



Early work for the Mathematics as a Gatekeeper to Engineering Project: A Review of Informal Learning, Engineering and Design Thinking Literature

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Abstract

Engineering education and engineering practice tend to be characterized by two complementary halves: engineering sciences and engineering design. While both are critical to engineering education and practice, the two are often treated separately – as separate classes, taught by separate faculty, and at times separate job functions. This project examines the relationships between these two “halves” by exploring the relationship between mathematical thinking and design thinking. The hypothesis guiding the research, informed by the investigator's experience with teaching first-year engineering students, is that the mathematical thinking processes students develop in their pre-college education may serve as cognitive obstacles students must overcome in order to develop the design thinking skills which are critical for engineering practice broadly as well as for creativity and innovation.

This hypothesis is investigated through a verbal protocol study, where undergraduate students will be asked to "think aloud" while attending to a design task. This data collection method allows the researchers to video record the students' thought processes and then analyze mathematical thinking and design thinking patterns, such as convergent and divergent thinking; fixation; estimation and modeling; and responses to ambiguity and uncertainty. Students from a variety of mathematics, design and engineering backgrounds will be asked to participate, to allow us to capture possible differences between approaches taken by students with more/less mathematics background and more/less design background. The verbal protocol data will be augmented by interviews with the students which will provide additional insights related to the design task as well as the students' other experiences, beliefs and attitudes. The video data will be used in the form of excerpts in a first-year engineering class as a teaching tool to help students develop design thinking skills, and will be used as a teaching tool in graduate engineering education courses to help graduate students develop research skills. The research findings can inform the design of educational activities – for example, the teaching of mathematics and engineering in pre-college settings; the sequencing of activities in first-year engineering courses; and the overall design of engineering curricula.

This project is still in its early stages. The paper and poster will focus on the development and selection of the design task used for the research study, and will include a review of other existing instruments for assessing students' understanding of design. In addition, the paper and poster will discuss recent educational interventions developed based on the research conducted to date.

Introduction

In the *Mathematics as a Gatekeeper to Engineering* study, we will observe how first-year and senior students from engineering, design and mathematics disciplines approach a common design task (the design of a playground for a fictitious neighborhood). We will look for distinctions and commonalities in the thinking processes and strategies used across groups. It is important for us to understand what literature exists about students' mathematics and design skills prior to entering college. This can guide our understanding about which practices or activities—whether they emerge through formal classroom instruction or informal experiences—help students form their understanding of mathematics and design thinking and their respective applications.

Through the use of the verbal protocol analysis (VPA) method, analysis of video data of each participant's design task experience and each student's follow-up interview data, we hope to develop a portrait of what types of mathematical and design thinking our design activity can elicit, and how the different types of thinking complement or conflict with the student's progress with the task. We are particularly interested in the type of design and mathematics concepts the students draw from as they complete the design task. This will help us understand whether and how mathematical thinking processes can serve as obstacles which the student must overcome.

The literature review in this paper will develop a portrait of engineering learning in informal settings by addressing perspectives in both formal and informal science, mathematics and engineering learning spaces. This literature provides a foundation for our work as it complements existing syntheses of what is known about engineering learning in formal environments.

The goals of the current study are to provide the engineering education community (researchers as well as P-16 educators) with an increased understanding of how students learn engineering content and engage in engineering thinking, and to understand how students' pre-college experiences impact their collegiate learning experiences. The hypothesis guiding the research, informed by the investigator's experience with teaching first-year engineering students, is that the mathematical thinking processes students develop in their pre-college education may serve as cognitive obstacles students must overcome in order to develop the design thinking skills that are critical for engineering practice broadly as well as for creativity and innovation. Therefore; it is important to understand the possible mathematical and design knowledge students have when they enter college from experiences both within and outside the classroom.

In precursive studies, researchers found that students' design behavior differed when they attempted problems for which the context was familiar to them¹. The design task in the *Mathematics as a Gatekeeper to Engineering* study is a modified version of the playground design task used in previous first-year and senior design - process comparison studies^(e.g.1,2). The implications of the previous study were of special interest to our project. Atman et al.¹ recommended that when designing studies similar to their work, context should be considered. Therefore; as we modified the design task to meet the needs of *Mathematics as a Gatekeeper to Engineering* study, we have carefully considered context specificity and complexity of the task as recommended by Atman et al.¹ This will further be discussed in the 'identifying the appropriate task' section.

