2006-1074: DEVELOPMENT OF AN INTEGRATED LEARNING FRAMEWORK FOR STEM LEARNING

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Abstract

As part of an NSF Math Science Partnership project targeting mathematics and science learning, our project is delivering a set of courses to high school mathematics and science teachers that integrates relevant mathematics, science, and engineering concepts and practice. These courses will promote conceptual competence in core content and key process behaviors in scientific inquiry, mathematical problem solving, and engineering design. A distinctive element of this effort is a commitment to design a coherent approach consistent with existing scholarship in the fields of STEM education. An early result of this effort has been the recognition by the project’s mathematics, science, and engineering faculty researchers of the need for an overarching learning framework that elucidates the commonalities, the distinctions, and the relationships between the learning and practice of mathematics, science and engineering. The starting point for development of this STEM learning framework is research on learning frameworks already developed by psychologists and by math, science, and engineering educators.

Preliminary work developing this framework has shown that although the process behaviors of mathematical problem solving, scientific method, and engineering design can be described using roughly similar overall frameworks, many elements of the frameworks do not map directly onto one another. We propose that clearly elaborating such similarities and differences in the process behavior frameworks for mathematics, science and engineering may illuminate difficulties in the integration of instruction for these fields. Further, we predict that by systematically engaging teachers in activities relating the nature of mathematics, science, and engineering practice over a long-term professional development experience, we will observe improvements in their ability to offer coherent mathematics and science programs in their schools leading to improved student preparation for STEM undergraduate programs.

Introduction

While central process behaviors in mathematics (mathematical problem solving), science (the scientific method), and engineering (engineering design) exhibit significant similarities, and certainly utilize significantly overlapping concepts, beliefs, attitudes and tools, there are some obvious differences. The purpose of this paper is to begin an exploration into these similarities and differences, with the long-term goal of developing an overarching learning framework that elucidates the commonalities, the distinctions, and the relationships between the learning and practice of mathematics, science and engineering. Such a framework would ultimately support the improvement of both cognitive and affective student outcomes. For example, if teachers recognize and teach the similarities of mathematical problem solving, scientific inquiry and engineering design, mutual reinforcement and improved integration among these disciplines is supported, and in addition, a student can more easily see the value of each approach to her ‘home’ discipline. An additional value of such a framework would be in providing a structure for translating the results of disciplinary research on knowing and learning from within these
disciplines of mathematics, science, and in engineering across disciplinary boundaries to mutually inform each other. In engineering, at least, this might prevent much ‘reinventing the wheel’ in engineering education research.

We will first present currently used frameworks for mathematical problem solving, scientific inquiry, and engineering design, compare and contrast structures and elements within these frameworks, and finally present our current working version of a unified framework for the learning and teaching of these fundamental STEM process behaviors.

A Framework for Learning and Teaching Mathematical Problem Solving

We look first at a framework for mathematical problem solving developed through empirical studies with research mathematicians by Carlson and Bloom\(^1\) and drawing on a history of analysis of problem solving extending from George Polya’s famous 1945 book *How to Solve It*\(^2\) to Alan Schoenfeld’s\(^3\) cognitive studies and frameworks. Carlson and Bloom’s framework characterizes problem solving along dimensions of the individual’s behaviors and attributes. The behaviors are engaged in a cyclical manner through the following four phases:

1. Observation of a problem, which may be posed by another individual or formed by the problem solver in response to other results, a puzzling real world situation or pattern, etc. This phase involves orienting oneself to the nature, elements, and structure of the problem.
2. Conjecturing solution paths involves imagining several possible plans of attack without actually carrying them out, quickly evaluating the potential effectiveness and requirements of each, then making a decision on how to proceed. This is often informed by previous problem solving efforts.
3. Execution of a plan involves proceeding with the chosen strategy and constantly monitoring progress, possibly reverting back to the planning phase if things are not going well or new information is discovered.
4. Checking and evaluation of whether the result makes sense and whether a viable solution has been achieved occurs near the end of a problem solving cycle. If the solution is not satisfactory, then one may discard the results of that plan and cycle back to the planning phase. If the solution method produces only a partial result or raises other questions, the problem solver may build on that by cycling forward to an additional planning cycle to make further progress.
Interacting with the problem solving phases are attributes of the individual including 1) their available resources such as knowledge and experience, 2) heuristics for solving various problems and an ability to modify them, 3) affective qualities such as curiosity, intimacy, and frustration, and 4) metacognitive skills such as monitoring for quality and effectiveness of thought processes.

While problem solving does not capture all forms of mathematical activity, frameworks to characterize other categories involve similar processes. For example, Harel’s DNR proof framework consists of three principles: Duality, Necessity, and Repeated reasoning. The duality principle links students’ ways of understanding (involving one’s interpretations, solutions, and evidence) to their ways of thinking (beliefs, problem solving approaches, and proof schemes). The necessity principle reflects the intellectual need to engage in the process of creating or understanding a proof, and the repeated reasoning principle reflects a need for practice reasoning. Thus the process of generating proofs involves a goal-oriented problem solving process with multiple affective components and is dependent on cognitive resources. Although the explicitly iterative aspect of the DNR framework (repeated reasoning) plays a pedagogical role, there is also an implicit iterative aspect for evaluation and refinement of the process of creating a proof in the problem solving component of the duality principle.

