

Do Basic Mathematical Skills Improve Spatial Visualization Abilities?

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Abstract

Spatial visualization skills are important in engineering graphics and computer-aided design courses, especially in solid modeling, which has gained popularity in recent years. Are basic mathematical skills a factor in improving spatial visualization abilities? Based on the need to develop these abilities, what mathematical courses should be set as prerequisites for engineering graphics courses? This paper presents a study of the relationship between basic mathematical skills and spatial visualization abilities. The Purdue Spatial Visualization Test – Rotation was given to students taking engineering graphics courses. The results were compared with scores on a basic mathematical skills test, a placement test that all new students took when they applied to the college where this research was undertaken. Statistical analysis of the data revealed the effects of basic mathematical skills on spatial visualization abilities.

1. Introduction

Since proficiency in engineering graphics is essential to engineering and technology students, improving their technical drawing skills has long been an interesting research topic for engineering and technology educators. During a discussion on curriculum development at Essex County College, some faculty members, believing that basic mathematical skills were fundamental for graphics apprehension, advocated the adoption of a mathematics course as a prerequisite to the graphics course. At the same time, others did not believe mathematical proficiency to be necessary for mastering graphics and computer-aided design (CAD) skills. The ability to visualize spatially is generally considered fundamental for graphics applications, especially for three-dimensional (3D) solid modeling using CAD software packages. Is there a relationship between basic mathematical skills and spatial visualization abilities? Also, is there a relationship between these skills and graphics scores? This paper investigated these relationships by correlating a standardized placement test with a standardized spatial visualization test, as well as with the graphics grades of the students at Essex County College.

2. The Spatial Visualization Test

Various tests have been developed to measure different types and levels of spatial visualization abilities¹. Among these tests, the Purdue Spatial Visualization Test – Rotation (PSVT-R)⁹ has been widely used by researchers. This study also chose the PSVT-R test to measure spatial visualization abilities. The PSVT-R test was designed to test rotational visualization ability. It consists of 30 multiple-choice problems. In each problem, an example first shows an object in its original and rotated views in isometric drawing. Then another object is shown in five different rotations, and the student is to choose the rotation that matches the given example. A sample problem similar to that of the PSVT-R is shown in Figure 1.

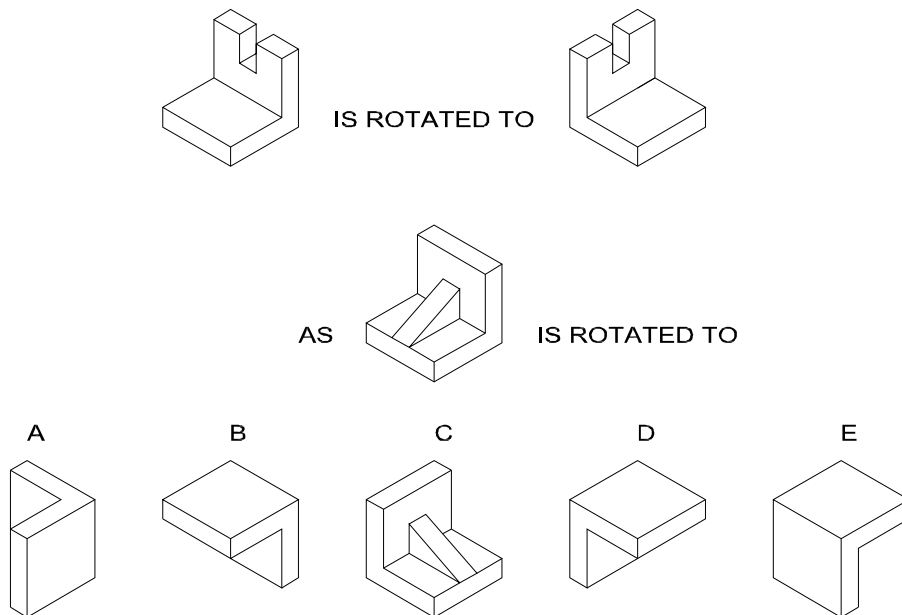


Figure 1. Example of a Rotational Spatial Visualization Problem

All of the 3D objects in the PSVT-R test consist of simple geometric shapes of either a cube or a cylinder with cut slots. The transformations are combinations of 90° and 180° rotations about the axes of the Cartesian coordinate system. The problems in the PSVT-R test can be categorized into four types according to their patterns and difficulties of rotation. These four types are: a 90° rotation about one axis, a 180° rotation about one axis, a 90° rotation about an axis and another