The literature review provides an overview of work which is similar to our project. It also shows our team where more work can be done to investigate students' approaches to mathematical and design tasks, whether and how mathematical thinking processes can serve as cognitive obstacles to applying design thinking skills in engineering contexts, and how first-year and senior students from various disciplines use mathematical and design thinking skills differently. Although our project is in its early stages, we end the paper with a discussion of the current status of the *Mathematics as a Gatekeeper to Engineering* project and will share early findings from two pilot studies.

Identifying an Appropriate Design Task

While identifying an appropriate design task, we were guided by two questions: 1. How does the structure of a task elicit both mathematical thinking and design thinking? 2. How can we ensure the wording of the task does not lean too heavily towards one type of thinking? These questions guided us as we analyzed open-ended design tasks for use in our study. Some of the design tasks we considered for this study were model-eliciting activities (MEA) such as the Travel Mode, Prosthetic Hand Pricing and the Image Tiling problems. MEAs are used in engineering courses to model real world problems, they are designed with mathematics at the core of the design processes and the objective is typically to design a process instead of a product. Initially these tasks were analyzed to determine what the participant was being asked to do and to what extent the participant would use mathematical and design thinking. Our research team opted not to recreate a design task but to add to data on pre-existing tasks. For each task we analyzed the following questions were considered:

- Is it accessible to engineering first-year students and graduating seniors?
- Is it accessible to mathematics first-year students and graduating seniors?
- Is it accessible to design first-year students and graduating seniors?
- Does it elicit mathematical thinking?
- Does it elicit design thinking?
- Will the students spend 1-3 hours on the activity?
- What other research is it related to?
- Does the design task resemble that of a practicing engineer?

After observations of engineering students engaged in an MEA, we were not convinced that these activities could elicit the broad range of design thinking activities we were interested in observing. These activities are heavily dependent on the student(s) developing a mathematical algorithm or a mathematical approach to solve the given problem. In order to understand how a task could elicit design thinking, we began to review literature on design thinking and collected studio problems from the industrial design program at the college. Studio problems are used to introduce concepts, vocabulary, and skills applicable to continued study in a variety of visual disciplines. There are typically used in the introductory design course where students are introduced to two-dimensional design fundamentals. It was important for our project's design task to incorporate concepts familiar to design students as these students would comprise a subset of our participant pool. Design students in the introductory course were typically assigned studio problems to complete individually over the course of 1-2 weeks and were given

little guidance throughout the process. A sample studio problem was obtained from a design student. This student was also a former engineering student who was able to provide insight into how the studio problems, which he was assigned, differed from the typical engineering problems he encountered in the past.

The team considered this student's insight as we continued to compare design tasks used in other studies as well as those used in classrooms. We decided that the design of a playground for a fictitious neighborhood was an appropriate task. This design task had been used in similar studies; therefore, we believed that provided validation for the design task. Additionally, this playground design task seemed to elicit more design thinking than the MEAs we reviewed. We were able to modify the wording of the task statement to encourage mathematical and design thinking. In the modified version of the task, participants were explicitly asked to provide dimensions for the equipment, provide a budget and provide a layout of the playground with the design equipment. See Figure 1 for the design task statement.

You live in a mid-size city. A city resident has recently donated a corner lot for a playground. You are an engineer who lives in the neighborhood. You have been asked by the city to design equipment for the playground. You estimate that the children who usually use the equipment will range in age from 1 to 10 years. However, occasionally some adults will also use this equipment. From the amount of space you have in the park, you estimate that you should design equipment to keep 12 children busy at any one time. You would also like to have at least three different types of activities for the children.

The equipment must:

- be safe for the children
- remain outside all year long
- not cost too much
- comply with the Americans for Disabilities Act, so handicapped children will be able to play also

The neighborhood does not have the time or money to buy ready-made equipment pieces. Your design should use material that is available at any hardware or lumber store. The equipment must be constructed in less than 2 months.

Please explain your solution as clearly and completely as possible. From your solution, someone should be able to build your playground without any questions. Your work should contain a detailed description of your design and any relevant diagrams and calculations. Provide a drawing of your design with final dimensions of equipment and the playground layout. Estimate both the costs and the benefits associated with your design. Please clearly state all assumptions which are needed in your analysis.

