AC 2009-296: DEVELOPMENT OF VISUALIZATION TOOLS FOR ONE AND MULTIPLE DOF DYNAMIC SYSTEMS

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Development of Visualization Tools for One and Multiple DOF Dynamic Systems

Introduction

A course in system dynamics is required in most mechanical and other engineering curricula. System dynamics deals with mathematical modeling and response analyses of dynamics systems. Over years the authors have observed that the students have difficulties with the fundamental concepts such as frequencies and damping, and some advanced topics such as vibration absorbers and multiple degrees of freedom systems, etc. The material in this course is very different than the students' previous courses, the majority of which are based on statics concepts. Even the course in dynamics usually focuses on the rigid body dynamics, and usually does not cover those fundamental topics in system dynamics.

Many modern-day engineering students are graphical learners. Multimedia content generally enhances student retention and interest [1]. In order to help the students better understand the concepts and topics in system dynamics, a series of MATLAB based graphical user interfaces (GUIs) and models have been developed. Multi-layered graphical user interfaces have been used in classroom teaching, including time and frequency response of first and second order systems due to a variety of different input conditions and initial conditions, for both one degree of freedom and two degrees of freedom systems. In this paper, selected example GUIs are introduced and displayed. The graphical user interfaces present data in a form so that students can immediately see the effects of changing system parameters as they relate to frequencies, damping, and even the principles of vibration isolation and vibration absorption. The paper also presents the student survey assessment on the usefulness of the tools in the enhancement of teaching in dynamic systems.

There has been considerable work done to exploit the use of computer graphics to clarify math and engineering subjects. For example, an early paper used MATLAB to illustrate solutions to hyperbolic differential equations [2]. The concept of using MATLAB for the animation of lumped parameter dynamic systems was demonstrated by Watkins et al [3]. Jacquot et al [4] created a series of MATLAB scripts that illustrate the solutions to partial differential equations commonly encountered in mathematics, engineering and physics courses. Recently there have been a number of papers describing the MATLAB and SIMULINK based GUIs related to response of dynamic systems due to a variety of different input conditions [5,6,7].

Compared with the published visualization tools for dynamic systems and other subjects, the developed MATLAB GUIs have some unique properties. They are multi-layered; both time response and frequency response for generally used input signals, such as step, impulse, ramp, and sinusoidal functions, can be displayed with the same interface by choosing corresponding pushbuttons. Besides the single degree of freedom 1st and 2nd order system, GUIs are also developed for two degrees of freedom systems with multiple inputs and initial conditions. The principle of vibration absorber is illustrated clearly by a GUI for two degrees of freedom system as well.

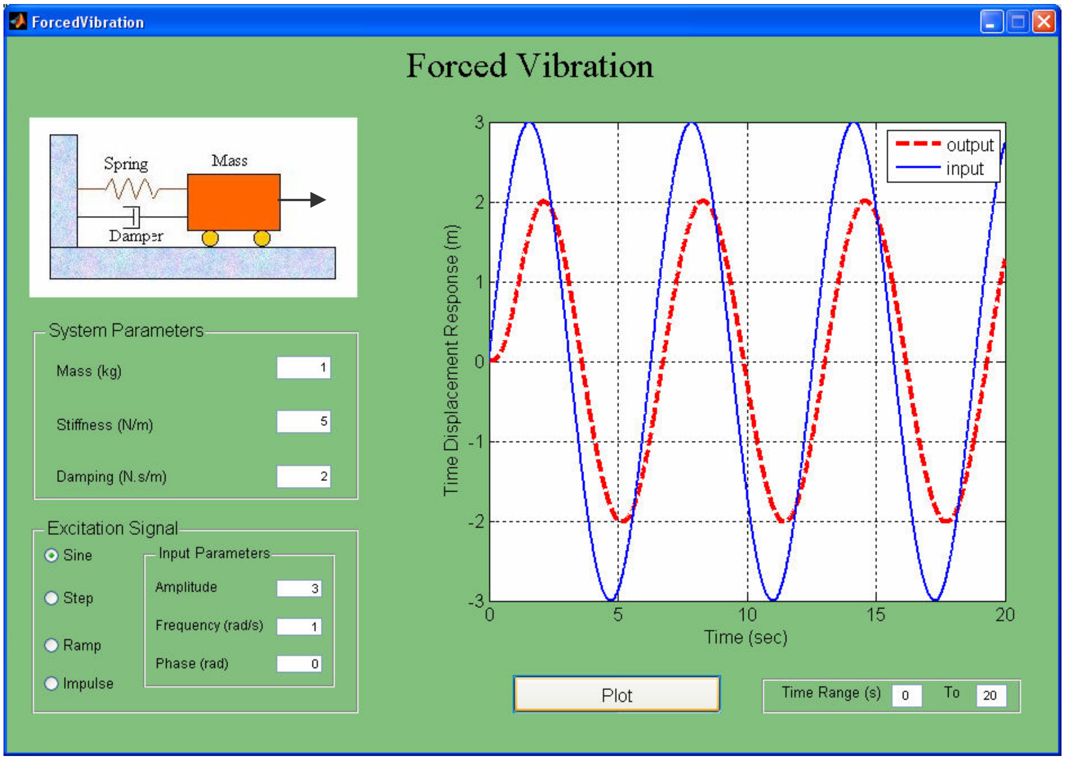
Single Degree of Freedom Dynamic System Characterization