Most frameworks for mathematical modeling are even more closely aligned with the problem solving framework whether they formulate modeling as an explicit process as in Buck or lay out general principles as in Smith & Bleloch. They all incorporate aspects of orienting to the problem (e.g., identification of variables and constraints and making simplifying assumptions and estimations), constructing and implementing the model (e.g., using algorithms and heuristics), evaluation (monitoring progress and verisimilitude and interpreting abstract entities in terms of the original context), and iteration (e.g., reconsidering assumptions and adding complexity).

We identify a fourth area of mathematical activity, statistical inference and hypothesis testing, with aspects of scientific inquiry outlined in the following section. In particular, a statistical Null hypothesis and a scientific hypothesis play similar logical roles. Loosely, they are used to
generate a prediction (in statistics this will be a sample distribution). After this, one devises a
test, collects data, compares observations against the prediction, and draws conclusions. As will
be discussed below, this process also has many similarities to the process of mathematical
problem solving.

**A Framework for Learning and Teaching Scientific Inquiry**

The second framework contributing to our integrated STEM learning framework is that of the
hypothetico-deductive reasoning cycle, as presented by authors such as Wallas\(^8\), Koestler,\(^9\) and
Lawson\(^10\), as illustrated in Figure 2. This pattern of hypothetico-deductive (HD) reasoning can
be seen at the heart of all scientific inquiry. The generation and deductive testing of
hypotheses/theories/conjectures is the central process behavior through which science advances
and science is ‘performed’. Human reasoning during discovery, problem solving, and invention
seems to operate in a hypothetico-deductive way, involving the following elements:
Figure 2. The Hypothetico-deductive reasoning cycle, after Wallas, Koestler, and Lawson

1. Making an initial puzzling observation or encountering a problem or a puzzling observation that contradicts current expectations about how the world should work. Importantly, current expectations are based on current mental models. Thus, puzzling observations and problems lead to the possibility that current mental models may need to be modified, extended, or even replaced.

2. Using analogical reasoning, analogical transfer, or abduction (i.e., a matching process), to spontaneously generate one or more hypotheses or problem solutions. In causal contexts, the reasoning process involves borrowing ideas that have been found to "work" in one or more past related contexts and using them as possible solutions/hypotheses/guesses in the present context.

3. Supposing for the sake of argument and test, that the hypothesis/conjecture under consideration is correct. This supposition is necessary so that a test can be imagined with relevant condition(s) that along with the hypothesis/conjecture allow the deduction of one or more consequences (i.e., expected results).

4. Conducting the imagined test. If the proposed ideas are in fact tested, the test must be conducted so that its expected result(s) can be compared with the observed result(s) of the test. The observed result(s) constitutes the evidence, which can be circumstantial, correlational or experimental in nature.

5. Comparing observed and expected results. This comparison allows one to draw a conclusion. A good match means that the hypothesis/conjecture is supported, but not proven, while a poor match means that something is wrong with the hypothesis/conjecture, the test, the deduction, or some combination.

6. Re-iterating the above steps until an alternative is generated, tested, and supported on one or more occasions and its competitors have been tested and rejected. It should be noted that the steps may occur on a subconscious level, the process may stop somewhere along the way, and/or mistakes may be made. Consequently, the steps represent a general plan for success—not a method that insures success.

A Framework for Learning and Teaching Engineering Design

Descriptions of the engineering design process have been presented in many sources and representations; we will begin with a structure used by Atman and co-workers. They divide the design process into a series of roughly sequential steps: (1) problem definition, (2) information gathering, (3) idea generation, (4) modeling (5) feasibility analysis, (6) evaluation, (7), decision, and (8) communication, as shown in Figure 3. Although this particular set of steps is not used universally (Fogler and LeBlanc, for example, include an “implementation” step, and Voland adds a “needs assessment” step), essentially all published engineering design algorithms can be fairly easily mapped to each other.

In addition to the process steps in most design algorithms, most representations of the design process are also explicitly cyclical (for example, Voland’s representation arranges the steps on a circular figure). In Figure 3, we illustrate the cyclic or iterative nature of engineering design by arrows: the result of essentially any step might be a decision to revisit an earlier step (for example, results of feasibility analysis might illustrate the necessity redefining the problem).
Although many of the published representations of the engineering design process exhibit significant overlap, only a few studies have been published about how engineers engage in the process of design and what conceptual bases they use in the design process, primarily by Atman and co-workers\textsuperscript{11-14}.