The administrator has more information and tools to help you address this problem if you need them. You must be specific in your requests. For example, if you would like a diagram of the corner lot for the playground equipment, you may ask for it now. If you think of any more information you need as you solve the problem, please ask. Remember, you have approximately 3 hours to develop a complete solution. The administrator will inform you about how much time is left as you work.

Figure 1: Modified Playground Design Task Statement. This figure illustrates the design task for the study

Literature Review: Learning Science, Engineering and Mathematics in Informal Settings

The learning experiences of students in informal settings are important to our work because these settings provide rich and flexible environments for students to develop mathematical thinking and design thinking skills, and these experiences impact students' learning and thinking in their other college and pre-college experiences. In this literature review, we discuss prior research that has been conducted to understand students' learning of mathematics, learning of engineering and learning of science. The topic of informal learning environments has received increased attention in the field during the past 10 years. Scholars have theorized and proven that students' learning is not restricted to formal settings³. Informal learning experiences are beneficial, rich in content and allow the student to create models and develop solutions that may not align with what is taught in formal schooling. When entering new learning environments, students are sometimes seen as blank slates when in fact they have valuable learning experiences, typically from informal learning spaces, which they bring to the classroom.

Currently, more extensive work has been done on learning of science and mathematics in informal environments than there is on learning engineering in informal settings. Research on engineering learning in informal settings is still in its infancy. However, mathematics and science are inherent to engineering and they form a foundation on which engineering education is built on. This facilitates the integration of engineering material into P-12 curriculum. Theoretical understandings of how students engage and learn mathematics and science have been used to shape preliminary thoughts and research studies on P-12 engineering education. Within this paper, there will not be discussion dedicated to work on the impact of technology on informal engineering education. Current research on technology learning in informal settings focuses on the design process, which will be discussed, along with other topics, in the engineering thinking and learning sub-section.

Science thinking and learning will be included in this review because research done on science learning in informal settings can help to inform engineering education. Often, at the P-12 level, some aspects of engineering and science are the same. In fact, the text *Learning Science in Informal Environments: People, Places, and Pursuits* addresses that the overall goals of engineering education and science are similar³. For the purposes of this literature review, findings discussed in the science learning sections will be loosely applied to engineering learning.

It is important to understand how students engage in mathematical thinking in informal learning environments in order to better understand ways to inform formal pedagogy. This understanding may also create avenues to identify opportunities for and instances of knowledge transfer from one venue to another. Research studies have shown that students may understand a mathematical concept in an out-of-school context but often struggle when they are tasked with solving a similar problem in a school setting when it has been taken out of its original context. Researchers have theorized that mathematics education should be taught with some cultural context, so that the students can relate to the problem.

Literature on mathematics and engineering learning will be discussed together throughout this review because of the similar foci of problem solving, modeling and the real-world context of problems. Since work and literature about engineering learning in informal environments is still

emerging, it was important that a literature review, such as this one, be done in order to properly describe the current state of the field. This review will also provide insight into the mathematical and design learning of students before they enter college. Insight in this area will help our team understand why we may see specific practices used by students when working through the playground design problem.

Informal Learning Spaces in Out-of-School Settings

Dorie et al.⁴ present a model which summarizes a variety of formal and informal learning environments in the paper titled *FILE: A Taxonomy of Formal and Informal Learning Environments*. They suggest that activities fall within a continuum between formal and informal. The authors identify eight common environments for informal and formal learning:

Table 1: Common Out-of-School learning environments and example environments

Common Learning Environments	Example Learning Environments
Curricular Learning	anything during normal school hours
Extracurricular Learning	tutoring, afterschool programs, design competitions
Outreach Learning	developed through an outside source
Professional Learning	workplace learning, professional societies, internships, co-ops
Service Learning	Engineers Without Borders, Engineering for a Sustainable World
Learning in Designed Settings	science centers, museums, zoos, aquariums
Learning from Media	books, television, games, social network, internet
Everyday Learning	play, family conversations

The authors state that there are times when the constraints of a formal environment can restrict what can be learned by the student. When this occurs, instructors should be aware that informal learning environments and activities exist which can re-engage the learning opportunity. A teacher might bring the strengths of informal learning environments into classroom setting by implementing innovative learning strategies and activities which may have been historically used in informal settings. For example, informal learning environments are often characterized by the choice that students have in directing the learning experience³. In a classroom environment, a teacher might also facilitate this sense of choice or self-directedness when students are given the opportunity to choose their own project topic or focus. We hope to continue conversations on how what we know about learning in these environments can 1) be appropriately integrated into more formal learning settings, 2) help us understand how students successfully transfer their knowledge from informal to formal settings 3) how learning in formal and informal settings can work complementarily and 4) how this knowledge transfer may help students overcome cognitive obstacles.