First and second order single degree of freedom systems are very common in engineering dynamic systems. Many engineering problems, like resistance-capacitance (RC) circuit, temperature rise in a thermometer, pressure change in a piston, can be modeled by ordinary differential equation of first order. On the other hand, engineering systems, like mass-spring-dashpot mechanical system and inductance- capacitance -resistance (LCR) circuit, can be modeled as ordinary differential equation of second order. Both 1st order and 2nd order systems form an important cornerstone of knowledge in system dynamics.

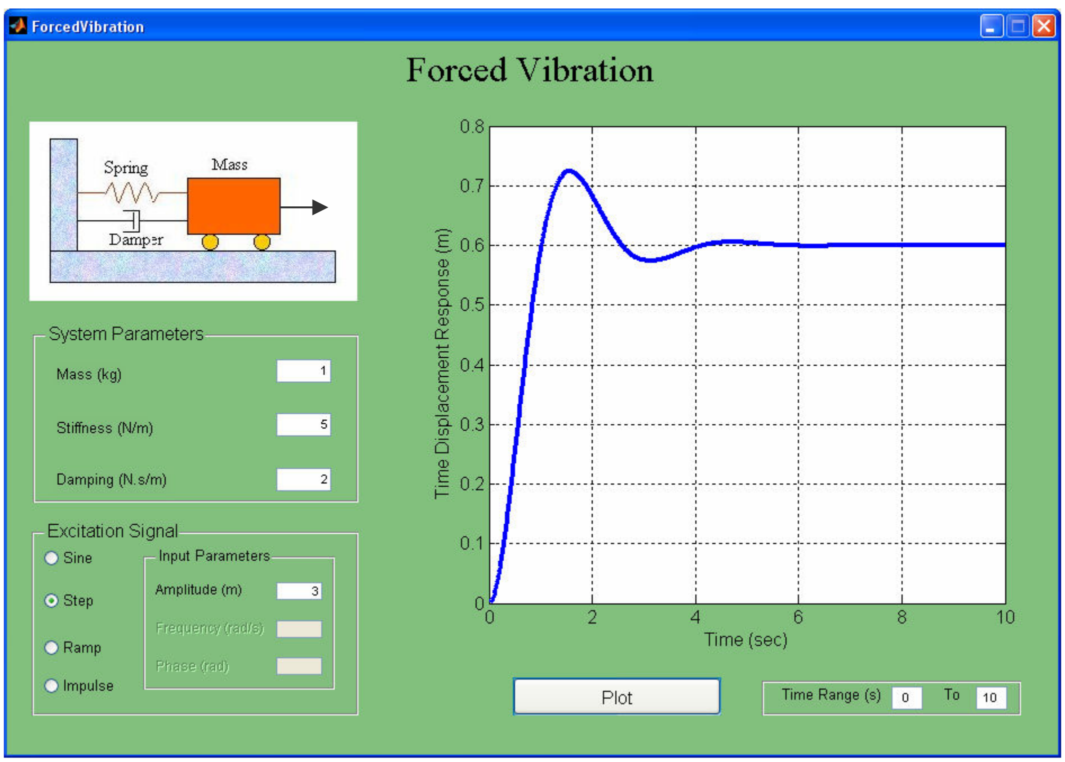
Only two MATLAB based GUIs of single degree of freedom 2nd order systems are displayed here. The 1st order system GUIs are similar to those of the 2nd order systems. The GUI for forced vibration is shown in Figure 1. The students are allowed to control system parameters and input forcing functions. System parameters include mass, damping coefficient, and spring constant. Generally used forcing functions, such as sinusoid, step, ramp, and impulse inputs, are available to choose. For an input function chosen, the needed parameters can be specified. For example, if the input is selected as sinusoidal function, as shown in Figure 1(a), amplitude, frequency, and phase angle of the input must be entered. The GUI reports the time response of the system and the input function as well. The time range of the response plot can be adjusted as desired. Similar to Figure 1(a), Figure 1(b) shows the case of step input with amplitude of 3 units.

