

## Dynamic Modeling and Analysis of a Crank Slider Mechanism

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### Abstract

Several commercially available software tools can be utilized for enhancing design and analysis tasks related to mechanisms and other dynamic systems. This paper delineates the results from a pilot independent study performed by junior level engineering undergraduate students. The study includes development of free-body diagrams and kinetic diagrams of individual components of the crank slider mechanism, development of nonlinear differential equation of motion of a crank slider mechanism driven by a DC (direct current) motor, and motion simulation using software programs. Standard numerical analysis techniques using MATLAB and the virtual prototyping environment provided by WORKING MODEL software are used. Students involved with the project have expressed enhanced understanding of the subject matter with the integration of the software tools. As expected, both simulation environments yield similar results, however, the visual display of the motion using WORKING MODEL provides an excellent correspondence between the abstract mathematics and a realistic animation of the physical reality. The success of the project and student feedback suggest that integration of these software tools will be beneficial for improving student performance in the Dynamics course that all mechanical engineering undergraduates have to undertake.

### I. Introduction

Computer simulation of dynamic systems is a valuable tool for engineering analysis and design. It allows for active experimentation, design modification, and subsequent analysis without investment in raw materials and supplies.

The 'chalk and talk' style of teaching and instruction attempts to transmit knowledge from the teacher to a passive recipient. There is a growing awareness among engineering educators that while this style of instruction is suitable for teaching engineering analysis it has some limitations when it comes to nurturing creativity, synthesis and engineering design<sup>1</sup>. Therefore, a prudent combination of teaching by lectures and active learning techniques are perhaps the ideal way to enhance student comprehension and creativity. Modern simulation software provides an efficient way of involving engineering undergraduate students in the active learning process. The reform movement in engineering education inspired by Engineering Criteria 2000 (EC 2000) of Accreditation Board of Engineering and Technology (ABET)<sup>2</sup> is consistent with this approach. It is attempting to integrate a continuous improvement cycle (Fig 1) with an experiential learning cycle (Fig 2) within engineering education<sup>3</sup>.

Traditionally, the “Dynamics” course is taught at the sophomore level in the Mechanical Engineering curriculum immediately following Statics. Emphasis is primarily on kinematics and kinetics of particles and rigid bodies. A typical course require the students to solve problems involving a particular state of motion for a particle or a rigid body (e.g., analysis of motion of a four bar linkage at a particular crank angle, given angular velocities and acceleration of some links and their dimensions and relevant forces and moments when appropriate). While these problems are mathematically elegant they reflect just an instance of motion. Conversations with students who are adept at the mathematical manipulations of solving such problems have revealed that they lack a complete understanding of the motion of the mechanisms and their manifestations. Similar observations can be found in some recent papers, where, some engineering educators reflect on their experiences of teaching “Dynamics” in a new interactive format<sup>4,5</sup>. Teamwork and computer-skills in solving “Dynamics” problems are emphasized to improve learning as well as developing skills needed in the workplace.

In this paper the project work that was performed by two junior level students as an independent study is discussed. The project involves the simulation study of a crank slider mechanism using both MATLAB<sup>6</sup> and WORKING MODEL<sup>7</sup> software.

## II. Development of Dynamic Equation of a in-line Crank Slider Mechanism

Figure 3 represent the schematic of a crank slider mechanism. Figures 4 -6 represent the free body diagrams and kinetic diagrams for the crank (assumed to be a circular disc), connecting rod and the slider (piston) of a general crank slider mechanism, where the input motion is provided to the crank by a suitable prime-mover (e.g., DC motor).  $D_1$ ,  $D_2$  and  $D_3$  are the centers of mass corresponding to the crank, connecting rod, and the piston, and  $l_{D_2}$  is the distance from the pivot point B (crank and connecting rod) and the center of mass of the connecting rod  $D_2$ .  $D_1$  coincides with the pivot point A (crank and ground). At the instant shown the crank is moving with a angular velocity  $\omega_1$  and angular acceleration  $\alpha_1$ .  $\phi$  and  $\theta$  are the crank angle and connecting rod angle with respect to the horizontal at the instant. The fixed link (ground) is numbered 0 and the crank, connecting rod and the piston are numbered 1, 2 and 3 respectively. 'l' and 'r' are the length of the connecting rod and radius of the crank respectively.  $I_{D_i}$  and  $m_i$  for  $i = 1,2,3$  represent mass moments of inertia and masses corresponding to links 1, 2 and 3. The linear and angular acceleration of the links are as in the Figures 4 -6.