Science Learning

The report *Learning Science in Informal Environments: People, Places, and Pursuits*³ (LSIE) provides an in depth history of the development of informal learning in the United States. In addition, the report also discusses frameworks which shape the authors' discussion on defining informal learning spaces. The Committee on Learning Science in Informal Environments ('the committee'), which prepared the report, identified the ways in which science in out-of-school environments contributes to overall knowledge of science related concepts. The committee's "charge" specifically included assessing the evidence of science learning across settings, learner age groups, and over varied spans of time; identifying the qualities of learning experiences that are special to informal environments and those that are shared (e.g., with schools); and developing an agenda for research and development"³.

Similar to the common learning environments described by Dorie et al., the committee identified "venues" for science learning. The described venues are everyday experiences, designed spaces, programs and science media³. Learning in these venues takes into consideration the broadness of learning as it relates not only to formally learned academic knowledge but also to lifelong science learning and to learning science within cultural contexts. The types of learning that can occur in informal science environment are categorized by the committee as follows³:

- **Lifelong learning:** "the acquisition of fundamental competencies and attitudes and a facility with effectively using information over the life course, recognizing that developmental needs and interests vary at different life stages." (p. 28)
- **Life-wide learning:** "the learning that takes place as people routinely circulate across a range of social settings and activities—classrooms, after-school programs, informal educational institutions, online venues, homes, and other community locales." (p. 28)
- **Life-deep learning:** the "beliefs, ideologies, and values associated with living life and participating in the cultural workings of both communities and the broader society" (p. 28). Learning is never void of culture.

Mathematics Learning

Mathematics learning and application in informal and formal environments has been explored through surveys, ethnographies, within the context of the family, and in the workplace. Additionally there is literature on how educators can assist in the transfer of knowledge learned in informal environments to formal educational environments³. A major theme in literature on mathematics learning is that the mathematics strategies used in the informal learning environments vary from those learned in formal education settings and that families contribute greatly to the students' mathematics learning. It is obvious from students' exposure to mathematics in family settings that they do not enter formal mathematics learning environments as blank slates but these students have prior mathematics knowledge. In fact, the literature posits that family mathematics can complement formal mathematics education from simple arithmetic through pre-algebra⁵.

There has been increased focus on using the cultural context within which mathematics is situated in order to inform how students can better learn mathematics concepts in the classroom. Therefore, it is important to study how mathematics is used in the real world, at home and

beyond. Mathematics in the workplace constitutes another form of informal mathematics learning and application. The main themes which arise from the literature on mathematics learning in informal environments are: problems solving, the mismatch between strategies used in formal and informal settings, the limited constraint rich problems in mathematics problem solving education and the transfer limitation of real-world math knowledge to in-class mathematics assignments.

Research on mathematical learning and thinking also examines students' beliefs about mathematics, such as⁶:

- Mathematics problems have one and only one right answer
- There is only one correct way to solve any mathematics problem-usually the rule the teacher has most recently demonstrated to the class
- Ordinary students can expect to understand mathematics; they expect simply to memorize it and apply what they have learned mechanically and without understanding.
- Mathematics is solitary activity, done by individuals in isolation
- Students who have understood the mathematics they have studied will be able to solve any problem in 5 minutes or less
- The mathematics learned in schools has little or nothing to do with the real world
- Formal proof is irrelevant to the processes of discovery or invention

Some of these beliefs can also be used to understand how students view engineering problems. Although mathematics is distinct from engineering, there is a need for engineering students and engineers to draw heavily from prior mathematics knowledge and apply that knowledge to solve problems. This integration of knowledge will be discussed in following section.