The MATLAB GUI for the initial condition response of a 2nd order dynamic system is shown in Figure 2. The students are asked to enter the initial displacement and initial velocity, in addition to the system parameters. Two pushbuttons give the choices for time response plot or frequency plot. Figure 2(a) shows the time response plot. The frequency response plots are shown in Figure 2(b), including both logarithmic magnitude curve and phase-angle plot. The GUI also reports the natural frequency, damping ratio, and damped frequency, in the left corner box on the interface.

The visual interface presents results in a way that students can immediately identify the effects of changing system parameters. For the developed GUIs, after the first solution is plotted, the system parameters and/or excitation can be altered to plot another system response. The two solutions can be compared to analyze the effect of different system parameters. The ‘Clear’ pushbutton can be clicked to clear all the graphical output from the graph. A good example to use this feature is to plot three different cases of system responses, i.e., under-damped system, critical damped system and over-damped system, in the same figure, as shown in Figure 3.

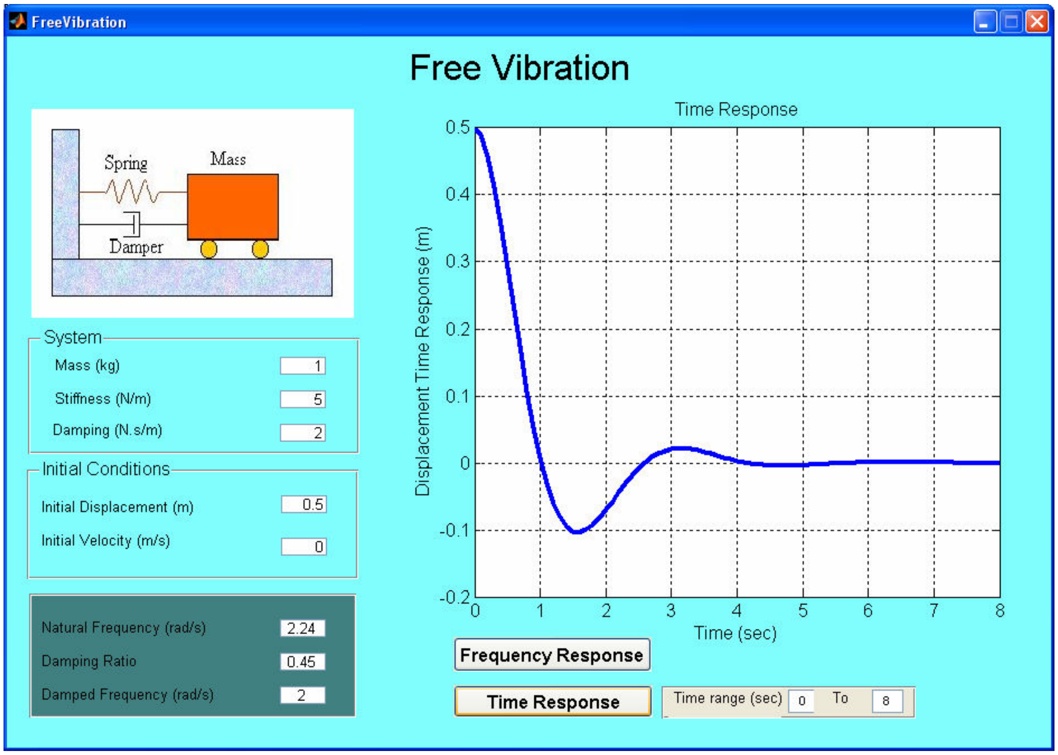


(a)

