Draining a Tank through Multiple Orifices: An Improved Lab Experiment in Fluid Mechanics

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Abstract

The junior-level draining of a tank experiment has been given more flexibility by employing sharp-edged, rounded-edge and straight-bore orifices. Steady-state experiments were carried out to determine the discharge coefficients for the orifices, yielding coefficients which ranged from 0.63-0.83. Transient experiments were performed to collect experimental height vs. time measurements that were compared to model data generated from a Bernoulli balance and the measured discharge coefficients. The experimental and model data agreed very well, demonstrating the validity of the procedures used in the experiment and in the development of the model.

Keywords

laboratory, fluid mechanics, experimentation, modeling, tank draining, orifice

Introduction

The undergraduate laboratory is an essential part of the engineering curriculum because it introduces the student to engineering equipment and hands-on activities while illustrating many of the concepts that are taught in the classroom. At the same time, lab is often used to build important soft skills such as teamwork and oral and written communication skills. In fact, undergraduate lab (along with capstone design) is very useful in satisfying ABET Student Outcomes 3 (an ability to communicate effectively with a range of audiences), 5 (an ability to function effectively on a team whose members together provide leadership, create a collaborative and inclusive environment, establish goals, plan tasks and meet objectives) and 6 (an ability to develop and conduct appropriate experimentation, analyze and interpret data, and use engineering judgment to draw conclusions).

There have been significant developments in the use of virtual teaching labs and this has led to arguments on the pros and cons of using virtual labs in place of physical labs. Mosterman *et al*. [1] argued that virtual laboratories reduce the time spent in lab, help students improve their skills and provide more satisfaction with the laboratory experience. Korestky *et al*. [2] noted that there is a place for both physical and virtual labs. They pointed out that virtual labs are better for experimental design, critical thinking and dealing with ambiguity, while physical labs are better for understanding lab protocols and specific content.

In 2018, Penney and Clausen [3] published a collection of physical fluids and heat transfer experiments that could be used either in the undergraduate laboratory or as classroom demonstrations. Each of the experiments was accompanied by a theoretical development of the important correlations that should be considered in analyzing the data. Furthermore, the experiments worked very well, giving reasonable agreement between the experimental results and the results from applying the correlations. One of these simple experiments was the draining of a small tank through a sharp-edged orifice and the calculation of the discharge coefficient, C_D, from either a steady state or transient experiment. Although the demonstration unit was very effective as a learning tool, the operation really worked best in a steady state overflow mode, where water from a hose was continually fed to the tank and allowed to overflow the top of the tank and into an overflow container and often creating a bit of a mess in the classroom. In related work, Baukal and Bussman [4] developed a simplified and very inexpensive method for measuring discharge coefficients and successfully applied the procedure to a number of nozzle drillings in the classroom.

In 2021, Savage *et al*. [5] presented results from a fully developed fluids laboratory experiment that drained a tank through a sharp-edged orifice. Tank height was measured using a Magnetrol® Eclipse® 705 guided wave radar transmitter [6] mounted inside the tank but could also be measured more simply by using a measuring tape attached to the side of the transparent tank wall. Typical tasks during an experiment included:

- Calibration of the wave radar transmitter by comparing by the height recorded by the transmitter with observed height on the measuring tape
- Steady state operation (continuous flow into and out of the tank to maintain a constant water height) in determining the discharge coefficient, C_D
- Transient operation to experimentally determine tank height with time during draining, to be compared to results obtained from a Bernoulli balance

The apparatus also had the flexibility, with a simple modification, of being able to use different orifices in addition to the sharp-edged orifice.

The purpose of this paper is to describe and present data from multiple orifices in the draining of the tank experiment presented by Savage *et al*. Steady state experiments are used to determine the discharge coefficient, C_D, for each orifice and transient operation is used to generate height vs. time data and to compare the data to MATLAB-generated results from the Bernoulli balance.

Experimental

Much of the apparatus and experimental procedures are the same as previously reported by Savage *et al*. [5] and are only briefly summarized here.