Engineering Learning

Research in the area of engineering learning in informal settings is still emerging. Traditionally engineering education has been viewed as the teaching and learning of traditional engineering disciplinary knowledge amongst college students in preparation for the needs of industry and the academe. The engineering education curriculum was shaped in part by the needs of industry, guided in the direction of funding and traditionally designed to resemble the French model of engineering curriculum which was built around the “basic sciences, technical subjects, and humanities, with theory taught before application”⁷. Naturally, the following questions arise for engineering education today: 1) What is engineering? , 2) Who is an engineer? and 3) How should engineering be taught? These questions should be answered in order to properly inform engineering teaching in pre-college and college settings. Since engineering is a constantly evolving field, the answers to these questions continue to change over time.

Engineers in society have many roles and typically are known for their ability to solve problems and meet needs in the most efficient manner possible by using different types of knowledge. This section of the literature review will focus on the following aspects of engineering learning: problem solving, engineering design process, engineering design and engineering modeling. There is some overlap within these four aspects of engineering. Some emergent engineering learning in informal settings literature will be used in conjunction with mathematics and science informal learning literature to create a portrait of informal learning opportunities in engineering. Also, engineering learning in formal settings will be addressed because it is important to

understand how students engage in engineering learning in all contexts because engineering learning in specific contexts is not completely understood.

Design & Design Education

Some engineers and educators, focus on design as a core and defining aspect of engineering education. In their paper, Dym et al. discuss the purpose of engineering education and engineers' understanding of the importance of design thinking. "Definitions of engineering abound, as do definitions of design"⁸. Design is a major focal point of engineering education and an attempt should be made to define it; additionally, design is an essential and complex cognitive component of engineering. The writers assert that the problem definition phase is the beginning of the design process as practiced by professionals, but educational content does not often teach this. This notion that the engineering strategies and thinking skills taught in schools are not similar to those used by a professional engineer is a common thread in many of the articles on engineering design and problem solving.

Researchers argue that the problems professional engineers face are "substantially" different than those students are given to solve⁹. At times, educators give students problems that are more defined (for example, parameters are already specified) than what they would encounter in professional practice; at other times educators provide students with immediate access to key resources, while practitioners would need to engage in additional work to determine what the right resources are and how to obtain those resources. Furthermore, students are often taught that they will solve engineering problems by exercising linear processes, that is, they ask questions, translate the problem into models, develop an equation to find unknown values and then confirm that the calculated values satisfy the conditions of the problem⁸⁻¹⁰. If students solely learn to problem solve in this manner, they are not allowed the experience that develops the ability to deal with uncertainty and ambiguity, which characterizes real engineering problems (including design and other types of problems)¹¹.

The objective of the study 'What could design learning look like?' is to populate a engineering design expertise continuum across various dimension in order to understand and describe learners' growth in terms of design thinking¹². The researchers believe that by understanding what a continuum of designer expertise looks like, and being able to assess where learners are on the continuum, they can use the findings to inform curriculum and inform assessment efforts. For example, being able to make sense of ambiguity (a trait associated with design expertise) is an example of a behavior which could be assessed at all levels of education.

Engineering Students in Engineering Design

Engineering students do not typically come to an engineering program with knowledge of the engineering design process as it will be taught in their engineering classes, or with skills that are associated with a good design process. For example, research suggests that more experience in engineering design leads to increased time gathering information on the design tasks¹⁰. More time spent gathering information typically leads to increased task completion time and an overall improvement of solution quality. In order to achieve quality in design solutions it takes knowledge, application and time. In the current study, we will look to see if we are able to observe these findings in our comparison of expertise between first-year students and senior

students completing a design task, as well as whether there are similar differences that relate to students' mathematics and design backgrounds (i.e. differences between mathematics majors vs. engineering majors vs. design majors). Similar work has been done previously by Atman and her colleagues, although they did not examine differences between mathematics, engineering and design majors.

Atman and her colleagues compared freshman and senior engineering design processes and identified three major stages: problem scoping, developing alternative solutions and project realizations (decision making and communication)¹³. This study used the verbal protocol analysis (VPA) method in order to gain insight to what the students were thinking as they engaged in the engineering design process. Atman and Turns posit that “the heart of the verbal protocol analysis is the point where the subjects solve the chosen problem while concurrently providing verbal protocols associated with their actions”². This is an appropriate method as it helps to characterize and distinguish how people engage in the engineering design process at varying levels of experience.

