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Abstract

This paper provides an introduction to three dimensional image edge detection and its relationship to partial derivatives, convolutions and wavelets. We are especially addressing the notion of edge detection because it has far reaching applications in all areas of research including medical research. A patient can be diagnosed as having an aneurysm by studying an angiogram. An angiogram is the visual view of the blood vessels whereby the edges are highlighted through the implementation of edge detectors. This process is completed through convolution, wavelets and matrix techniques. Some illustrations included will be vertical, horizontal, Sobel and wavelet edge detectors.

I. Introduction

To help motivate this paper, we provide an introduction to some interesting problems in image processing implementing matrix techniques, partial derivatives and convolutions. Section (2) provides an introduction to matrix and partial derivatives and how they are applied to the pixels to obtain the gray level value. Section (3) introduces a few specific examples such as the vertical, horizontal and Sobel edge detectors. Section (4) provides the reader with a series of illustrations that demonstrate edging techniques in three-dimensional image processing.

II. Some Notions and Notations

A monitor displaying an image may contain approximately 1024 rows and 512 columns of pixels. Of course the number continues to grow everyday as technology progresses. Then each pixel location designated by the coordinates, (x_1, y_1), contains a gray level value indicating the shade of gray within the image at that point. The values are usually on a scale of 0 to 255 whereby 0 corresponds to pure white and 255 correspond to black. The value of the gray level at this lattice point, (x_1, y_1), will be designated by $f(x_1, y_1)$.
However before we continue with the edge detection analysis, we first review a few matrix and calculus techniques. We first recall the familiar dot product for two vectors, **x**, **y**, to be $\mathbf{x}\bullet\mathbf{y}=\sum_{i=1}^{2} x_i y_i$. From this dot or inner product we define the norm to be $\|x\|^2 = \sum_{i=1}^{2} x_i y_i$. Then we obtain the familiar and very important result to many applications that the cosine of the angle between the two vectors, **x** and **y**, satisfy the

equation that $\cos(\theta)=x \bullet y/(\|x\|\|y\|)$. We know the maximum value for the cosine occurs when the two vectors coincide giving a value, cosine(0)=1. This is an important observation in edge detection and will latter be brought forward. We now introduce the partial derivative formulas,

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x},$$

and

$$\frac{\partial f(x,y)}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}.$$

The distance between pixel locations will be defined to be 1 so all of the increments in the partial derivative formulae will be equal to one. This then gives,

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1},$$

and

$$\frac{\partial f(x,y)}{\partial y} \approx \frac{f(x, y+1) - f(x,y)}{1}.$$

We now denote the function, f(x,y), to be the gray level values between neighboring pixels in the horizontal and vertical directions respectively giving us the formulas, $f(x_1+1,y_1) - f(x_1,y_1)$ and $f(x_1,y_1+1) - f(x_1,y_1)$. The spatial locations, x_i and y_i can only take on integer values given by their integer locations.

III. Convolution and Edge Detectors

We first introduce the usual calculus definition for convolution given by the formula,

$$h(x,y) * f(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(k_1,k_2) f(x-k_1, y-k_2)\, dk_1 dk_2,$$

and its discrete version by the formula,

$$h(n_1,n_2) * f(n_1,n_2) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} h(k_1,k_2) f(n_1-k_1, n_2-k_2).$$

We now reduce the discrete convolution to be a 3 by 3 matrix, which will play the role of a convolute and select our function, $h(n_1,n_2)$, to have the matrix values,

$$\mathbf{h}=\begin{pmatrix} h(-1,1) & h(0,1) & h(1,1) \\ h(-1,0) & h(0,0) & h(1,0) \\ h(-1,-1) & h(0,-1) & h(1,-1) \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix}.$$

The arguments (n_1,n_2) in $h(n_1,n_2)$ of the first array are easily remembered by noting that they are the needed lattice point coordinates referred to as a Cartesian coordinate system. This is illustrated in Figure 1. Clearly the reduced array for $h(n_1,n_2)$ is part of the complete array where $h(n_1,n_2)$ is part of the complete array and is equal to zero whenever $|n_1|$ or $|n_2|>1$. We next use the indexing shown in Figure 1 and convolve the function, $h(n_1,n_2)$ with the function, $f(n_1,n_2)$, and obtain

$$h(n_1,n_2)*f(n_1,n_2)=\sum_{k_1=-1}^{1}\sum_{k_2=-1}^{1}h(k_1,k_2)f(n_1-k_1,n_2-k_2)$$

$$=f(n_2-1,n_2+1)-f(n_1+1,n_2+1)+ f(n_1-1,n_2)-f(n_1+1,n_2+1)+ f(n_1-1,n_2+1)-f(n_1+1,n_2-1).$$

