _ Moderate
- _ High

8. **You would like to melt ice which is at 0°C using hot blocks of metal as an energy source. One option is to use <u>one</u> metal block at a temperature of 200°C and a second option is to use <u>two</u> metal blocks each at a temperature of 100°C. Each individual metal block is made from the same material and has the same mass and surface area. Assume that the heat capacity is not a function of temperature.**

   a. **If the blocks are placed in identical insulated containers filled with ice water, which option will ultimately melt more ice?**

      _ Either option will melt the same amount of ice.
      _ The two 100°C blocks
      _ The one 200°C block.

Indicate how confident you are that you have selected the correct answer.

_ Total guess
_ Low
_ Low-Moderate
_ Moderate
_ Moderate-High
_ High

   b. **Because…**

      _ 2 blocks have twice as much surface area as 1 block so the energy transfer rate will be higher when more blocks are used.
      _ Using a higher temperature block will melt the ice faster because the larger temperature difference will increase the rate of energy transfer.
      _ The amount of energy transferred is proportional to the mass of blocks and the change in block temperature during the process.
      _ The temperature of the hotter block will decrease faster as energy is transferred to the ice water.

Indicate how confident you are that you have selected the correct answer.

_ Total guess
_ Low
_ Low-Moderate
_ Moderate
_ Moderate-High
_ High

c. Which option will melt ice more quickly?

- _ Either option will melt ice at the same rate.
- _ The two 100°C blocks.
- _ The one 200°C block.

Indicate how confident you are that you have selected the correct answer.

- _ Total guess
- _ Low
- _ Low-Moderate
- _ Moderate
- _ Moderate-High
- _ High

d. Because…

- _ 2 blocks have twice as much surface area as 1 block so the energy transfer rate will be higher when more blocks are used.
- _ The higher temperature block creates a larger temperature gradient which will increase the rate of energy transfer.
- _ The temperature of the hotter block will decrease faster as energy is transferred to the ice water.
- _ The rate heat transfer is proportional to the surface area of blocks and the temperature difference between the blocks and ice.

Indicate how confident you are that you have selected the correct answer.

- _ Total guess
- _ Low
- _ Low-Moderate
- _ Moderate
- _ Moderate-High
- _ High

9. **You have a glass of tea in a well-insulated cup that you would like to cool off before drinking. You also have 2 ice cubes to use in the cooling process and an equivalent mass of crushed ice.**

   a. **Assuming no energy is lost from the tea into the room, which form of ice (cubes or crushed ice) added to your tea will give a lower final drink temperature?**

      _ The crushed ice.
      _ The ice cubes.
      _ Either will lower the drink temperature the same amount.

Indicate how confident you are that you have selected the correct answer.

_ Total guess
_ Low
_ Low-Moderate
_ Moderate
_ Moderate-High
_ High

   b. **Because…**

      _ Energy transfer is proportional to the mass of ice used.
      _ Crushed ice will melt faster and will transfer energy from the tea faster.
      _ Ice cubes contain less energy per mass that crushed ice so tea will cool more.
      _ Ice cubes have a higher heat capacity than crushed ice.
      _ Crushed ice has more surface area so energy transfer rate will be higher.

Indicate how confident you are that you have selected the correct answer.

_ Total guess
_ Low
_ Low-Moderate
_ Moderate
_ Moderate-High
_ High

10. An engineering student has two beakers containing mixtures of dye in water. The first beaker has a 1% dye solution (1 gram of dye in 100 grams of water) and the second beaker has an equal volume of a 2% dye solution (2 grams of dye in 100 grams of water). The student places 2 identical sponges in the 1% dye solution and 1 sponge in the 2% dye solution.

a. Which of these combinations will absorb more dye?

_ The two sponges in the 1% solution will absorb more dye.
_ The one sponge in the 2% solution will absorb more dye.
_ Both systems will absorb the same amount of dye.

Indicate how confident you are that you have selected the correct answer.

_ Total guess
_ Low
_ Low-Moderate
_ Moderate
_ Moderate-High
_ High

b. Which of these combinations will initially absorb dye at a faster rate?

_ Two sponges in the 1% solution will absorb dye at a faster rate.
_ One sponge in the 2% solution will absorb dye at a faster rate.
_ Both systems will absorb dye from solution at the same rate.

Indicate how confident you are that you have selected the correct answer.

_ Total guess
_ Low
_ Low-Moderate
_ Moderate
_ Moderate-High
_ High