Apparatus

Figure 1 (left) shows a photograph of the tank (a clear acrylic tube that had a height of 10 ft (3.1 m) and an inside diameter of 3 in (7.6 cm)) as mounted to metal scaffolding, along with the computer used to collect data from the wave transmitter. Figure 1 (right) shows a close-up of the 4 in (10.2 cm) x 2 in (5.1 cm) rubber no-hub coupling at the bottom of the tank, which surrounds a 2 in (5.1 cm) x 1 in (2.5 cm) female threaded coupling. The 1 in (2.5 cm) PVC plugs containing the orifices are then threaded (hand tight) into the bottom of the tank. A simple plastic valve with a garden hose connection was mounted approximately 3 ft (0.9 m) from the bottom of the tank to permit the introduction and regulate the inlet flow of water. Water from the tank flowed into a 5 gal (18.9 L) receptacle and then into a floor drain. A stopwatch was used for timing the flow of water during steady-state operation and an Erlenmeyer flask was used to collect water from the exit of the tank to determine the flow rate. A simple garden hose was used to bring water to the tank.

Figure 1. Tank (Acrylic Tube) Used for Draining (left) and
the Bottom of Tank with Orifice (right)

Figure 2 shows top views of five orifices used in the experiments, as machined from 1in (2.5 cm) PVC plugs. The bottom views of the plugs were all the same—just a hole. Details on the orifices follow:

- Orifice A is a standard $\frac{3}{16}$ in (4.76 mm) sharp-edged orifice with a 30° exit angle. This orifice was used in the work by Savage *et al*.
- Orifice B is a $\frac{1}{8}$ in (3.18 mm) sharp-edged orifice with a 45° exit angle.
- Orifice C is a $\frac{3}{16}$ in (4.76 mm) rounded-edged orifice with a $\frac{3}{16}$ in (4.76 mm) radius exit.
- Orifice D is $\frac{3}{16}$ in (4.76 mm) straight-bore orifice with a $\frac{5}{8}$ in (15.88 mm) stepped recess.
- Orifice E is a $\frac{3}{16}$ in (4.76 mm) straight-bore orifice.

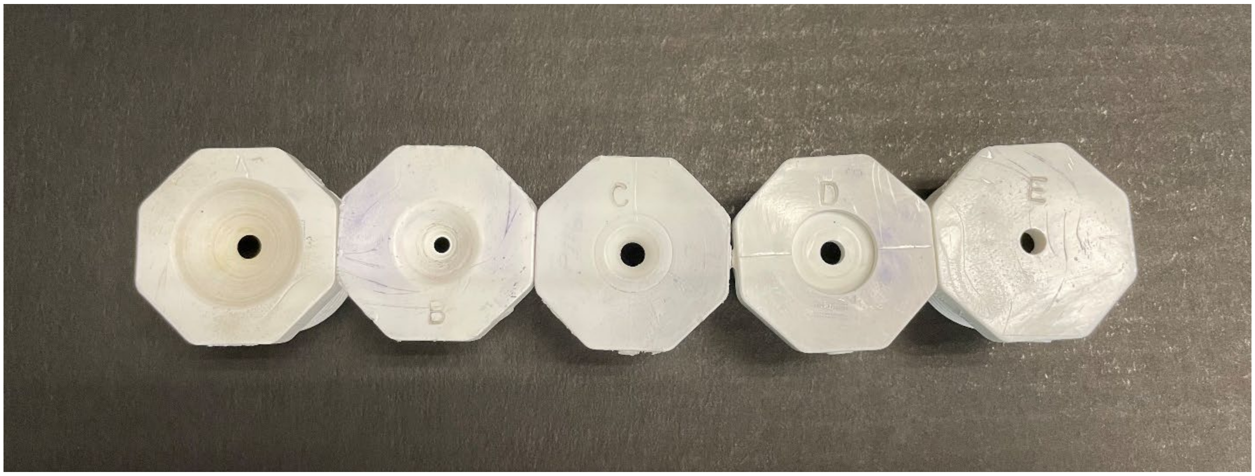

Figure 2. Top Views of the Orifices Used in the Experiments

Experimental Procedure

A steady state experiment was used to determine the flow rate through the system which, in turn, was used to calculate the discharge coefficient, C_D. After adjusting the flow rate to maintain a constant level in the tank, the flow rate was measured using a stopwatch and Erlenmeyer flask. To monitor the tank level with time, the tank was filled to a height of about 9 ft (2.7 m) and then allowed to drain while the wave radar transmitter recorded water height with time. The data were then used in comparing experimental data with data generated from a Bernoulli balance model.