The piston ( link 3) exhibits pure translation along the x-axis. From the F.B.D and kinetic diagram in Figure 4, the dynamic equation for the piston may be represented as :

$$\sum F_x = m_3 a_{D3x} \quad \Rightarrow \quad F_{23x} = m_3 a_{D3x} \quad (1a)$$

$$\sum F_y = 0 \quad \Rightarrow \quad F_{03y} + F_{23y} = 0 \quad (1b)$$

The connecting rod (link 2) exhibits general plane motion. From the F.B.D and the kinetic diagram in Figure 5, the dynamic equation for the connecting rod can be represented as:

$$\sum F_x = m_2 a_{D2x} \Rightarrow F_{12x} + F_{32x} = m_2 a_{D2x} \quad (2a)$$

$$\sum F_y = m_2 a_{D2y} \Rightarrow F_{12y} + F_{32y} = m_2 a_{D2y} \quad (2b)$$

$$\sum M_B = \sum (\mu_k)_B \Rightarrow F_{32x} l \sin\theta + F_{32y} l \cos\theta = (m_2 a_{D2x}) I_{D2} \sin\theta + (m_2 a_{D2y}) I_{D2} \cos\theta + I_{D2} \alpha_2 \quad (2c)$$

Equation 2c equates the applied moment about point B to the kinetic moment about point B <sup>8</sup>.

The crank exhibits pure rotation about pivot A. From the F.B.D. and the kinetic diagram in Figure 6, the dynamic equation for the crank can be represented as:

$$\sum F_x = 0 \Rightarrow F_{01x} + F_{21x} = 0 \quad (3a)$$

$$\sum F_y = 0 \Rightarrow F_{01y} + F_{21y} = 0 \quad (3b)$$

$$\sum M_A = I_{D1} \alpha_1 \Rightarrow T - F_{21x} r \sin\phi + F_{21y} r \cos\phi = I_{D1} \alpha_1 \quad (3c)$$

### III.Simplified Simulation Model

It is necessary to perform complicated velocity and acceleration analysis for link 2 (connecting rod) that exhibits general plane motion to completely solve for the motion of the mechanism. For simplification the connecting rod may be replaced by a lumped mass ( $m_B$ ) at the crank connection and mass ( $m_C$ ) at the piston connection. To ensure that the assumption is dynamically equivalent the following criteria should be satisfied as far as possible:

- the center of mass must be at the same location of the original connecting rod ( $D_2$ ),
- the total masses must be equal i.e.,  $m_B + m_C = m_2$ ,
- the moments of inertia with respect to the center of mass must be the same.

Also, by appropriately counter-weighting the crank the inertia effect due to the rotating part of the connecting rod mass ( $m_B$ ) may be nullified. The net moment of inertia of the crank with this assumption will be referred to as  $I$  and the sum of  $m_C$  ( translating part of the connecting rod mass) and  $m_3$  ( piston mass) will be referred to as  $m$ .

This assumption simplifies the velocity and acceleration analysis considerably. The acceleration of  $m$  (piston mass + portion of connecting rod mass) can be expressed after some trigonometry and algebra as :

$$a_x = a_{D3x} = -r\alpha_1 [\sin\phi + (r/2l)\sin 2\phi] - r\omega_1^2 [\cos\phi + (r/l)\cos 2\phi] \quad (4)$$

The DC motor that will be driving the mechanism is assumed to have the following characteristics:

$$T = k_1 - Bk_2 \quad (5)$$

where  $k_1$  and  $k_2$  are motor constants. The relationship indicates the input torque varies with crank speed which is realistic.

The simplified dynamic model can be expressed after some algebra as <sup>9</sup>:

$$K_1\ddot{\phi} + k_2\dot{\phi} + K_2(\dot{\phi})^2 - k_1 = 0 \quad (6)$$

where,  $\dot{\phi} = \frac{d\phi}{dt} = \omega_1$  and  $\ddot{\phi} = \frac{d^2\phi}{dt^2} = \alpha_1$ ;

$$K_1 = I + mr^2[\sin\phi + (r/2l)\sin 2\phi](\sin\phi + \cos\phi \tan\theta)$$

$$K_2 = mr^2[\cos\phi + (r/l)\cos 2\phi](\sin\phi + \cos\phi \tan\theta),$$

$$\text{and, } \tan\theta = \frac{r\sin\phi}{(l^2 - r^2\sin^2\phi)}.$$

To carry out the computer simulation studies the following parameter values were chosen:

Crank radius,  $r = 0.4$  m, connecting rod length  $l = 0.16$  m, reciprocating mass  $m = 0.6$ kg and a crank moment of inertia of  $0.5$  kg-m<sup>2</sup>. The constants involved with the DC motor were chosen as  $k_1 = 95$  Nm and  $k_2 = 1$  Nm-s.

