Didactic Dimensions of the Future Total Worth Model

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Abstract

The future total worth (FTW) model [1] offers a new method for teaching engineering economics. It is easy to integrate into existing syllabi, and it provides the theoretical foundation for instructors to address commonly occurring, practical questions such as:
1. Why is the minimum attractive rate of return (MARR) used to compute present worth (PW) and equivalent annual worth (EAW)?
2. What is the definition of equivalence within an industrial context?
3. How are PW and EAW interpreted?
4. Why does choosing the largest PW or EAW maximize a company's FTW?
5. How should alternatives with different lives be compared?

The next section briefly reviews the FTW model, and following sections provide the answers to the above questions.

FTW Model

A firm's FTW is its total capital at the end of the planning horizon. The FTW model computes this capital as a function of project selection, thereby enabling the decision maker to select the project that maximizes FTW. Maximizing FTW is an intuitively appealing investment objective, and the FTW model leads to the use of popular criteria such as PW or EAW.

There is ample precedent for the use of FTW as an economic criterion. Oakford and Theusen [2] explain the basic investment logic of the FTW model presented below, and then use a different analytical process to reach similar conclusions. Swalm and Lopez-Leautaud base a textbook [3] on this criterion, but their modeling approach does not consider capital budgeting. This leads to the recommendation that the average internal rate of return (IRR) of all future investments should be used as the discount rate, rather than some form of marginal rate, such as the minimum attractive rate of return (MARR). This section explains the effect of capital budgeting on the investment process, and then it summarizes theoretical results available elsewhere [1].

Investment Process

Suppose that the mutually exclusive investments shown in Table 1 are under consideration by a firm having $100 of investment capital at time 0. The objective is to chose the alternative that maximizes the capital at the end of the planning horizon, time 3, the firm's FTW.

Alternative A initially requires $10, leaving $90 available for other investments, as shown in Table 2. Similarly, alternative B allows $91 for other investments, and the null alternative results in all $100 remaining available. All alternatives have a common funding base of at least $90 for other currently unknown investments, and it is assumed this first $90 will

Table 1. Cash Flows			
Time	A	B	Null
0	-10	-9	0
1	7	3	0
2	5	4	0
3	0	5	0

be consumed during capital budgeting by better projects that provide an average high rate of return (AHRR), say 15%.

Table 2 . Investment Capital				
Time	A	B	Null	Common
0	90.00	91.00	100.00	90.00
1	110.50	107.60	114.50	107.60
2	131.93	127.74	131.33	127.74
3	151.51	151.90	150.85	150.85

There are a limited number of good investments, so any funds above the common amount ($90 at time 0) will be used by projects of marginal quality relative to those selected with the first $90. In this example, average marginal rate of return (AMRR) of those projects equals 10%.

Developing a mathematically tractable model requires assuming that projects other than alternatives A and B last for one year. Moreover, projects characterized by the AHRR precisely consume the common amount, and projects having an AMRR exactly consume any funds over the common amount. These assumptions usually are not met in practice, so this investment process only approximates reality, as is the case with many engineering models. Theusen's simulations [4] suggest that the resulting model is sufficiently accurate to select among alternatives in a way that maximizes FTW.

These assumptions allow computing the investment capital corresponding to each alternative at time 1. Choosing alternative A implies that the remaining $90.00 of investment funds available at time 0 will be invested in non-marginal projects that will yield an AHRR of 15% and then be augmented by the first return of $7, so the funds at time 1 resulting from choosing A are

$$F_{A,1} = 90.00(1.15) + 7 \tag{1}$$

or $110.50. Choosing alternative B results in the first $90.00 of investment funds growing at 15%, the marginal amount of $1.00 (91.00 - 90.00) increasing at 10%, and then receiving the cash flow of $3, so

$$F_{B,1} = 90.00(1.15) + (91.00 - 90.00)(1.10) + 3 \tag{2}$$

or $107.60. The null alternative results in the first $90.00 growing at 15% and the marginal amount of $10.00 (100.00 - 90.00) increasing at 10%, so

$$F_{O,1} = 90.00(1.15) + (100.00 - 90.00)(1.10) \tag{3}$$

or $114.50.