Model Development

Model development for this experiment was previously shown by Penney *et al.* [3] and is repeated here to aid the reader. The basic Bernoulli balance, with no work in the system and negligible friction losses, is described by Wilkes *et al.* [7] as

$$\frac{v_1^2}{2g} + z_1 + \frac{p_1}{\rho g} = \frac{v_2^2}{2g} + z_2 + \frac{p_2}{\rho g} \tag{1}$$

For application in this experiment, point 1 was selected as the fluid level in the tank, and point 2 was selected as the location of the *vena contracta*, labeled with the subscript *vc*, which is located about one-half of an orifice diameter from the orifice entrance. Since both ends of the tank were open to the atmosphere, $p_1 = p_2$. The velocity at the top of the liquid in the pipe, v_1, may be neglected, and the *vena contracta* is at zero height, so that $z_{vc} = 0$. With these simplifications, Equation (1) may be rearranged to solve for the velocity at the *vena contracta*, v_{vc}:

$$v_{vc} = \sqrt{2gz_1} \tag{2}$$

The flow through the orifice may be described by the equation

$$Q = A_{vc}\, v_{vc} \tag{3}$$

However, the area of the vena contracta is difficult to measure. Thus, the discharge coefficient, $C_D = \frac{A_{vc}}{A_o}$, is introduced so that Equation (3) may be rewritten as

$$Q = C_D A_0 v_{vc} \tag{4}$$

where A_0 is the area of the orifice, equal to $\frac{\pi d_o^2}{4}$. Thus, C_D may be calculated as

$$C_D = \frac{Q}{A_0\sqrt{2gz_1}} \tag{5}$$

for the steady state system, where the volumetric flow rate is calculated as the volume of water collected, divided by the time of collection ($Q = \frac{V}{t}$) at steady state.

In considering the time-dependent system where height changes with time, the simplified Bernoulli balance of Equation (2) must be combined with the mass balance,

$$\frac{dm}{dt} = m_1 - m_{vc} \tag{6}$$

For a draining tank, $m_1 = 0$, since there is no water flowing into the tank. Furthermore, $\frac{dm}{dt}$ may be written as $\rho A \frac{dh}{dt}$, and m may be written as $\rho v A$. Thus, Equation (6) becomes

$$\rho A_1 \frac{dh}{dt} = -\rho v_{vc} A_{vc} \tag{7}$$

Once again, A_{vc} is unknown. Reintroducing C_D yields

$$\rho A_1 \frac{dh}{dt} = -\rho v_{vc} A_o C_D \tag{8}$$

Combining Equations (2) and (8) yields

$$\frac{dh}{dt} = -\frac{A_o C_D}{A_1}\sqrt{2gh} \tag{9}$$

Separating variables and integrating Equation (9) from $h = h_0$ at $t = 0$, and $h = h$ at $t = t$ yields, with rearrangement

$$h = \left(\frac{C_D t A_o \sqrt{2g}}{-2A_1} + \sqrt{h_0}\right)^2 \tag{10}$$

Finally, taking the square root of each side yields

$$\sqrt{h} = \frac{C_D t A_o \sqrt{2g}}{-2A_1} + \sqrt{h_0} \tag{11}$$

Thus, a plot of $\sqrt{h}$ vs. t will yield a straight line, the usual method of presenting this type of data.

Experimental Data, Results and Discussion

Table 1 shows the steady state experimental data collected for each of the five orifices and the resulting discharge coefficients found from Equation (5). In applying Equation (5), the diameter of the orifice (d_o) was required, as well as the steady state height in the tank (z_1), the volume of water collected (V) and the time for collection (t). The calculated discharge coefficients (C_D) ranged from 0.64 for the $\frac{3}{16}$ in (4.76 mm) sharp-edged orifice to 0.83 for the $\frac{3}{16}$ in (4.76 mm) straight-bore orifice.