90° rotation about another axis, and a 180° rotation about an axis and a 90° rotation about another axis. The degrees of difficulty of rotation increase from type I through type IV, and are also quite evenly distributed on the test (six problems in type I and eight problems each for types II through IV.) Logically, the more difficult the rotations are, the harder the visualization would be. Yue's study¹⁷ of 64 students in engineering graphics and CAD classes showed that there were significant differences among the mean PSVT-R test scores for the four types of rotations [$F(3, 252) = 11.38, p < .01$]. Further analysis using Scheffe's method showed that the mean score differences are significant between types I and II and between types I and IV, but not for other pairs. Therefore, the simplest single 90° rotation is easier to visualize than other more difficult types of rotations, but the mean score differences for the more difficult rotations are not statistically significant. Further research is needed to design and improve standardized spatial visualization tests.

3. The Placement Test

New students at Essex County College are required to take a placement test to assess their educational level and determine their placement in developmental courses. The COMPANION Placement Test (CPT) developed by the College Board, also known as ACUPLACER in its computerized form, is used to assess the basic skill levels of a student in reading, writing, and mathematics. It is a standardized multiple-choice test (except for the essay portion) consisting of four parts: Essay (40 minutes), Reading Comprehension (45 minutes for 35 questions), Elementary Algebra (40 minutes for 35 questions), and Arithmetic (40 minutes for 35 questions). The total time allotted is 195 minutes with 15 minutes at the beginning for background information and 15 minutes at the end for reviewing the entire test. The raw score of 0 to 35 is scaled to 20 to 120 in the score report. Based on their performance in the placement test, students may be required to take developmental courses as shown in Table 1.

Table 1. Companion Placement Test Scores and Developmental Courses Placement²²

Arithmetic	Elementary Algebra	Math Course Placement
0-22 (20-68)	N/A	MTH 086/087
23-35 (69-120)	17-22 (48-72)	MTH 092/093
23-35 (69-120)	23-30 (73-108)	MTH 100
23-35 (69-120)	31-35 (109-120)	College Level Math Course

Note: The raw scores are listed first and the scaled scores are shown in parentheses.

The mathematics course requirements and sequences for the engineering and technology programs are shown in Table 2.

Table 2. Mathematics Course Requirements and Sequences ²³

Course	Title	Credits	Prerequisite
MTH 086/087	Introductory Algebra	4.5	Placement
MTH 092/093	Elementary Algebra	4.5	Placement or MTH 086/087
MTH 100	Introductory College Mathematics	4	Placement or MTH 092/093
Engineering Technology Programs:			
MTH 113	College Algebra with Trigonometry	4	MTH 100
MTH 114	Unified Calculus I	3	MTH 113
MTH 213	Unified Calculus II	3	MTH 114
Engineering Programs:			
MTH 119	Pre-Calculus I	4	MTH 100
MTH 120	Pre-Calculus II	4	MTH 119
MTH 121	Calculus with Analytic Geometry I	4	MTH 120
MTH 122	Calculus with Analytic Geometry II	4	MTH 121
MTH 221	Calculus with Analytic Geometry III	4	MTH 122
MTH 222	Differential Equations	4	MTH 221

4. Test Samples

Essex County College (ECC) is a two-year urban community college located in the downtown of Newark, the largest city in New Jersey. The student population of ECC has a high percentage of minorities including 51% of African Americans and 17% of Hispanics.

Since fall 1999, many students in various classes have taken the PSVT-R test. The students in the *ENR 103 Engineering Graphics* classes were used as samples for this study. *ENR 103* is an entry-level course required for all engineering and technology majors. It carries 2 semester credits and 3 contact hours. The prerequisite for *ENR 103* is *MTH 092/093*. Most students in the *ENR 103* classes took the placement test a short time prior to enrolling in *ENR 103* and had not yet taken college level mathematics courses. Therefore, the mathematical skills shown in their placement test represented their level upon entering *ENR 103*. It was also the first time that these students took the PSVT-R test, hence eliminating any possible re-test effect.