**Comparison of the Process Frameworks**

As a starting point of comparison, the process frameworks represented in figures 1 through 3 show some obvious similarities: they are each cyclical or iterative, with some element of checking or evaluation driving the iteration. In addition, they each proceed broadly from identification/definition of a problem, need, or discrepancy between expectation and reality, include some element of creativity, imagination or conjecture, and move from the concrete/contextualized/situated world to the abstract/generalized world through (the process of...
abstraction) and back (the process of application). Obvious differences between the frameworks include their supporting conceptual bases, the ultimate product of the respective processes, and their terminologies and nomenclatures.

The concept structures
It is useful to consider how the concept structures of mathematics, science, and engineering not only inform their respective process behaviors, but also inform each other. Figure 4 represents a preliminary representation of the mutual interactions of engineering, mathematics, and science concepts.

While the concept structures of mathematics (mathematical concepts) and science (scientific theory) were relatively easy to identify, it was initially somewhat difficult to identify the comparable ‘concept structure of engineering.’ Engineering design is always an integrative activity—practitioners call on knowledge and relationships acquired from schooling, from experience, from training, and from their ‘sense’; they will integrate concepts from science, mathematics, social sciences, economics and esthetics in the performance of their discipline’s ‘process behavior’. There are perhaps some concepts most engineers would recognize as ‘universal’ to engineering, - conservation (of mass, energy, charge, momentum) for example, - but these are certainly not unique to engineering. We propose that the ‘concept structure’ of engineering corresponding to the mathematical concepts and scientific theory structures, is in fact, a structure comprising heuristics and values: heuristics such as Occam’s razor, and values such as meeting societal needs. In this we agree with and build from Koen’s assertion that “the engineering method is the use of heuristics to cause the best change in a poorly understood situation within the available resources.” We view heuristics as the elements with which the method of design is accomplished, and “best” implying the value structure. Working from this statement, then engineers ‘use’ heuristics in the process of engineering design in a manner similar to the way scientists ‘use’ theory in the process of scientific inquiry. We note that although mathematics and science are most certainly also preformed within a value
structure, what makes the value structure of engineering unique among the three is that it derives from and is in explicit service to the non-engineering society at large.

The products of the processes
We focus this comparison primarily on the contrast between the product of scientific inquiry and engineering design. The product of scientific inquiry – especially ‘pure’ research – is new or improved scientific theory. The immediate ‘customer’ for the ‘product’ of the process is the scientific community itself. In contrast, the product of engineering design is a new or improved artifact or process. The immediate customer of the process is more than likely not to be a member of the engineering community, since the need for the new artifact or process is normally driven fairly directly by a societal need. While scientific models are normally developed in order to support the understanding or explanation of a phenomenon, engineering models are in addition developed to support the creation, design, use, adaptation or improvement of and artifact or process.” Since engineering design is concerned with generation of new systems, process, and artifacts, while science is primarily concerned with extant systems and processes\textsuperscript{18}, engineering design aims more at predicting future behaviors of both extant and new processes and systems, than at explaining phenomena\textsuperscript{19}. Issues concerning schools and teachers

Ongoing and Future Work

This work is part of a large project that is aimed at improving high school math/science teacher preparation and ultimately, at improving student outcomes We predict that by systematically engaging teachers in activities relating the nature of mathematics, science, and engineering practice over a long-term professional development experience, we will observe improvements in their ability to offer coherent mathematics and science programs in their schools leading to improved STEM outcomes for grades 9-16 (attitudes and beliefs, grades, standardized test scores, choice of STEM courses, choice of, retention in, and success in STEM college majors). The
approaches we are taking are based as much as possible on the research literature. Although that literature is growing rapidly, there are still important gaps, and less inter-disciplinary communication than might be best.

This work has been motivated by conversations among the project’s science, mathematics, and engineering faculty. Such conversation has led us to conclude that a unique opportunity exists to enrich the teaching and learning of STEM disciplines at all levels through better communication between, and integration of the research on knowing and learning across STEM disciplines. While recognizing that the disciplines have unique aspects to their concepts, their key process behaviors, and their nomenclatures, we also predict that in the long run, emphasizing commonalities will lead to better student learning in STEM fields. We propose that elucidating both the differences and commonalities in the process behavior frameworks of mathematics, science and engineering may illuminate difficulties in the learning of mathematics, science and engineering. Development of an integrated framework will provide a necessary platform from which we can address fundamental questions in STEM knowing and learning:

1. What are the key factors (cognitive and behavioral processes, affective dispositions, resources, etc.) involved in the STEM process behaviors of mathematical problem solving, scientific inquiry, and engineering design?
2. What are the common elements of these process behaviors? How do the key factors differ with expertise and experience level?
3. What are the indicators of goodness or quality in the products of these STEM process behaviors? How are these influenced by their ultimate purpose?
4. How are the key factors of the STEM process (mathematical problem solving, engineering design, scientific inquiry) related to the success or quality of the resulting STEM product (theory, artifact/process design, mathematical concept or proof)?
5. What skills explicitly taught in one context are transferred by STEM students to other contexts?

Bibliography