Table 1. Steady State Experimental Data and Resulting Discharge Coefficients

Orifice	d_o, in	z_1, ft	V, cm^3	t, s	C_D	std
A $\frac{3}{16}$ in sharp-edged orifice	0.1875	2.75	423	9.05	0.65	
	0.1875	2.75	360	7.86	0.63	
	0.1875	2.75	296	6.50	0.63	
	average				**0.64**	**0.007**
B $\frac{1}{8}$ in sharp-edged orifice	0.1250	3.50	194	7.48	0.72	
	0.1250	3.50	193	7.31	0.73	
	0.1250	3.50	259	9.88	0.72	
	average				**0.72**	**0.005**
C $\frac{3}{16}$ in rounded-edged orifice	0.1875	3.50	375	6.65	0.69	
	0.1875	3.50	360	6.36	0.70	
	0.1875	3.50	390	6.95	0.69	
	average				**0.69**	**0.002**
D $\frac{3}{16}$ in straight-bore orifice w/ stepped recess	0.1875	3.00	315	6.83	0.61	
	0.1875	3.00	305	6.41	0.63	
	0.1875	3.00	268	5.41	0.66	
	0.1875	3.00	321	6.81	0.63	
	average				**0.63**	**0.02**
E $\frac{3}{16}$ in straight-bore orifice	0.1875	3.50	420	6.16	0.84	
	0.1875	3.50	400	5.85	0.84	
	0.1875	3.50	435	6.60	0.81	
	0.1875	3.50	380	5.61	0.83	
	average				**0.83**	**0.01**

Figures 3-7 show plots of the square root of the tank height, $\sqrt{h}$, with time, t, for the transient experimental runs and the Bernoulli equation model, found by solving Equation (9) using MATLAB. Equation (11) may also be solved without the use of MATLAB, if desired. Each of the MATLAB solutions used the average discharge coefficients shown in Table 1. In examining the plots, Equation (11) predicts a straight line and the experimental runs all essentially showed straight line behavior. Small deviations from the model and experimental data were observed for the $\frac{1}{8}$ in (3.18 mm) sharp-edged orifice but the agreement was generally quite good for all of the experiments.

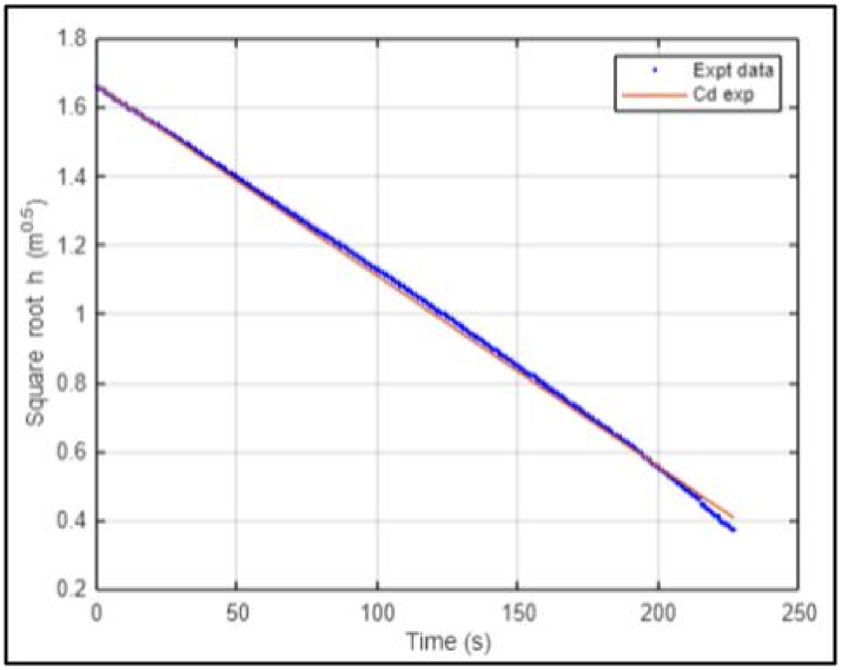

Figure 3. Comparison of the Experimental and Calculated Square Root of Height vs. Time for a $\frac{3}{16}$ in (4.76 mm) Sharp-edged Orifice, C_D = 0.64