#### IV. Software Implementation and Simulation Results

##### MATLAB

Equation (6) embodies the motion dynamics of a crank slider mechanism driven by a DC motor in the absence of friction and other dissipation effects. The equation is highly nonlinear and therefore closed form solution is difficult. This provided an opportunity to introduce simple numerical integration techniques to the students for solving for the crank motion as it moves through the transients to a steady state. Only Euler's method of numerical integration was used with a time step of 1 milli-second but the students were made aware of other more accurate methods of performing the numerical integration. A MATLAB program (m-file) was developed to carry out the simulation for 30 cycles (a little over 2 seconds of simulated time) of the crank starting from rest. The results are shown in Appendix A. Figure 7 illustrates the motion of the crank as its velocity rises from 0 to around 5443deg/s (95rad/s). Figure 8 demonstrates that the steady state (although there are small fluctuations around 95rad/s crank velocity) is reached around the 7th or 8th cycle, after which the 'Time Vs. Cycles' graph achieve a more constant slope. Figure 9 shows the motor torque against the number of cycles of crank motion. As expected the torque value is 95Nm at time  $t = 0$  and gradually diminishes to small oscillation around zero as the mechanism settles down to a steady motion pattern after the 7th or 8th cycle.

##### WORKING MODEL

WORKING MODEL 2-D was also used to simulate the motion of the crank slider mechanism. The crank was modeled as a circle with the motor at the center of mass. The connecting rod was modeled as a thin rectangular member attached by pin joints to the crank and piston (slider). The slider was also connected to a straight grooved channel along the x-axis by a slot joint that

allowed motion along the channel but restricted any other linear or rotational motion (See Figure 10). After the mechanism was assembled, using the "properties dialog box" appropriate values for mass moments of inertia, masses, torque function etc. were assigned. Euler method of integration with a time step of 1 milli-second was also chosen from a menu of different numerical integration techniques allowed by the software to ensure similarity with MATLAB simulation. The connecting rod inertia was inputted as a very low number (zero is not allowed) to conform as far as possible to the simplified model used for MATLAB simulation. The connecting rod mass was assumed to be incorporated in the crank inertia and the piston mass values used. Results of the simulation were displayed using "measurements dialog box" provided in the software environment. Besides displaying results during run time using the "measurements dialog box", the WORKING MODEL software provides the capability of exporting data from the simulation to suitable data processing applications. For generating the graphs in Figures 11 and 12, the data was exported to EXCEL and plotted. The results are similar to those obtained using MATLAB.

## V. Conclusion

As expected, the results of the simulation are similar using both methods. The students who were involved in this independent study have reflected that the simulation study enhanced their learning in several ways. Listed below are some of the significant observations made during and after the completion of the project:

- Insight into the connection between design and dynamic analysis. Students could easily change parameters to see how the system behavior was modified.
- Assimilation of ideas from different sections of a typical " Dynamics " textbook. The students had to use concepts of free body diagram development, velocity and acceleration analysis, development of equations of motion that are in different chapters in standard textbooks. The exercises that follow each chapter typically examine concepts introduced in the specific chapter, which does little towards holistic assimilation of the information.
- The choice of the mechanism also allowed discussion along three significant motion patterns that planar rigid bodies exhibit, namely, pure translation (slider), pure rotation (crank) and general plane motion(connecting rod).
- The simulation study provided the instructor to introduce concepts from nonlinear dynamics and numerical analysis. By comparing the simulations and by varying the time steps of the integration the students also learned the limitations of numerical integration techniques.
- The WORKING MODEL software provided a realistic animation of the motion and the students could readily appreciate the connection between the input torque at the crank and the output motion of the slider. This allowed the students to observe the entire motion of the mechanism not only throughout its 360-degree rotation but also from a transient unsteady pattern to a more permanent steady pattern. Solving kinematics and dynamic problems for a particular instance of motion as required in a typical textbook does little to provide the students a realistic portrayal of the motion. In this regard the virtual prototyping capability of WORKING MODEL provides an excellent correspondence with reality and thereby stimulates student learning.

- The simulations also provided a basis for comparing results obtained by students at different instances of motion by hand calculations. This made the mathematical manipulations they were required to learn during the project more meaningful.
- Lot of industries uses the software tools utilized in the project for their simulation and analysis work. Exposure to the software not only adds to student comprehension and learning but also prepares them for their future jobs.