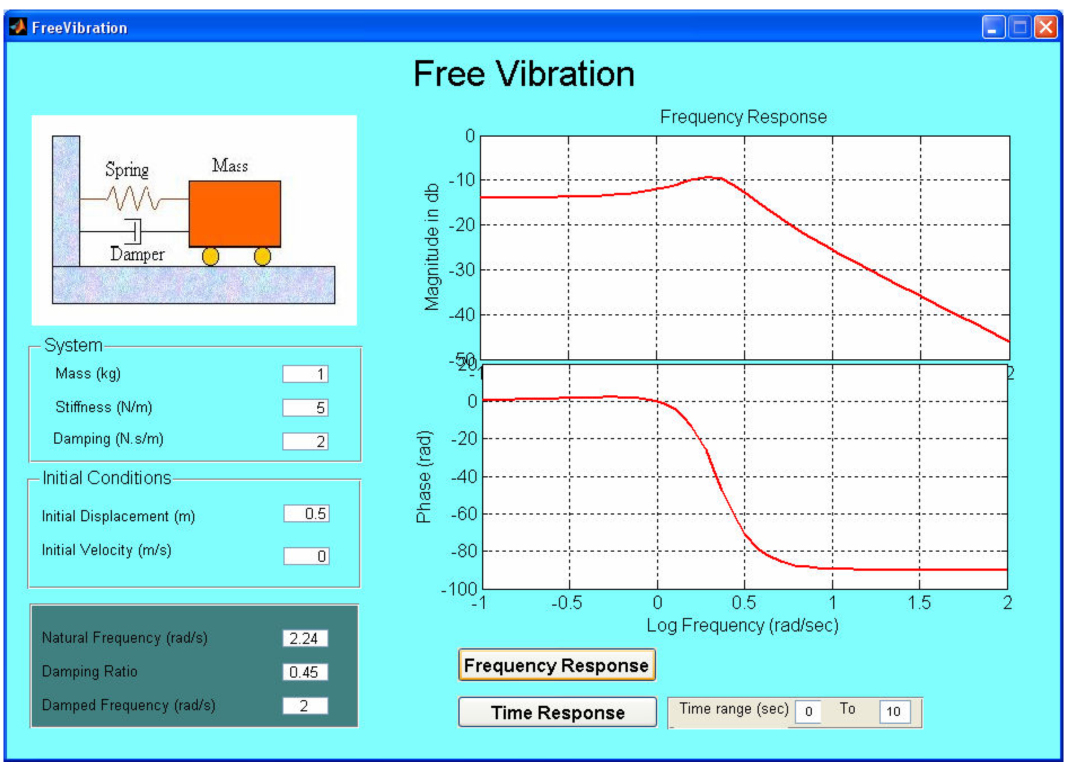


(b)

Figure 2 – Multifunctional GUI for the forced vibration of a 2nd order system: (a) sine force excitation; (b) step input



(a)



(b)

Figure 2 – Multifunctional GUI for the free vibration with initial conditions of a 2nd order system: (a) time response; (b) frequency response