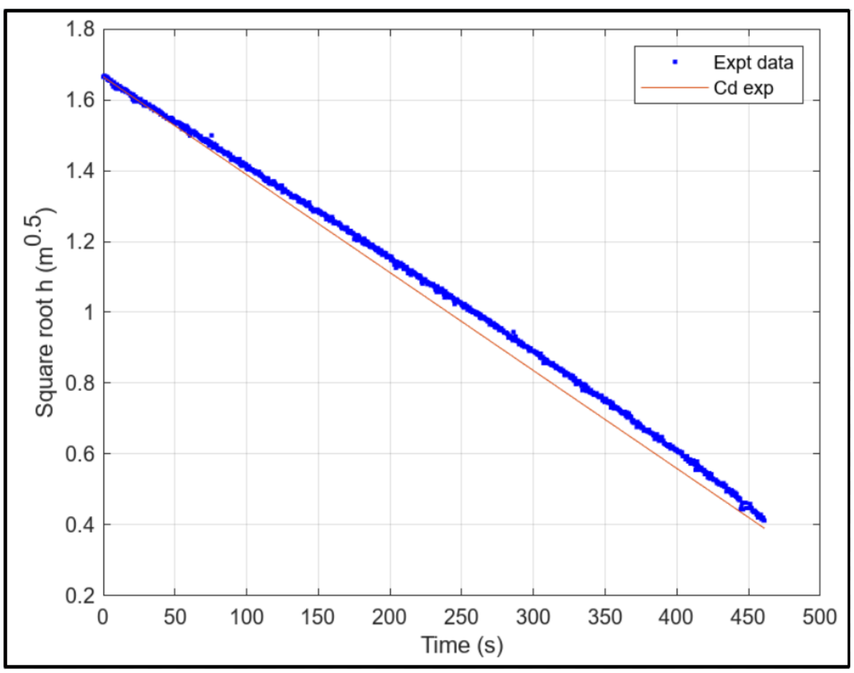

Figure 4. Comparison of the Experimental and Calculated Square Root of Height vs. Time for a $\frac{1}{8}$ in (3.18 mm) Sharp-edged Orifice, C_D = 0.72

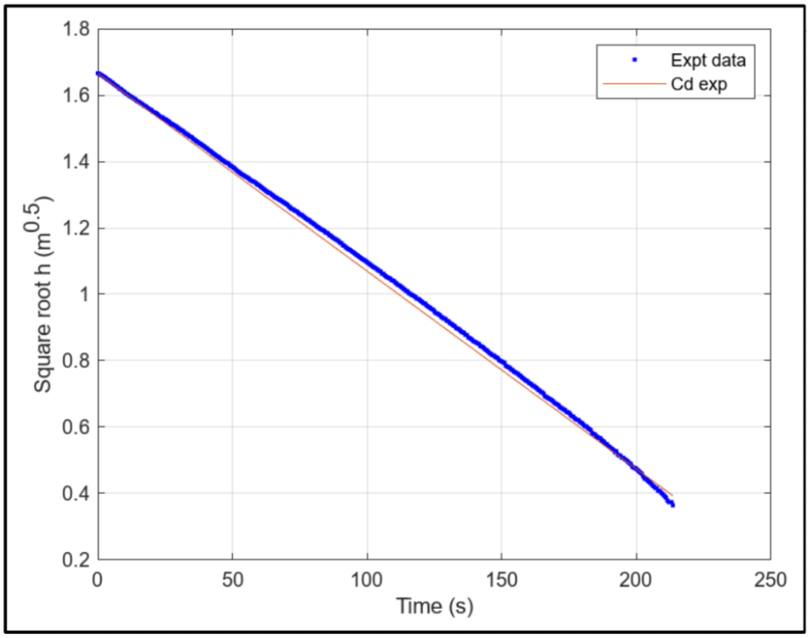

Figure 5. Comparison of the Experimental and Calculated Square Root of Height vs. Time for a $\frac{3}{16}$ in (4.76 mm) Rounded-edged Orifice, $C_D = 0.69$