## VI Acknowledgment

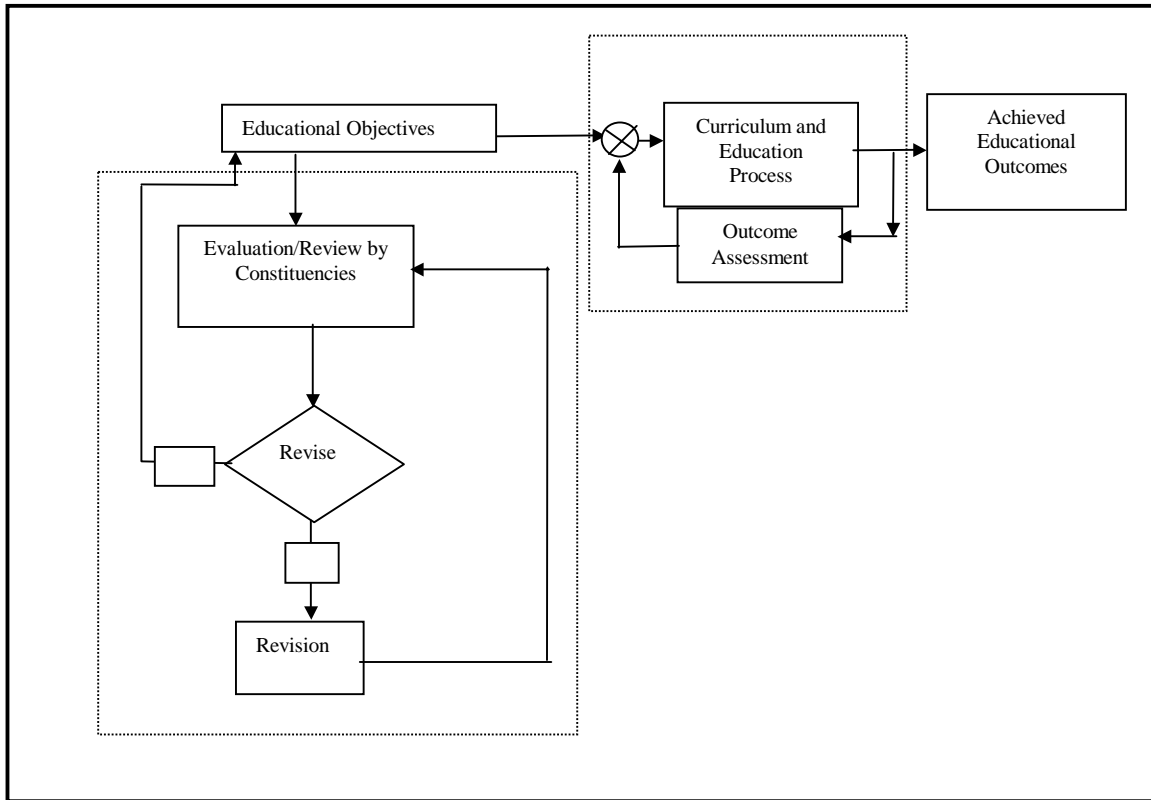
The two students that were involved in the independent study reported are Mr. Adolfo E. Andino and Mr. Luis J. Forteza of Turabo University in Puerto Rico, where the author worked as Assistant Professor prior to taking up his current position at the University of Maryland Eastern Shore. The students patiently learned through trials and tribulations two new software programs, in particular, Working Model, which the author of this paper learned with them. While these students strongly encouraged integration of the software tools to enhance learning in a "Dynamics" course, they also helped the author realize the extent to which some of the concepts involved (nonlinear dynamics, numerical analysis etc.) can be introduced in a regular dynamics class in the sophomore year.