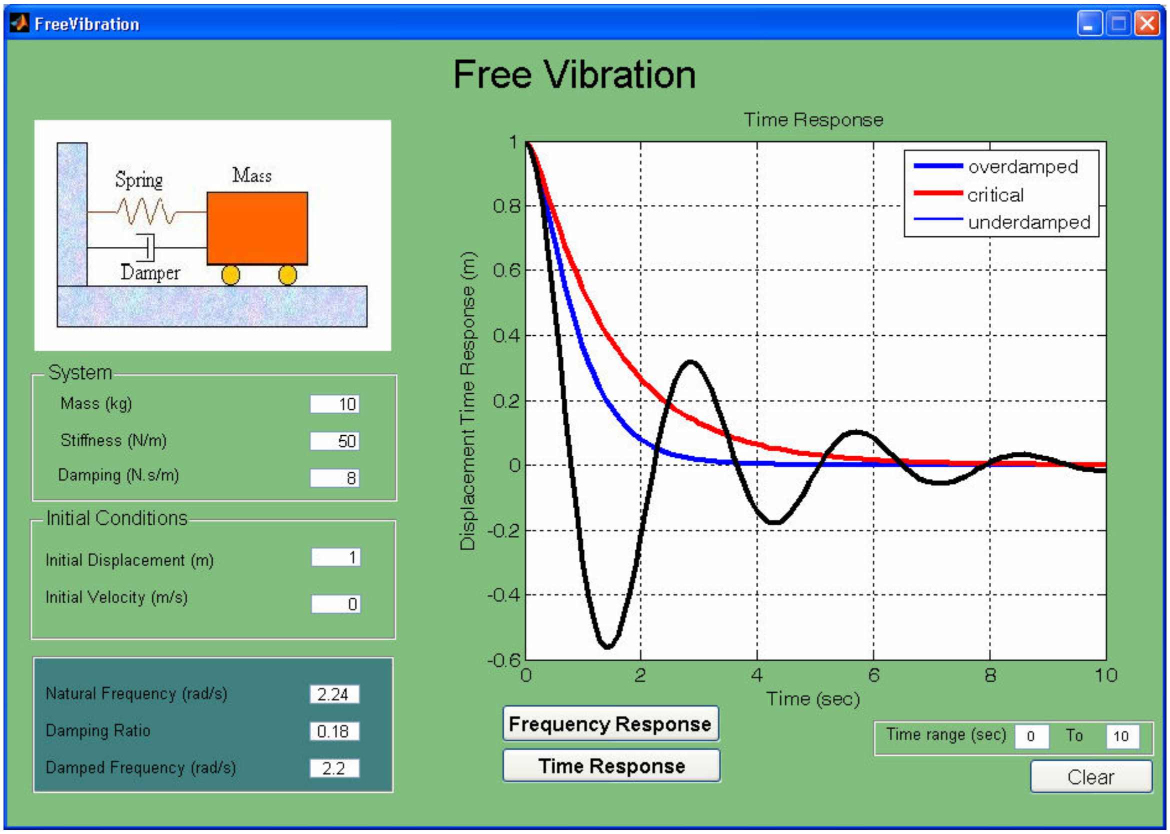


Figure 3 – Responses of three different cases of a system

Two Degrees of Freedom Dynamic System Characterization

Modeling in one degree of freedom is not always appropriate in practice. Many practical vibration applications require a model having more than one degree of freedom in order to describe the important features of the system response [8]. Consider a mass-spring-damper system with two degree of freedom. Let m_1, m_2, b_1, b_2 and k_1, k_2 be the mass, damping coefficient and stiffness of the system in Figure 4. x_1 and x_2 are the displacements of the two masses of the system when external force excitations, $u_1(t)$ and $u_2(t)$, and/or initial conditions are applied on masses m_1 and m_2, respectively. The external excitation can be either zero or any of the four standard inputs, namely sinusoidal, step, ramp or impulse functions.

The system can be mathematically modeled as the following matrix equation.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} b_1 & -b_1 \\ -b_1 & b_1+b_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1+k_2 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad (1)$$

This equation can be solved to obtain the nature of the system. Due to the complicated nature of external force, in many cases, analytical calculations of such systems can be difficult.

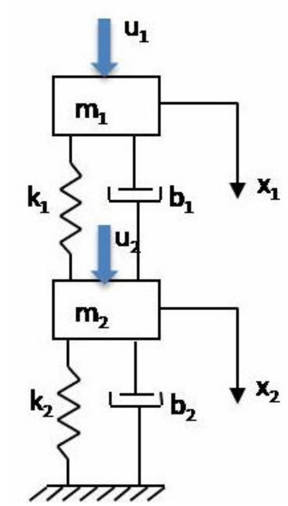


Figure 4 - Two degree of freedom mechanical system

The GUI for solving this equation has two parts, solution with only external excitations and solution with only initial conditions (initial displacement and velocity). The complete or total solution is the sum of the two solutions.

The state-space equation for this system can be written as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{2}$$

where *x* is the space vector, *u* is the input vector, and y is the output vector of the system in time domain denoted by *t.*

Based on the state space representation, a system with the transfer function 'sys' for a two degrees of freedom system can be defined in MATLAB as

sys=ss (A, B, C, D) (3)

where A is the state matrix of the system, B is the input matrix, C is the output matrix and D is the direct transmission matrix.