To explain the researchers' finding that senior engineering students developed higher quality solutions, they claim that more time spent on a project does not yield a higher quality solution rather it is how designers spend their time which is directly related to the quality of their work¹³. In this study, senior students on average asked for information 25 times compared to the freshmen average of 14.2 requests. Seniors made more transitions or iterations between major design stages than did freshmen¹³. This data show that gathering information is an integral component of the design process. Students should gain experience learning how to gather information. Their ability to gather information can be assessed by quantifying information requests (as done in the study) and by observing how information was used towards the problem solution.

Through extended formal and informal engagement in engineering design, students seem to begin to experience a paradigm shift in thinking about engineering design. Students no longer perceive design as a linear process but as an iterative process. Similar to the critique about mathematical problem solving learning, students are not typically assigned constraint rich engineering design problems to solve. This limits the students' need to spend more time gathering information. It could be due to this practice that students are trained not to ask deeper questions when solving design problems. When a student and an expert are given the same problem, some research suggests that the students ask for more details less often than expert practitioners.¹⁰ A possible explanation for this is that the student might believe that if the problem is to be solved, they must already have all the information they need to provide a solution. This perspective might be developed based on how they are trained to solve problems in previous classes, particularly mathematics classes. A follow-up interview asking the students why they did not request more information would be helpful in understanding this finding.

Problem solving

Downey et al.¹⁴ posit that engineering design and problem solving are one in the same. However, many other researchers draw a distinction between the two. One argument for this is that problem solving does not always yield a design but rather it could be an improved process or way of thinking. Others argue that design is distinct from problem solving (or at least a distinct form of problem solving) because of the emphasis on problem defining – rather than just solving the problem, in design one must identify, understand, scope, and define the problem. Therefore, we discuss problem solving literature as distinct from design in this section of the paper.

In general, problems which are solved by professionals are different than problems which students solve in their formal education. Work is being done, as research and curriculum work cyclically to inform each other, to close that gap. As addressed earlier in this review, most professionals and researchers believe that the engineering problems given to students to solve should more closely resemble those experienced by professional engineers. The students should be comfortable with constraint rich ill-defined problems. They “must develop adequate conceptual frameworks and apply those frameworks in solving complex ill-structured problems”.⁹

Jonassen et al.⁹ engaged in a grounded theory study with the goal of building a case library of engineering stories from the profession. The collection of these stories will allow for the identification of the characteristics of everyday engineering workplace problem which make them constraint rich. This information can be used in the design of more authentic problems for students⁹ which will better prepare the students for workplace engineering problems. Engineers from a professional society were asked to share information about typical problem they solved. From this study twelve themes emerged (see Table 2) which can help define some of the parameters of workplace engineering problems as well as (a) the types of problems we might give students to work on and (b) the different ways that students might frame or treat the design tasks that we give them (i.e. different ways that students might understand and approach the task for the Mathematics as a Gatekeeper to Engineering study presented in Figure 1).

Table 2: Twelve Parameters of Workplace Engineering Problems 9

Workplace problems are ill-structured	Ill-structured problems include aggregates of well-structured problems.	Ill-structured problems have multiples, often conflicting goals
Ill-structured problems are solved in many different ways	Success is rarely measured by engineering standards	Most constraints are non-engineering
Problem solving knowledge is distributed among team members	Most problems require extensive collaboration	Engineers rely on experiential knowledge
Engineering problems often encounter unanticipated problems	Engineers use multiple forms of problem representation	Engineers recommend more communication skills in engineering curricula

In their paper, Jonassen et al.⁹ summarize many approaches to preparing students to be better workplace problem solvers. This work could have been extended by focusing some of its attention on the need for engineers to communicate with other professionals who represent different cultures (i.e. ethnic cultures, business units or areas of specialty). Since literature shows that engineering design and problem solving becomes more team-based as the projects become more complex and students begin to experience a “shared struggle”¹⁴, more development on what collaboration looks like in everyday engineering problem solving, could also have been analyzed more deeply in study.