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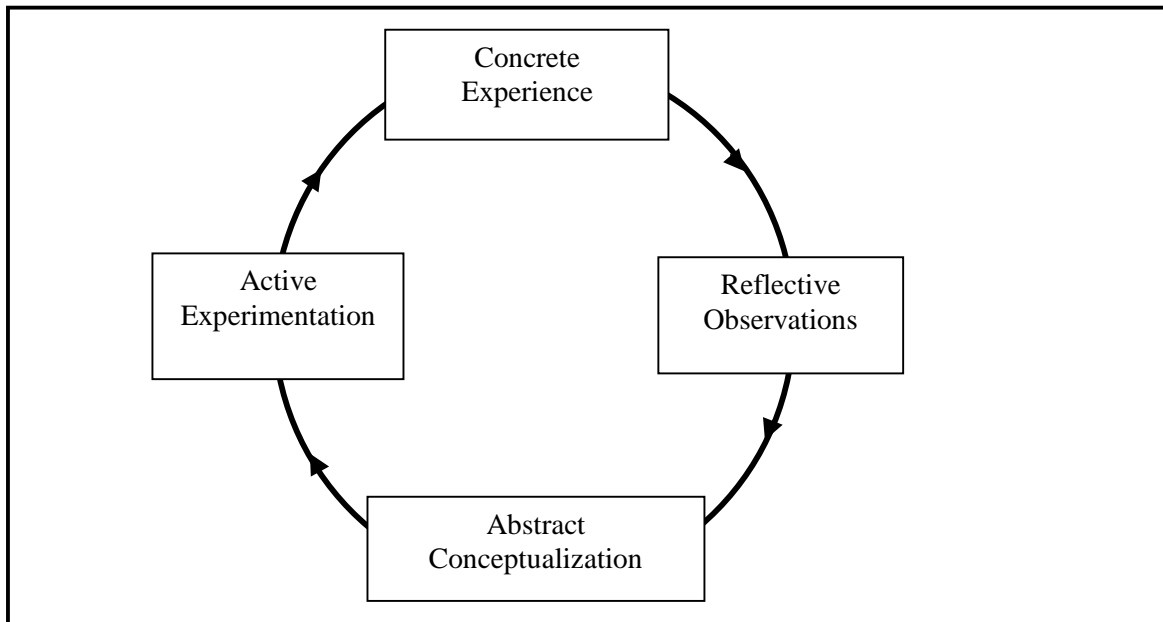
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## ABHIJIT NAGCHAUDHURI

Abhijit Nagchaudhuri is currently an Associate Professor of Engineering and Aviation Sciences at the University of Maryland Eastern Shore. Prior to joining UMES he worked in Turabo University in San Juan , PR as well as Duke University in Durham North Carolina as Assistant Professor and Research Assistant Professor, respectively. Dr. Nagchaudhuri is a member of ASME, SME and ASEE professional societies and actively involved in teaching and research in the fields of engineering mechanics, robotics and systems and control. Dr. Nagchaudhuri received his bachelors degree from Jadavpur University in Calcutta, India with a honors in Mechanical Engineering in 1983, thereafter, he worked in a multinational industry for 4 years before joining Tulane University as a graduate student in the fall of 1987. He received his M.S. degree from Tulane University in 1989 and Ph.D. degree from Duke University in 1992.



**Figure 1 : Continuous Improvement Cycle in EC 2000**



**Figure 2 : Experiential Learning Cycle**

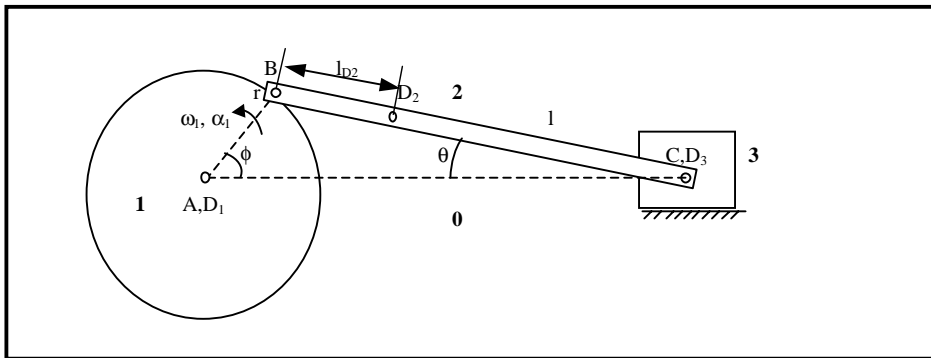


Figure 3: Crank Slider Mechanism Schematic

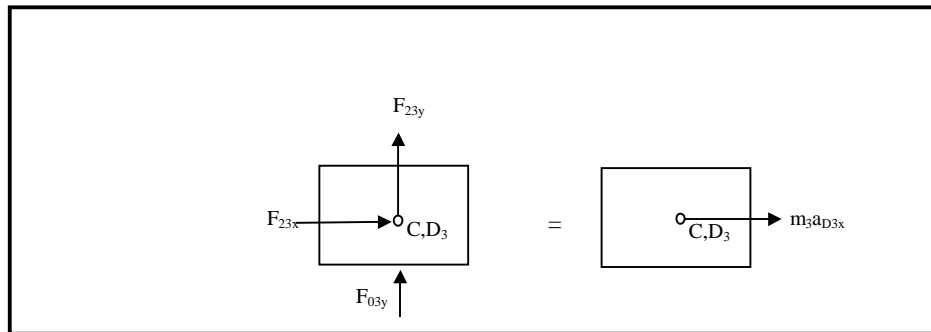


Figure 4 : Free body and kinetic diagram of the Piston (Pure Translation)