The response subjected to initial displacement and velocity of such a system zero external excitation can be obtained in MATLAB using 'initial' function as

Y_i = *initial* (*sys, x0, t*); (4)

where *x0* is the initial condition matrix, and Y_i is the matrix to hold the numerical solution.

Similarly, the response due to external excitation with zero initial conditions of the system can be obtained in MATLAB using the 'lsim' function as

$$Y_f = lsim\ (sys,\ u,\ t);\qquad (5)$$

where u is the external excitation vector, and Y_f is the matrix to hold the numerical solution.

The complete response of the system is the sum of Y_i and Y_f. This GUI has been developed to manipulate all these equations to find the final solution in a user friendly manner, as shown in Figure 5. The users enter the system parameters such as masses m_1 and m_2, damping coefficients b_1 and b_2, and stiffness k_1 and k_2. The initial conditions of the system should also be entered. For the external excitations, the users can select any one out of four standard inputs, such as step, impulse, sine, and ramp. In some cases, either initial conditions or the external excitations can also be zero. With all these information provided, all the calculations are done by the GUI automatically. A final graphical solution can be obtained just by clicking the 'Plot' pushbutton.

As previously indicated, comparison of two systems with varying the system parameters and excitations is also possible in this GUI. Similarly, the time range of the plot can be varied to get the solution of desired time range by changing the limits in the bottom right of the GUI.