We now investigate this last result only to find that it gives the difference of three columns of pixel values in the horizontal direction. If one checks the literature[6,7], we find that this is the approximation used in the horizontal direction in several leading software image-processing packages. The function, $h(n_1,n_2)$, is called the kernel of the convolution and when we change its values, we obtain different edger's. The edge is the portion of the image where there is a sudden change in gray levels. The edger implemented selects a particular feature in the image, which is beneficial to the particular application. The kernel for vertical edging is given by

$$\mathbf{h}=\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{pmatrix}.$$

A more sophisticated edger is the Sobel edger, which uses the gradient to approximate the edges. Since the gradient includes both horizontal and vertical components, two kernels are employed given by the matrices,

$$\begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix}.$$

IV Illustrations using Edge Detectors

Figure 2 and 8 illustrates the alphabets O and N respectively in three dimensions. We then employ a vertical edge detector on the alphabets O and N respectively in Figures 3 and 9. Again horizontal edge detectors are applied and illustrated on O and N in Figures 4 and 10 respectively. The Sobel edge detector is then applied to O and N and illustrated in Figures 5 and 11. We conclude the illustrations with a wavelet constructed using the Gaussian together with its application on the letter N and again illustrated in Figure12.

Figure 1. The lattice array

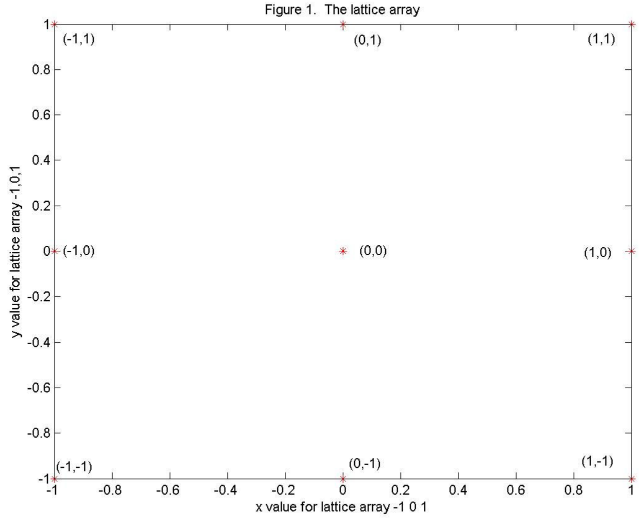


Figure 2. The Letter O

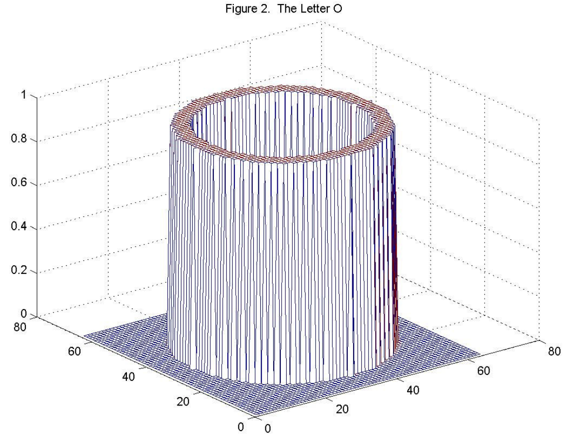


Figure 3. A vertical edge detector on the Letter O

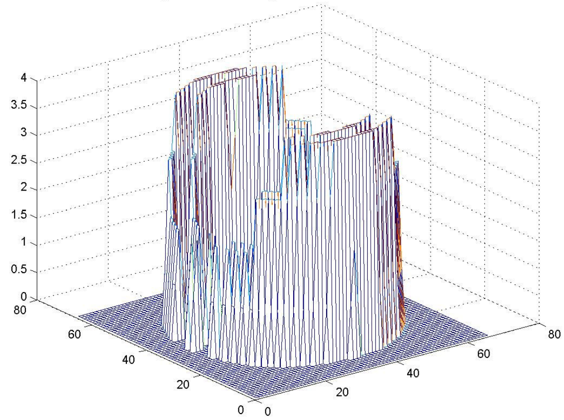


Figure 4. A horizontal edge detector on the Letter O

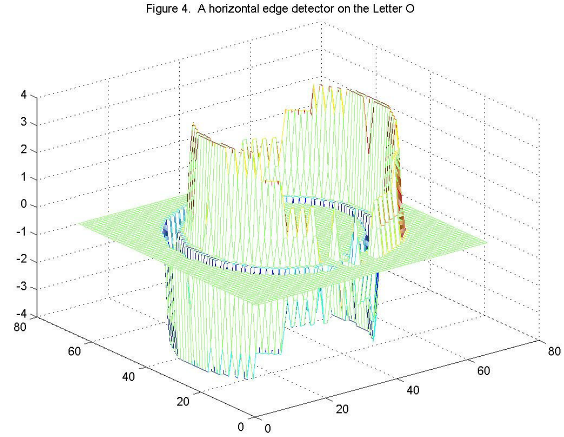


Figure 5. Soble edge detector on the Letter O

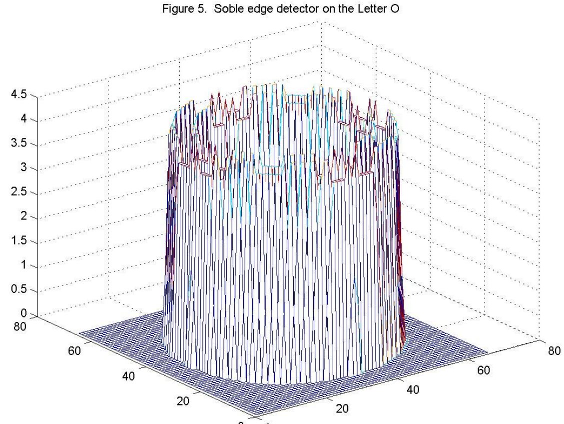


Figure 6. The Wavelet constructed as derivative of Gaussian in x

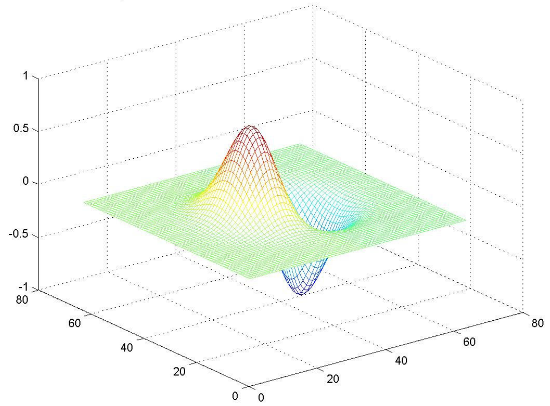


Figure 7. Wavelet transform of Letter O

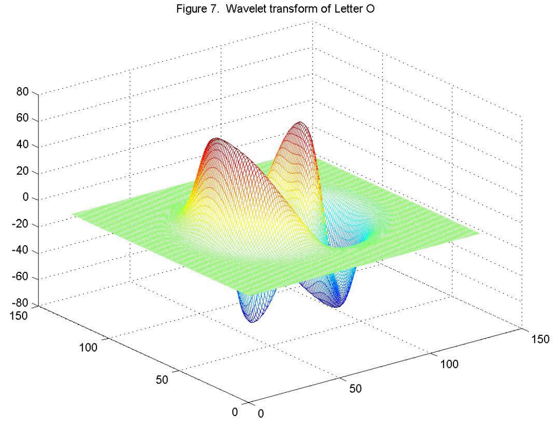


Figure 8. The Letter N

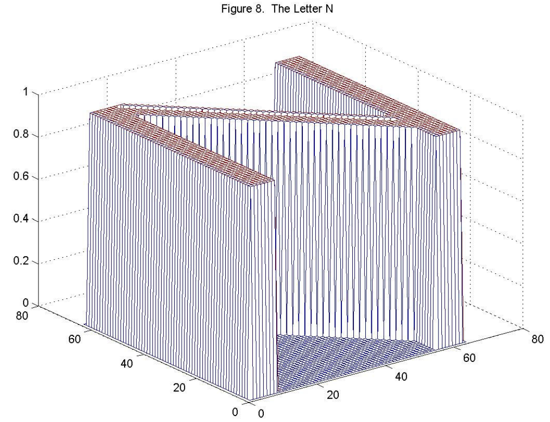


Figure 9. A vertical edge detector on the Letter N

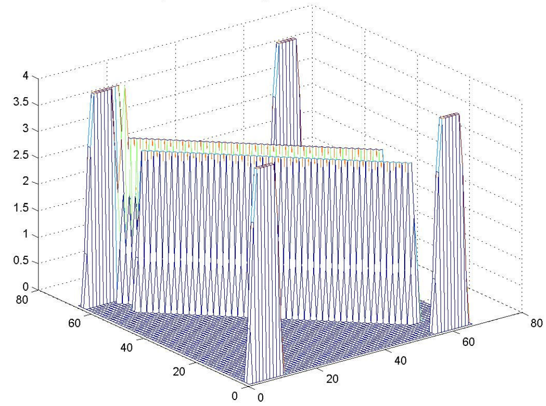


Figure 10. A horizontal edge detector on the Letter N

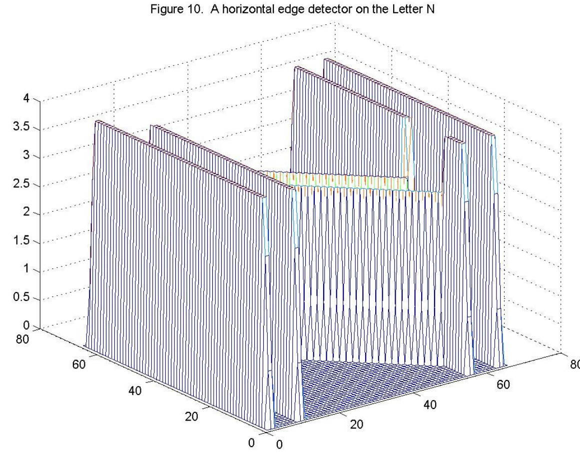


Figure 11. Soble edge detector on the Letter N

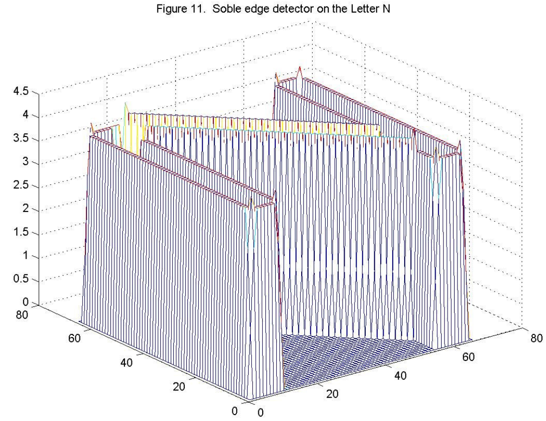
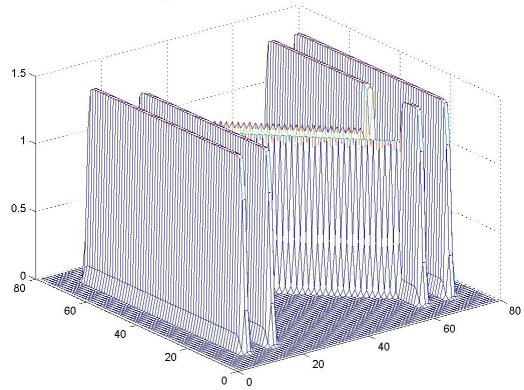


Figure 12. Wavelet transform on the Letter N



As seen, Figures 2 and 8 show the alphabets O and N respectively. We realize that edges of images contain much of the recognizable content for any image. A house versus a water tank is clearly visible to human eyesight by looking at its geometrical shape. The edges dictate the shape. Thus if we have a vertical edger the vertical edges will be emphasized and a horizontal edger will emphasize horizontal edges. Mathlab using the default window for graphics will show the vertical edges somewhat “choppy” on the letter O and also somewhat “obscure” on the letter N. A horizontal edger also “obscures the letter O with better viewing on the letter N. However the Soble edger does better on the letter O and very good on the letter N. The wavelet transform does very well on the letter N. Thus the particular edger selected will dictate the content of the photo.

We conclude these applications with the realization that the type of image can yield results that are clearly visible for human sight. The Sobel edge detector on the letters O and N are clearly visible to our eyesight. However adjustments not readily apparent to human sight such as the wavelet transform on the letter O can have far reaching consequences when compared to “normal” vs. “abnormal” physical phenomenon such as the aneurysm situation.

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