The test sample included 57 students from seven *ENR 103* classes spanning two years from fall 1999 to fall 2001. The students in the test sample took the CPT test when they enrolled in the college as new students, and took the PSVT-R test at the beginning of the semester in *ENR 103* classes. Those students who did not take both the CPT test and the PSVT-R test were not included in the test sample. The two test results were statistically analyzed to investigate the relationship between basic mathematical skills and spatial visualization abilities. Comparisons were also made between the CPT test and the graphics scores, and the PSVT-R test and the graphics scores.

5. Test Results

The test results and the statistical analyses by the correlation coefficients and the t -ratios are as follows.

5.1. Relationship between Basic Mathematical Skills and Spatial Visualization Abilities

The mean score on the elementary algebra portion of the CPT test was 66 or 46%. The percentage score for the CPT test is obtained by subtracting 20 from its scaled score range of 20 to 120. The mean score on the PSVT-R test was 19 or 63%. The correlation coefficient was 0.0041. There was little statistical relationship between the algebra scores and the PSVT-R test scores [$t(57) = 0.03$, $p > .05$].

The mean score on the arithmetic portion of the CPT test was 72 or 52%. The correlation coefficient between the arithmetic and visualization tests was 0.31. The correlation was statistically significant at a .05 level of significance [$t(57) = 2.49$, $p < .05$], but not statistically significant at a .01 level of significance.

Therefore, the two standardized tests used in this study showed no evidence that elementary algebra skills could improve spatial visualization abilities. The study showed a possible relationship between arithmetic skills and spatial visualization abilities, but not a strong relationship.

5.2. Relationship between Spatial Visualization Abilities and Engineering Graphics Grades

The grades at Essex County College are A (4.0), B+ (3.5), B (3.0), C+ (2.5), C (2.0), D (1.0), and F (0.0). The students who withdrew from the classes were not included in the test samples.

The mean grade in the graphics classes was 2.80 or 70%. The correlation coefficient between the visualization test and the graphics grades was 0.33. The correlation was statistically significant at a .05 level of significance [$t(47) = 2.41$, $p < .05$], but not statistically significant at a .01 level of significance. Therefore, the study showed a possible relationship between spatial visualization skills and engineering graphics grades, but not a strong relationship.

In a previous study with smaller sample sizes¹⁷, the author also found a significant correlation between the PSVT-R test scores and *ENR 103* grades, as well as the grades of CAD courses.

Different from standardized tests, many factors determine the course grade of a student. Missing classes and assignments, variations in tests and examinations, and other factors all contribute to grading. Nevertheless, statistical analysis might still show some general trends.

5.3. Relationship between Basic Mathematical Skills and Engineering Graphics Grades

The correlation coefficient between the elementary algebra scores of the CPT test and the graphics grades was 0.52. The correlation was statistically significant [$t(47) = 4.15, p < .01$].

The correlation coefficient between the arithmetic scores of the CPT test and the graphics grades was 0.48. The correlation was also statistically significant [$t(47) = 3.80, p < .01$].

It seems that solid basic mathematical skills will help students get better grades in engineering graphics courses.

6. Conclusion

The study investigated the relationship between basic mathematical skills and spatial visualization abilities using two standardized tests: the COMPANION Placement Test (CPT) and the Purdue Spatial Visualization Test – Rotation (PSVT-R). The test samples were students in an entry-level engineering graphics course at a two-year urban community college. The study showed a possible relationship between very basic mathematical skills, the arithmetic scores on the CPT test, and spatial visualization abilities. However, there was no significant correlation between the scores on the elementary algebra part of the CPT test and the PSVT-R test. Further investigation into the relationship between spatial visualization abilities and engineering graphics grades revealed a possible relationship, but not a strong correlation. The study also showed a significant correlation between basic mathematics skills, both arithmetic and elementary algebra, and engineering graphics grades. It seems that conventional wisdom still prevails: a student with solid basic mathematical skills stands a better chance of getting a good engineering graphics grade.

Spatial visualization abilities may involve comprehensive factors and be developed in informal ways, such as childhood play with construction toys, art and drafting sketches, computer and video games, sports activities, and real-life experiences with 3D objects. More research and evidence is needed to find a relationship between spatial visualization abilities and other possible factors.

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