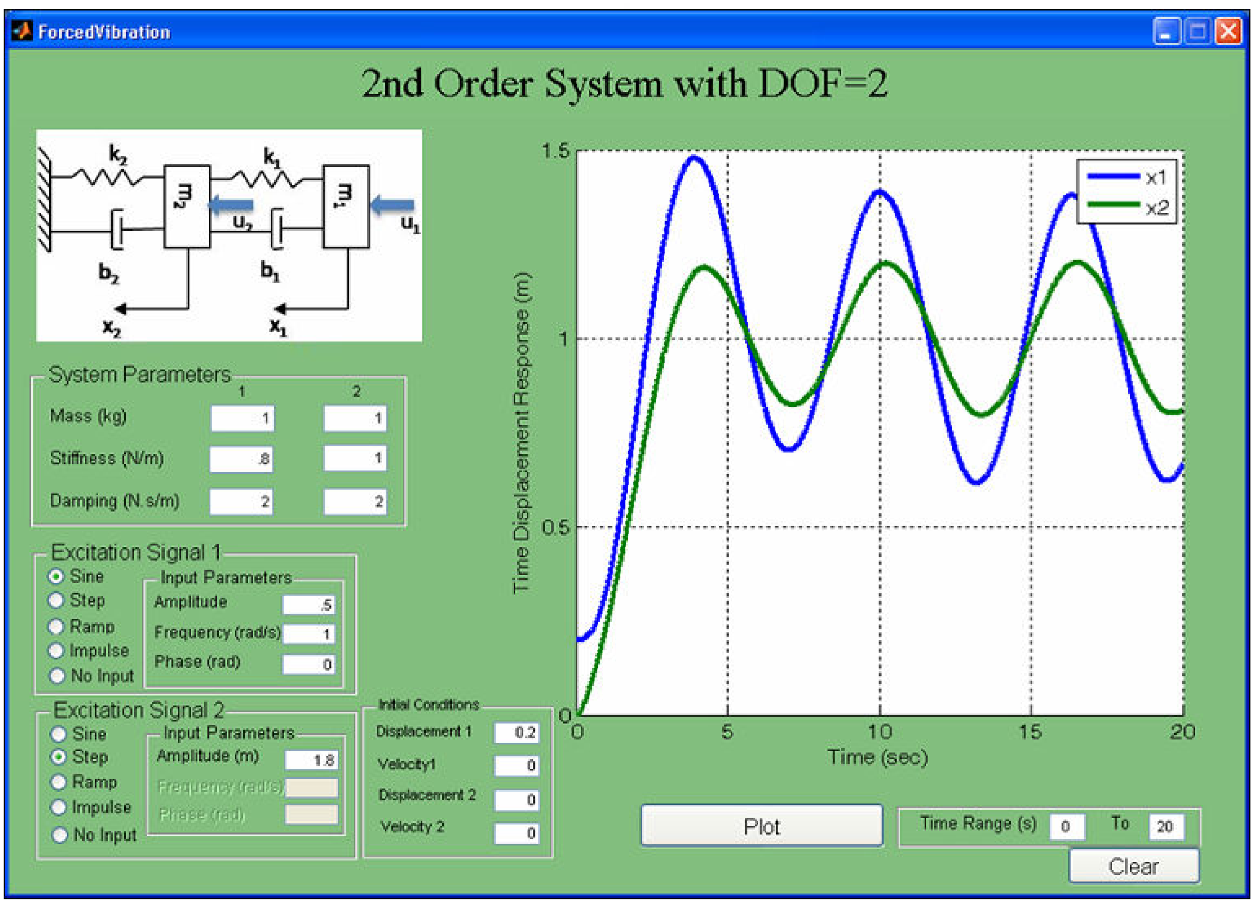


Figure 5 – GUI for the dynamic response of two-DOF 2nd order system subjected to initial conditions and external excitations.

Vibration Absorber

When a major system, such as a rotating turbine or compressor, is excited by an external force or displacement at a given frequency, it is possible to modify the vibration if the resulting vibration amplitude is too high. Vibration amplitude can be significantly reduced by using an auxiliary mass on a spring tuned to the frequency of the excitation. Such an auxiliary mass system with very little damping is called a dynamic absorber or vibration absorber.

Consider a system in Figure 6 with primary mass *m*, stiffness *k* and damping coefficient *b*. For the auxiliary absorber, m_a is the mass, k_a is the stiffness, and b_a (≈ 0) is the damping coefficient. The condition for zero vibration in the primary system is

$$k_a - m_a\omega^2 = 0 \qquad (6)$$

where ω is the known constant frequency of the external excitation.

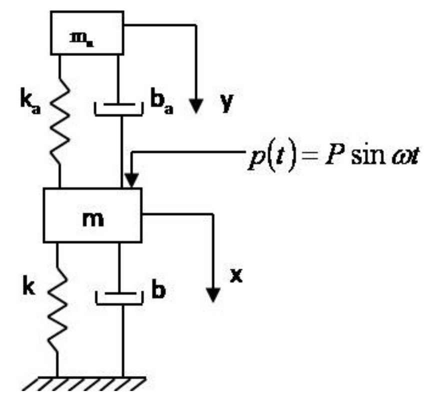


Figure 6 - Vibration Absorber

The dynamic absorber is a practical example of controlling vibration amplitude in single degree of freedom systems. However, the addition of the absorber to the system changes that system to a two degree of freedom system. Because of this, the associated GUI is similar to the previous GUI for second order system with two degrees of freedom. The modified CUI is shown in Figure 7. The only difference is that the natural frequency of dynamic absorber is equal to that of the external sinusoidal applied to the primary mass. The user is allowed to enter the parameters such as mass, stiffness and damping of the primary system and the sinusoidal force applied on the primary system. Mass and damping for the vibration absorber should also be provided by the user. The GUI reports the stiffness of the dynamic absorber and plots the vibration of both masses with respect to time. The zero displacement of the primary mass can be clearly observed in the GUI plot.