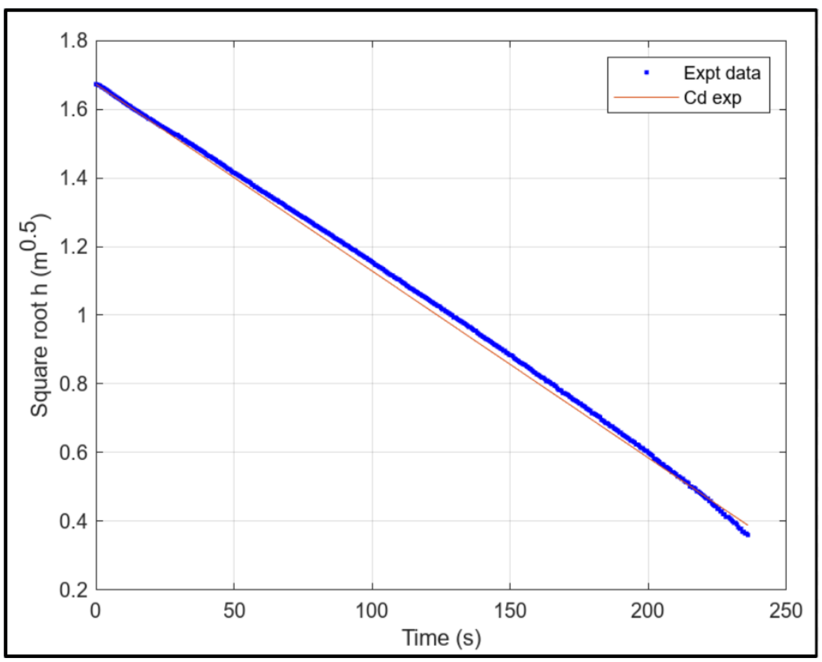

Figure 6. Comparison of the Experimental and Calculated Square Root of Height vs. Time for a $\frac{3}{16}$ in (4.76 mm) Straight-bore Orifice with a Stepped Recess, $C_D = 0.63$

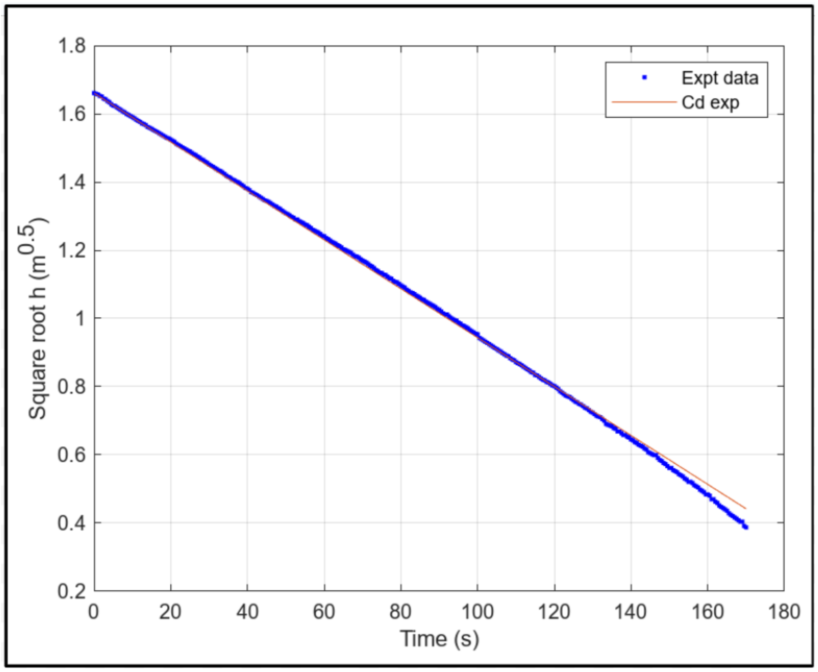

Figure 7. Comparison of the Experimental and Calculated Square Root of Height vs. Time for a $\frac{3}{16}$ in (4.76 mm) Straight-bore Orifice, $C_D = 0.83$