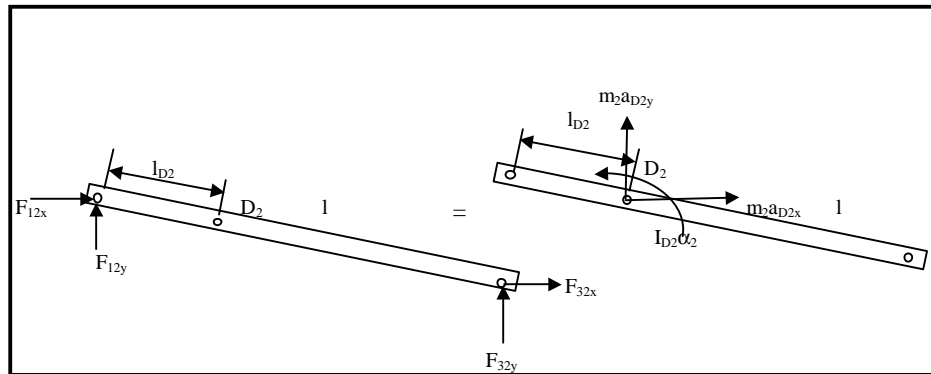


Figure 5 : Free body and kinetic diagram of Connecting Rod (General Plane Motion)

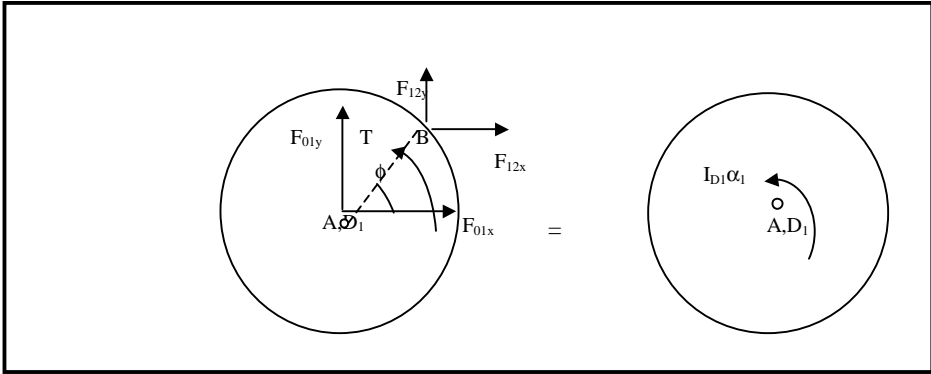


Figure 6 : Free body and kinetic diagram of Crank ( Pure Rotation)



APPENDIX A (RESULTS FROM MATLAB SIMULATION)

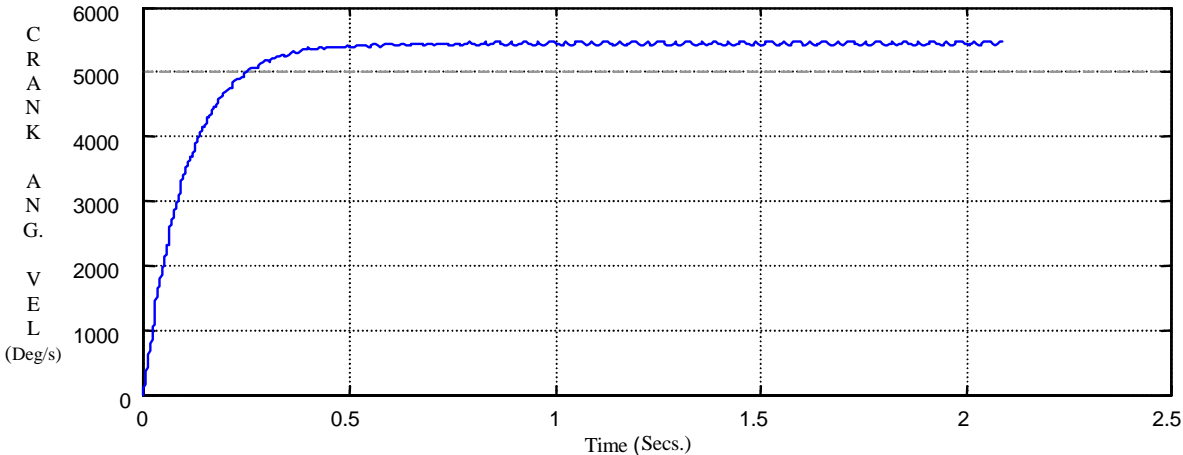


Figure 7 : Variation of  $\omega$  (Crank angular velocity with time)