At time 1, there will be investment capital of at least $107.60 regardless of which alternative is chosen, so the first $107.60 of investments will grow at the AHRR of 15%. Any funds above $107.60 are consumed by less attractive investments that provide an AMRR of 10%. The investment capital at time 2 if A is chosen is

$$F_{A,2} = \$131.93 = 107.60(1.15) + (110.50 - 107.60)(1.10) + 5, \tag{4}$$

and similar computations are made for the other alternatives.

At time 3, this process is repeated. Notice that the project A's two year life does not affect the logic; it merely results in a cash flow of $0 at time 3, so the FTW for A is:

$$FTW_A = F_{A,3} = \$151.51 = 127.74(1.15) + (131.93 - 127.74)(1.10) + 0 \tag{5}$$

The FTWs for the three alternatives are $151.51 for A, $151.90 for B, and $150.85 for the null. Project B provides the largest FTW, so it is chosen.

Selection Procedure and Model

If all costs and benefits of alternatives are known and reinvestment occurs as described above, then it seems reasonable to use the *FTW procedure*:

1. Set the length of the planning horizon. This frequently equals the life of the longest alternative. If it should be shorter, then the last cash flow of any longer project would equal the cash flow obtainable by liquidating the project.

2. Assume that future benefits and costs increase or decrease capital that is reinvested, and that common amounts earn the AHRR and amounts beyond that earn the AMRR.

3. Choose the alternative producing the largest FTW, the amount of capital at the end of the planning horizon.

Developing a formula for FTW requires assigning symbols to the parameters of the investment problem explained above, writing the expressions using those symbols instead of numbers until a pattern emerges, and then algebraically rearranging the result. Let the following symbols be used, illustrated using values from the preceding example:

K_j Common capital at time j, e.g., K_0 equals 90.00 and K_1 equals 107.60.
c_j Value of a cash flow at time j, e.g., c_0 for alternative B equals -9.
A Initial investment capital at time 0, e.g., \$100.
a Average high rate of return or AHRR, e.g., 15%.
m Average marginal rate of return or AMRR, e.g., 10%.
n Length of planning horizon, e.g., 3.

Then the equation for future total worth can be shown to be:

$$FTW = \begin{aligned} &A(1+m)^n + (a-m)[K_0(1+m)^{n-1} + K_1(1+m)^{n-2} + \cdots + K_{n-2}(1+m) + K_{n-1}] \\ &+ c_0(1+m)^n + c_1(1+m)^{n-1} + c_2(1+m)^{n-2} + \cdots + c_{n-1}(1+m) + c_n \end{aligned} \quad (6)$$

In the example,

$$FTW_A = \begin{aligned} &100(1.1)^3 + (0.15-0.10)[90.00(1.10)^2 + 107.60(1.1) + 127.74] \\ &- 10(1.1)^3 + 7(1.1)^2 + 5(1.1) + 0 \end{aligned} \quad (7)$$

or \$151.51, as before.

Economic Criteria

Equation (6) is primarily of theoretical importance, since the values of K_j cannot be determined without performing year-by-year computations. However, notice that the top line in equation (6) is the FTW of the null alternative for which the cash flows equal \$0:

$$FTW_O = A(1+m)^n + (a-m)[K_0(1+m)^{n-1} + K_1(1+m)^{n-2} + \cdots + K_{n-1}(1+m)] \quad (8)$$

Only the bottom line of equation (6) varies from alternative to alternative, and this leads to the concept of future worth and other convenient economic criteria for the industrial environment. Students readily grasp FTW as a legitimate investment criteria, so it is merely necessary to define the more commonly used criteria in terms of it and to show that each one produces the same choice among alternatives as FTW.

Future Worth

The *future worth* of a project P is defined as the change in FTW caused by selecting the project

instead of the null alternative:
$$FW_P = FTW_P - FTW_O \tag{9}$$
This implies that:
$$FW_P = c_0(1+m)^n + c_1(1+m)^{n-1} + \cdots + c_{n-1}(1+m) + c_n \tag{10}$$
For example, the future worth (FW) of alternative B is
$$FW_B = 1.05 = -9(1.1)^3 + 3(1.1)^2 + 4(1.1)^1 + 5, \tag{11}$$
so selecting project B produces a FTW that is $1.05 larger than choosing the null alternative.

Rewriting equation (9) as
$$FTW_P = FTW_O + FW_P \tag{12}$$
makes it clear that FW is directly proportional to FTW, so selecting the project with the largest FW results in the choice of the alternative having the largest FTW. Thus FW is referred to as a surrogate criteria. Two commonly used surrogate criteria are present worth (PW) and equivalent annual worth (EAW).