Mathematical & Engineering Modeling

Children and adults who use math in street vending^{15,16}, basketball¹⁷ carpentry¹⁸, carpet laying¹⁹, design structures²⁰ or engineering model eliciting activities²¹ often develop provisional mathematical models in order to solve problems that are inherent to their activities. Modeling is often used in practice-linked math understanding by children and adults alike. Gainsburg considers mathematical modeling to be a process by which the problem solver 1) translates a real-world or phenomena into mathematical terms, 2) works out the mathematical model and 3) translates the results back into the real world context²⁰. In street vending activities children developed strategies, which can be considered models in order to make profit which were typically different than those that non-selling students used in school¹⁵ to solve the same problem. One can argue that mathematical modeling in engineering is the same as mathematical modeling in mathematical contexts. Modeling in both contexts help the problem solver to visualize or provide more insight into the given problem.

Family Mathematics and Family Engineering

Mathematics and engineering situated within family’s decision making and activities allow for the development of unique mathematical skills for the child. Families serve as effective learning settings for students. Goldman and Booker posit that “parents are the primary, most committed and effective educators of their children”²². It is in this environment students may gain initial exposure to mathematics, engineering and problems solving in out-of-school settings through budgeting, grocery shopping, and family cell phone plan shopping²². Problem solving and trouble-shooting that is situated in the family setting provides mathematically relevant problems that that each member can connect with²². Depending on the structure of the family, the children become participants or witnesses in the trouble-shooting process. Familial mathematical problem solving depends on and allows for the each member to “approach problems with different ways of bringing math to the solution”²³. Successful problem solving as a family is dependent on the various skills that each member of the family possesses.

Goldman and Booker’s research uncovered problems situated in family contexts, such as a case where when a problem arose the family’s response was to use some form of mathematics to get to some optimal solution. In this way the family members identify mathematics as one of the tools they must use to successfully solve the problem. In contrast, the same family problem may be assigned as a homework or in-class problem in in the formal school environment and students struggle to solve the problem²². In addition, scholars agree that mathematics is an embedded practice in our everyday lives and people successfully develop mathematical models within informal contexts such as in candy selling¹⁵ carpet laying¹⁹ and calculating statistics in

basketball¹⁷ but there is still an inability by some to transfer the skills and knowledge learned in these informal settings to classroom settings.

The problems which were solved by the families in the reviewed studies were given attention because the families valued the end goal of problem solving experience⁵. Also, the families rarely considered their problem solving and decision making as mathematical²². Goldman and Booker examined three cases of everyday mathematics and mathematical problem solving situated within the family context²²³. Their ethnographic study focused on the involvement of parents in order to understand “math” at home. Overall, they observed varying levels of differences in how families related to math. The results of this study identified three types of problem solving observed in family settings:

- **Modeling-** talking through the problem solving process in the presence of the child
- **Prompting-** after modeling the parent will ask the child to attempt a solution
- **Distributed problem-** the problem is shared across the family members

The researchers summarized that “families do math regularly, [they] don’t think it as such and that family problem solving is a combination of life practice and school math forms”²². In a similar study by Goldman et al.⁵ the goal was to identify the contexts that created opportunities for mathematics learning in family situations. Their results identified four features of math engaged problem solving in the family.

- Everyday problems lead to math
- Family values guide the problem identification
- Family member engage in “multiple maths” in problem solving
- Math thinking is distributed across the members of the family

From these studies, there is a similar thread which we have coined ‘familial mathematics cultures. Familial mathematics cultures represent the uniqueness of families and how each family may have differing approaches to solving the same problems. This mathematical methods for solving problems could be passed from parent to child, almost like a genetic trait would be.

Both works conclude that family math can be used to compliment school math through pre-algebra. These works, however, focus on the most supportive and almost ideal family math contexts. For example in various communities parents are not “the primary, most committed, and effective educators of their children”and must be supplemented by the teacher. One response to this critique is that “connections among parents, students, and teachers are prominently important.”²⁴

The literature about family engineering is emerging. With the most recent adaptation of the Family Math and Science modules for Engineering, we expect more literature in relevant forthcoming studies. A not -et published work on the role of parents in engineering education provides a detailed review of the literature of parents engagement in engineering and science and frames the literature with four common themes. The parent is viewed as²⁴:

- Engineering career motivator
- Engineering attitudes builder

- Students' achievement stimulus
- Scientific/Engineering thinking guide

Family Engineering was created with the goal of “actively engaging elementary-age youth in exploring engineering activities and career opportunities with their parents”²⁶. This work basically describes the Family Engineering activities and events and their validation. These activities are a formalized way to engage families –parents and children alike- in engineering activities. The activities and their impact were validated through feedback surveys, evaluation questions and field test events. Heil et al.²⁶ reported that in organized family engineering activities 1) attendance at the events positively impacted knowledge and interest in engineering and 2) the program provides proven models engineering outreach for the community leaders. With the study of engineering learning in informal environments gaining interest by schools and families, we expect to see more literature on family engineering over time.