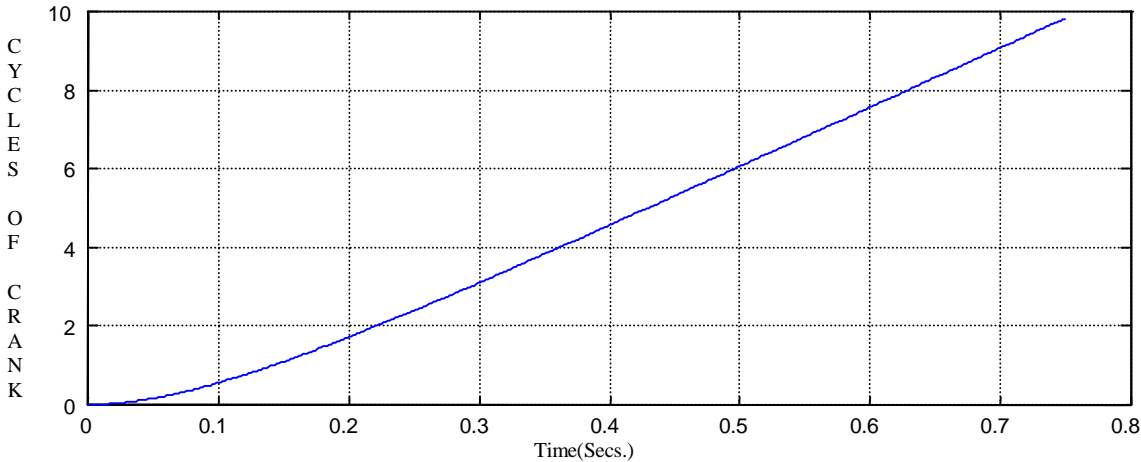


Figure 8 : Cycles of Crank Vs. Time ( from transient to steady state)