Present Worth

The more popular present worth measure is defined as the single cash flow at time 0 that produces the same FTW as the original project. The formula for PW is derived by setting the FTW of a project equal to the FTW of a single cash flow of amount PW at time 0 to obtain:
$$PW = c_0 + c_1(1+m)^{-1} + c_2(1+m)^{-2} + \cdots + c_{n-1}(1+m)^{-(n-1)} + c_n(1+m)^{-n} \tag{13}$$
Equations (9) and (13) imply that
$$PW = FW(1+m)^{-n}, \tag{14}$$
so PW is proportional to FW. Thus PW results in the same ranking of projects as FW and FTW.

Equivalent Annual Worth

A project's equivalent annual worth is defined as equal cash flows of amount EAW occurring at times 1, 2, ..., n that produce the same FTW as the project. Setting the FTW of equal amounts EAW at times 1, 2, ..., n equal to the FTW of a project results in
$$EAW = PW(A|P, m, n) \tag{15}$$
or
$$EAW = FW(A|F, m, n). \tag{16}$$
EAW is proportional to PW and FW, and hence to FTW, so it results in the same ranking of projects as FTW.

Classroom Applications

The concept of FTW has several useful classroom applications [5] in addition to helping students understand economic criteria. Some of these are discussed below.

Equivalence and Interest Rates

It is not unusual for equivalence to be defined as "having the same effect." The vagueness of the term "effect" must be addressed by a de facto description of FTW or by analogies to a savings account. Students, particularly better ones, sometimes feel uneasy at this point in the course. This is due to a gap in their education: Formulas developed within the context of banking accounts suddenly are being used in an obviously different industrial environment. The students ultimately

must accept the instructor's word that evaluating these formulas using the minimum attractive rate of return produces the desired results.

The FTW framework allows presentations to be a well-ordered blend of theory and application typical of other engineering courses, such as statics. Now two sets of cash flows can be defined as equivalent if they result in the same FTW. For example, suppose that one or more flows e_f through e_g are equivalent to a set of original cash flows c_0 through c_n. Equations (10) and (12) indicate that the FTW of the equivalent is

$$FTW_E = FTW_O + e_f(1+m)^{n-f} + \cdots + e_g(1+m)^{n-g}, \qquad (17)$$

and the FTW of the original cash flows is

$$FTW_C = FTW_O + c_0(1+m)^n + c_1(1+m)^{n-1} + \cdots + c_{n-1}(1+m) + c_n. \qquad (18)$$

The definition of equivalence implies that FTW_E equals FTW_E so that

$$e_f(1+m)^{n-f} + \cdots + e_g(1+m)^{n-g} = c_0(1+m)^n + c_1(1+m)^{n-1} + \cdots + c_{n-1}(1+m) + c_n. \qquad (19)$$

At this point the banking analogy becomes useful. Equation (19) shows that two sets of industrial cash flows are equivalent if their compound amounts are equal when evaluated at the AMRR. Similarly, dividing both sides of equation (19) by $(1+m)^n$ shows that two sets of industrial cash flows are equivalent if their discounted amounts are equal using a discount rate of AMRR. In short, banking formulas do indeed work in the industrial environment, provided that the AMRR is used as the interest rate.

Sometimes the AMRR is referred to as the MARR, but the two are slightly different. The MARR is the lowest value of IRR acceptable to management, whereas the AMRR is the average IRR of the last projects actually accepted or rejected during capital budgeting process over the course of the planning horizon. For example, if IRRs for marginal projects should be between 7% and 9%, then the AMRR would be 8%, whereas the MARR would be 7%. Thus the MARR technically is a lower bound of the AMRR, but the small difference in these values probably has little effect on rankings of projects. The AMRR is used to compound cash flows because differences in alternatives cause differences in the amounts available for reinvestment. This affects marginal alternatives where capital growth occurs at the AMRR.