Table 2 shows the calculated times from Equation (11) to completely drain the tank with each of the orifices. These times for complete draining correspond reasonably well with the extrapolated results from Figures 3-7, but the results from the equation were always a bit lower. At least part of this difference can be attributed to the use of Matlab in attaining a numerical solution of Equation (9). The Matlab solutions with each orifice tended to fall below the experimental values as the tanks reached complete drainage and underestimated the drainage time by about 15%. By contrast, the analytical solutions of Equation (9) was a better representation of the experimental data.

Table 2. Calculated Times to Completely Drain the Tank Using the Different Orifices (Initial height: 9 ft (2.74 m))

Orifice	C_D	Time for Complete Draining, s	
		From Equation (11)	From Figures 3-7
A $\frac{3}{16}$ in sharp-edged	0.64	300	250
B $\frac{1}{8}$ in sharp-edged	0.72	595	520
C $\frac{3}{16}$ in rounded-edged	0.69	276	230
D $\frac{3}{16}$ in straight-bore with stepped recess	0.63	303	260
E $\frac{3}{16}$ in straight-bore	0.83	230	190

Conclusions and Future Work

The addition of sharp-edged, rounded-edge and straight-bore orifices to the tank draining experiment increases the flexibility of the experiment that was carried out by Savage *et al*. [5] so that the experiment can be satisfactorily used from semester to semester without worrying about the students copying reports. The experiment generated discharge coefficients which ranged from 0.63-0.83 and showed very good agreement between transient experimental data and Bernoulli model data. The model is flexible in that it can be solved algebraically using Equation (11) or numerically using Equation (9) with computer programs such as MATLAB. A MATLAB solution of Equation (9) which produces a plot of fluid height vs. time is a very good application of MATLAB and a good alternative way to illustrate the lab results.

The experimental protocol and the ability to easily machine and then test a number of orifices open up additional opportunities for laboratory or classroom study, including:

- Using experimental data along for various orifice configurations to predict discharge coefficients for other orifice configurations—the students might very well gain an appreciation of the factors that most affect changes in the discharge coefficients
- Using multiple orifice sizes of the same types (for example, $\frac{1}{8}$ in (3.18 mm), $\frac{3}{16}$ in (4.76 mm), $\frac{1}{4}$ in (6.36 mm) sharp-edged orifices) in predicting and then determining the differences in discharge coefficients and resulting draining times.

These studies can be done experimentally or just by working with the model equations as predictive tools.

Nomenclature (SI units shown)

Latin Letters

A_0	Area of the orifice, m^2
A_1	Area of the tank pipe, m^2
A_{vc}	Area of the vena contracta, m^2
C_D	Discharge coefficient
$C_{D\ ideal}$	Discharge coefficient using the ideal value of 0.61
$C_{D\ experimental}$	Discharge coefficient that is experimentally found
d_0	Inside diameter of shape-edged orifice, m
d_1	Inside diameter of the cylindrical tank, m
g	Gravitational constant, m/s^2
h	Height of water in the tank, m
h_0	Initial height of water in the tank, m
m_1	Mass at fluid level in the tank, kg
m_{vc}	Mass at the *vena contracta*, kg
p_1	Pressure at point 1, fluid level in the tank, Pa
p_2	Pressure at point 2, selected as the location of the *vena contracta*, *vc*, Pa
Q	Volumetric flow rate out of the orifice
t	Time, s
v_l	Velocity of the fluid level tank, m/s
v_t	Velocity of the water in the drain tube, m/s

v_{vc}	Velocity at the vena contracta, m/s
z_1	Height of the free surface in the tank as marked by the tape measure, m
z_2	Height of the free surface in the tank where the water exits, m

Greek Letters

ρ	Density of water, kg/m^3

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