Discussion

Future work in creating the portrait of engineering informal learning in informal environments could focus on how students and teachers can transfer strategies and knowledge learned in informal environments to formal environments. The following questions could be explored: When solving an engineering design problem that is situated within a context outside of the classroom, it would be interesting to observe what types of mathematical and design skills and knowledge the students rely on. Will they defer to skills and knowledge learned in informal environments (i.e. through family math and problem solving) or formal environments (i.e. classroom instruction) or some combination of both? Do students have the mathematics and problem-solving experience to apply their knowledge to diverse “real-world” problems?

With respect to teaching students the design process and problem solving skills, one of the major themes across the literature on engineering, mathematics and science learning is that the tasks and design experiences in many formal settings do not mirror “real-world” design challenges. Many authors posit that more work needs to be done in order for class projects to mirror real-world projects but rarely provide insight on how it can be accomplished. As research continues on how students engage in engineering learning in informal environments, it is expected that similar work will be done on engineering knowledge transfer to the classroom.

Much of the literature reviewed in this paper seems to present the perspective that the purpose of all design courses is to prepare the student for the workplace, when there could be other course aims. Within the P-12 learning space, in order for teachers to incorporate engineering concepts, the lesson must also meet other non-engineering related curriculum standards. So, the goal of incorporating design might be to solely expose the students to a design process, or solely to motivate learning of science and/or mathematics content. If that is the case, then it is quite possible the design process learned during a student's pre-college years may not resemble any aspect of that which will be learned in the college design course. For this reason, it is important to identify some aspects of the design process which must be introduced to students of all levels. From the literature, major aspects of the design process that should be addressed are: iteration, working with constraints, working with some level of uncertainty, and defining problems rather than simply solving problems.

Current Status of the Research Project

Currently, we have run one pilot with a first year participant and second pilot with the fifth year senior participant. We have seen that the first year student heavily focused on using the design process, which he learned in his introduction to engineering course. He mentioned that it was different than what he used in highschool but even during this study he noticed that he was able to modify the prescribed process to meet the needs of his current design task. Initial observations on the student are that his work was heavily focused on the mathematical components of the exercise. The participant also made sure that he was aware of and worked within the constraints. The actual design of the layout seemed secondary to the student as compared to correctly dimensionalizing the playground equipment.

In the second pilot, a senior student, who already earned a non-engineering degree but is now earning a undergraduate degree in mechanical engineering participated. He is currently in the same introduction to engineering class as was the first-year participant. The more senior student spent the first two hours considering constraints, gathering data, asserting assumptions, and brainstorming. During the last hour, the participant focused heavily on meeting the requirements in the design task and performing calculations. As each piece of equipment was accurately dimensionalized and accounted for in the budget, it was carefully drawn on the layout of the playground by the participant.

A few interesting comparisons to be made between the first-year and the senior participant are:

- Both the first-year and senior asked for additional information. The senior typically did not allow the new information to derail his current train of thought. He saved the new information until he needed it.
- The senior student asked for information about the opinions of the community. He considered their opinion in his design decisions. The first-year student did not, although he expressed how important it was to him and to engineering design.
- At some point both students got ‘stuck’. This happened during trigonometric calculations for the senior and when the first-year student considered the ADA design requirement.
- Both students referred to their previous experiences with playgrounds as they participated in this design activity.

From these pilots the research team has increased interest in observing students among cohorts who have varying design training (e.g. first-year students with engineering design service-learning experience vs. first-year students with engineering design service-learning experience) The data from these two pilots—video and audio of VPA and interview questions—are currently being analyzed. Also, the students provided feedback on the structure of the study and the design task. This information along with feedback received from the project’s advisory committee will inform the the final study design to be implemented with our future participants.

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