Null Alternative

The null alternative can defined simply as the rejection of all alternatives for which cash flows have been estimated. However, a closer examination helps to interpret the surrogate criteria. Figure 1 shows that projects 1, 2, …, 19, 20 are currently selected and have an AHRR. Let projects 21, 22, and 23 be marginal projects, of which projects 21 and 22 currently are selected. The

Figure 1. Project Selections
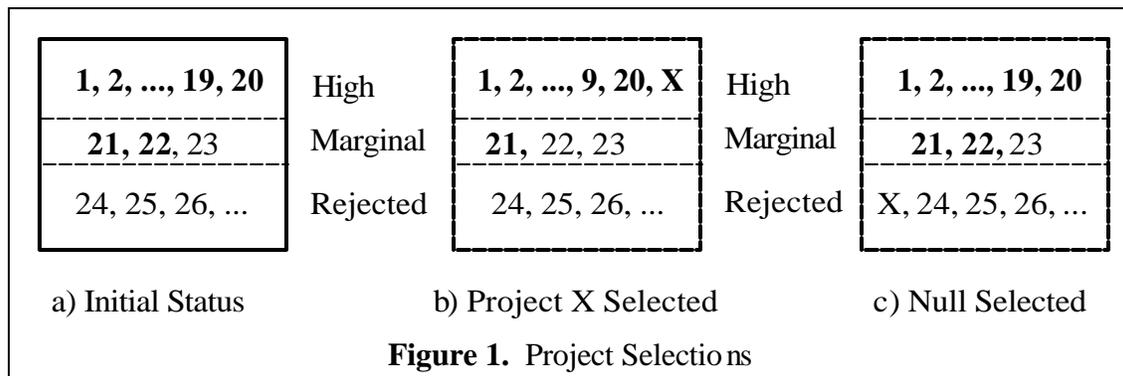
remaining projects have been rejected. Now consider a new project X. If it is selected, then it will displace some marginal project due to capital limitations, such as project 22. If it is rejected, then the null alternative is selected and the selections remain in their initial state, so project 22 is still selected. Thus selecting the null alternative implies that some (usually unknown) marginal project will be selected in place of the alternative under consideration.

Interpreting Economic Criteria

The definition of FW implies that the FW of a marginal alternative is $0. This also can be seen from equation (10), since marginal alternatives have an IRR of AMRR and FW equals $0 when evaluated at a project's IRR. The definition of FW also implies that a project with a FW of $10 results in a FTW of $10 more than a marginal alternative, and a project with a FW of -$10 produces a FTW of $10 less than a marginal alternative. Thus FW measures the FTW of a project relative to that of a marginal alternative.

Similarly, the PW of a marginal alternative is $0. Equations (12) and (14) imply that

$$FTW_P = FTW_O + PW_P (1+m)^n , \tag{20}$$

so selecting project P produces the same FTW as choosing a marginal project instead of P to obtain FTW_O, plus having PW_P more or fewer dollars to invest today in an account paying the AMRR. The EAW of a marginal project also is $0, and equations (12) and (16) infer that

$$FTW_P = FTW_O + EAW_P (F|A, m, n) , \tag{21}$$

so selecting project P produces the same FTW as choosing a marginal project instead of P to obtain FTW_O, plus having EAW_P more or fewer dollars to invest today at times 1, 2, …, n in an account paying the AMRR.

Alternatives with Different Lives

Maximizing FTW requires determining the time at which total capital is computed. This concept of a well defined planning horizon is useful in evaluating alternatives with different lives. Notice, for example, that projects A and B in Table 1 last 2 and 3 years, respectively. Project A's returns from earlier years continue to be reinvested during year 3 until project B is complete. As long as all costs and benefits are known, then it merely is necessary to compute FTW or use one of the surrogate criteria. If there are unknown cash flows or differences in cash flows, then sensitivity analysis or heuristics are appropriate.

Having FTW as the explicit criteria helps students in applying the surrogate criteria of EAW to projects with different lives. EAW is defined in terms of FTW, and its definition requires its being evaluated for the entire planning horizon. As a consequence of its definition, it is proportional to PW and hence to FTW. For example, applying equation (15) to projects A and B results in

$$EAW_A = PW_A (A|P, 10\%, 3) , \tag{22}$$

and

$$EAW_B = PW_B (A|P, 10\%, 3) . \tag{23}$$

If EAW_A should be evaluated as

$$EAW_A = PW_A (A|P, 10\%, 2) , \tag{24}$$

then the proportionality with PW and hence with FTW would be destroyed. Students initially find it odd that EAW_A is evaluated over 3 years, but realize the necessity of the definition if FTW is the primary criteria and proportionality is to be maintained.