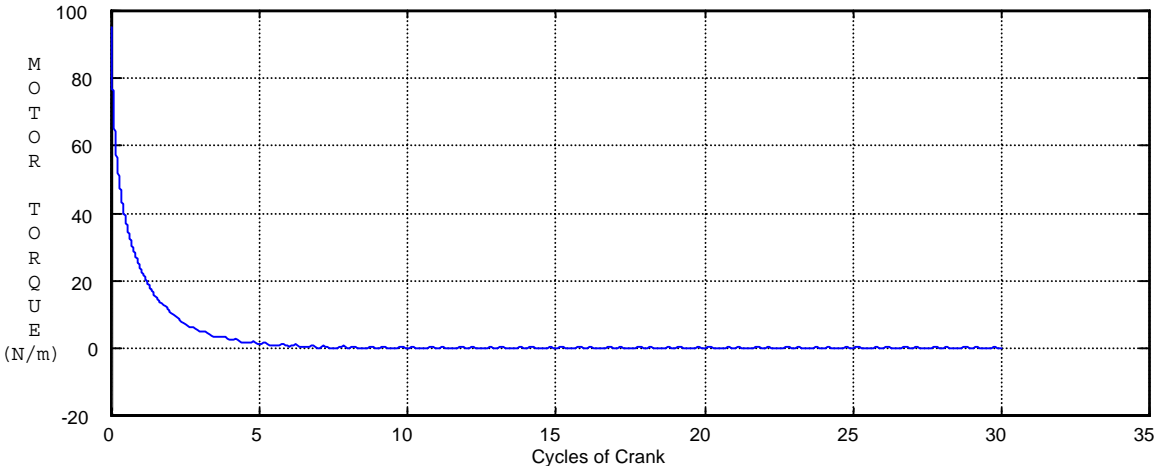


Figure 9 : Torque variation over cycles of crank motion

APPENDIX B: WORKING MODEL REPRESENTATION AND RESULTS

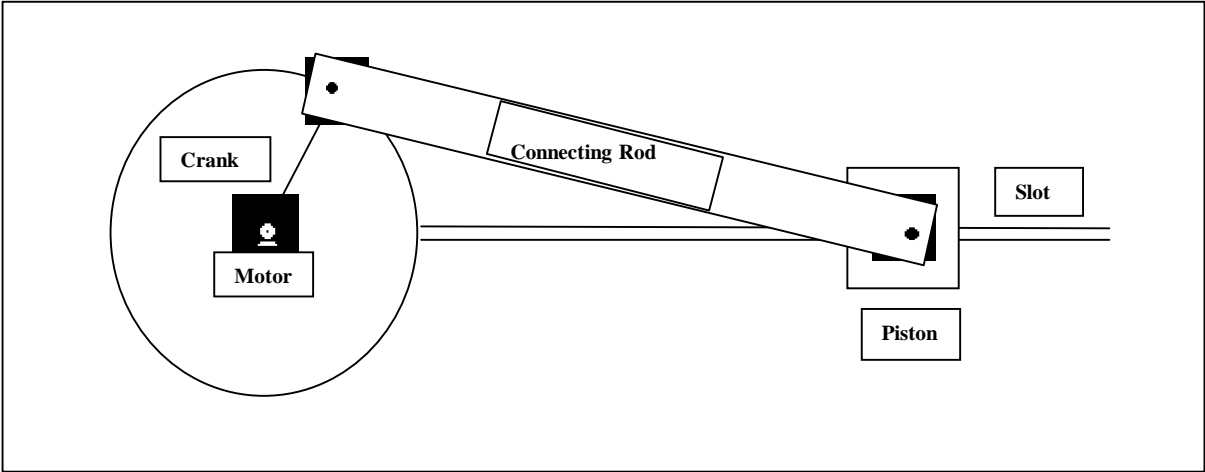


Figure 10 : Working Model Representation

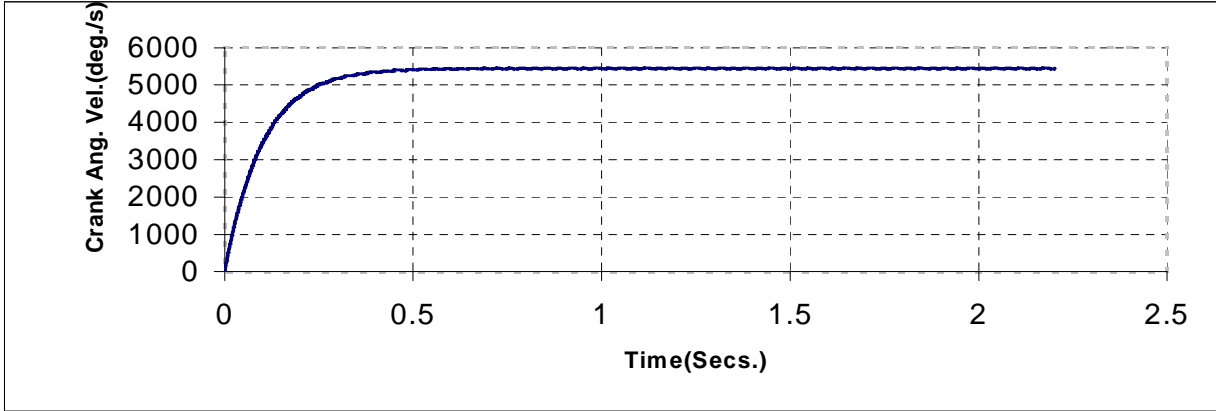


Figure 11 :  $\omega_1$  (Crank Ang. Vel.) Vs. Time

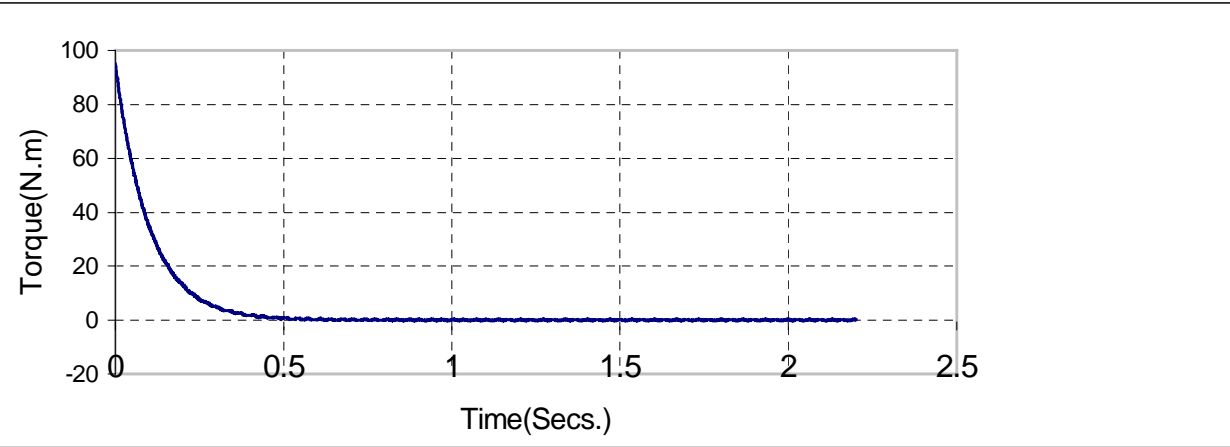


Figure 12 : Motor Torque variation with time