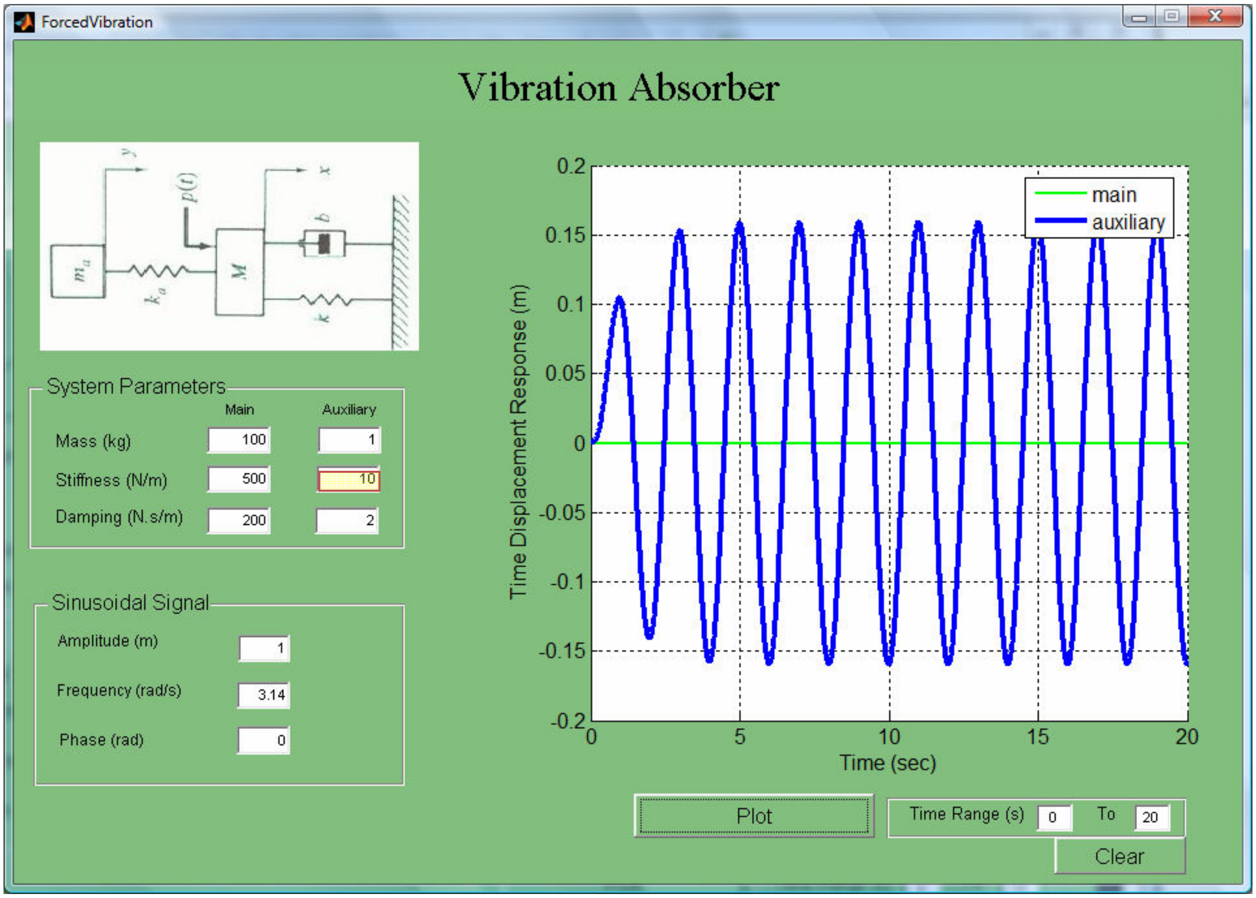


Figure 7 – Use of an auxiliary mass as a vibration absorber to reduce vibration amplitude of the primary mass of a one DOF system

Student Assessment of GUIs

The series of developed 1-DOF and 2-DOF system response GUIs have been used in classes of dynamic systems, and mechanical vibrations as well. Student satisfaction survey on the use of GUIs was performed in summer 2007 and fall 2008 semesters. The students answered questions on a Likert scale [9]. The format of the five-level Likert item is: 1-strongly disagree, 2-disagree, 3-neither agree or disagree, 4-agree, and 5-strongly agree. The result is shown in Table 1.
The students indicated that they felt that the GUIs helped them to visualize the mathematical solutions with greater ease and provided a deeper understanding of the underlying concepts.

Table 1: Student Satisfaction Survey

Number of Students	Average Rating	Standard Deviation
40	4.1	0.6

Conclusion

Multi-layered graphical user interfaces have been developed and employed in classroom teaching. The GUIs include time and frequency responses of first and second order systems due to a variety of different input signals and initial conditions, for both one degree-of-freedom and two degree-of-freedom systems. The implementation of the series of the MATLAB GUIs help students to understand important concepts and theories behind complex differential equations, such as the relationships between the inputs and outputs, solutions in time-domain and frequency-domain, three vibration modes for the systems, and the effects of system parameters and input selections on output responses, two degrees of freedom mechanical systems, and the principles of vibration absorbers. Student response was very strong concerning the need of the GUIs to help foster a deeper understanding of course materials.

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