Integrating FTW into Existing Syllabi

Professors who have developed course syllabi, examples, homework problems, and tests have very legitimate concerns regarding any revisions that might be required by a new pedagogy. Provided below is what has been found to be an efficient way to integrate FTW into existing syllabi. It recognizes that two distinct phases in teaching engineering economics consist of (1) mastering the use of banking formulas and then (2) learning how to apply those mechanics to the industrial environment.

Students commonly learn how to use factors such as $(F|P, i, n)$ or $(P|A, i, n)$ within the context of bank accounts. It is a straightforward matter to show that the final balance in an account having initial assets A, interest rate i, and cash flows c_0 through c_n is given by

$$B_n = A\,(1+i)^n + c_0\,(1+i)^n + c_1(1+i)^{n-1} + \cdots + c_{n-1}(1+i) + c_n. \tag{25}$$

Then define banking equivalents as two sets of cash flows that produce the same final balance, and developing any factor merely becomes a matter of rearranging the balance equation.

Figure 2. Uniform Series

For example, Figure 2 shows a uniform series at times $r+1$, $r+2$, …, s. To determine its equivalent at time r, first write the balance equation for the cash flows,

$$B_{n,\,Cash\ Flow} = A\,(1+i)^n + U(1+i)^{n-r-1} + U(1+i)^{n-r-2} + \cdots + U(1+i)^{n-s}, \tag{26}$$

and then for the equivalent,

$$B_{n,\,Equivalent} = A\,(1+i)^n + E_r(1+i)^{n-r}. \tag{27}$$

By definition,

$$A\,(1+i)^n + U(1+i)^{n-r-1} + U(1+i)^{n-r-2} + \cdots + U(1+i)^{n-s} = A\,(1+i)^n + E_r(1+i)^{n-r}, \tag{28}$$

and solving equation (28) merely requires rearranging terms, factoring, and using the formula for the resulting geometric series. Once students observe this process once or twice, it is not necessary to repeat the details each time, and the focus can shift to applying the factors.

It is useful to compare projects A and B in Table 1 by stating that the source of the investment funds is a savings account and that all returns are deposited in that account. Then choose the better investment by determining which alternative's final balance is larger. This sets up the transition to the industrial environment. The formula for FTW is more complicated than the balance equation, but the principles are the same. Instead of choosing the project with the larger final balance, choose the project with the larger FTW. The formula for FW is identical to the formula for the compound amount of cash flows, so all of the factors obtained in the banking environment can be used in the industrial environment and the appropriate interest rate is the AMRR. Similarly, industrial equivalents are sets of cash flows that result in the same FTW, instead of the same final balance, and all previously developed factors can be used.

The material on FTW and economic criteria requires roughly one hundred minutes of lecture. This time can be recaptured by the unified development of the factors provided by the bal-
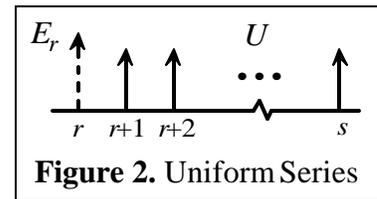
ance equation, plus the ease with students understand equivalence and economic criteria. A classroom-tested syllabus and instructional material are available via the world wide web [6].

Summary

A traditional approach to teaching engineering economics consists of developing concepts and factors in a banking environment and then applying them in the industrial environment. However, reinvestment in banking accounts occurs using well defined interest rates, whereas industrial reinvestments are made in a variety of different, currently unknown projects. Each of these projects typically receives funds and then provides returns, essentially acting as a savings account paying an interest rate equal to its IRR. The difference in these environments can create doubt regarding the appropriateness of transferring results from one to the other.

The FTW model offers a bridge between these two environments that is easy to integrate into existing syllabi. It provides a clear understanding of equivalence, economic criteria, and a variety of other issues such as which interest rate to use for industrial analyses or how to compare alternatives with different lives.

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Biography

Dr. John H. Ristroph is a Professor of Engineering and Technology Management and a registered professional engineer in Louisiana. His B.S. and M.S. are from LSU, and his Ph.D. is from VPI&SU, all in industrial engineering. He has taught engineering economics for over thirty years, and enjoys bringing his research into the classroom. His practical experience includes serving as Director of Policy and Planning for the Louisiana Department of Natural Resources where he performed numerous large-scale economic analyses.