Economic Simulations for Risk Analysis

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Introduction and Overview

Errors in estimates of cash flows are the rule rather than the exception, so inclusion of risk analysis in an engineering economics course is essential to a student's becoming a practicing engineer. Teaching this topic, however, can be difficult due to the lack of readily available tools. This paper presents a student-friendly software system named Econsim that performs economic simulations. It is a Microsoft Excel 2000™ workbook containing easy-to-use macros and functions that perform simulations and provide reports. Ready-to-run copies of workbooks with examples are freely available from the author at ristroph@louisiana.edu.

Provided below is a concise explanation of simulation concepts and their implementation in Econsim. Specific topics include a general treatment of risk analysis, basic simulation concepts, random numbers and generators, and then Econsim's simulation logic. This is followed by observations regarding the system's use in the classroom. The presentation is suitable for use as student handouts.

Risk Analysis and Simulation

A primary question involving most cash flows in not whether they will be incorrect, but rather by how much will they be incorrect. In routine analyses involving relatively small cash flows, potential errors frequently are ignored, and each cash flow is estimated using an average value known as an *expected value*. The expected value of a randomly varying cash flow C is the sum of each of its possible values c_j multiplied by its probability of occurrence p_j,

$$E(C) = c_1\, p_1 + c_2\, p_2 + \cdots + c_n\, p_n\,, \qquad (1)$$

where the probabilities sum to 1.0. For example, if a cash flow is estimated to be \$30,000 with probability 20% and \$40,000 with probability 80%, then its expected value is:

$$\$38{,}000 = 30{,}000(0.2) + 40{,}000(0.8) \qquad (2)$$

Using expected values for routine decisions works because errors tend to average out. For example, suppose that a company has about 100 small projects of roughly the same size. If each project's actual present worth independently varies by 20% about its estimated value, statistical theory indicates that the actual value of the total present worth of all projects varies by only 2% about the estimated total present worth. In general, if there are N projects each having a percentage variation of PV, then the total percentage variation TPV is:

$$TPV = PV / N^{1/2} \qquad (3)$$

Companies also have a few large projects where there is little opportunity for errors to average out, and the effects of the errors can be damaging. Sensitivity analyses can help determine breakeven points or limits beyond which a project is no longer desirable. Simulation is another helpful tool that can be performed using Excel, as described in the following sections.

Basic Simulation Concepts

Table 1 shows a simple problem involving a common economic measure, the internal rate of return (IRR). The cash flows in cells B4 through B7 (B4:B7) have the IRR of 9.70% shown in B8. The entry in cell B8 is = IRR(B4:B7, 0.1), where B4:B7 is the location of the cash flows and 0.1 is an initial estimate of the IRR.

Table 1. Cash Flows

	A	B
1	**Cash Flow Model**	
2		
3	Year	Cash Flow
4	0	–10,000
5	1	4,000
6	2	4,000
7	3	4,000
8	IRR	9.70%

Now suppose that the estimates of cash flows are thought to be accurate to within 10%. For example, consider year 0 where 10% of 10,000 is 1,000. Any value from -11,000 to -9,000 is equally likely to occur. Similarly, each cash flow for years 1 through 3 can independently vary from 3,600 to 4,400.

Various procedures allow random numbers to be generated on the computer. Econsim contains a subroutine named Uniform that generates equally likely observations on any interval. For example, one set of four randomly generated cash flows might be (-9,531, 4,138, 3,942, 3,897) with an IRR of 12.49%, and another might be (-10,566, 3,768, 4,204, 4,317) with an IRR of 8.20%. Computing IRR's for many sets of random cash flows produces a statistical sample of *observations* of IRR that are summarized by Econsim in the report shown in Figure 2.

The upper left corner shows that the report is based on 2,000 observations of the economic measure (IRR) in row 8, column 2 (cell B8) of Table 1. The IRR's range from 0.27% to 20.65%, with an average of 9.86% and a standard deviation of 3.80%. The table beneath the summary statistics provides additional details. For example, IRR's in the vicinity of 5.93% occur 16% of the time, and IRR's less than 9.33% are observed 46% of the time. These relative frequencies and cumulative relative frequencies are plotted under the table.

Simulation provides insight into the variability of a project's potential performance and hence its risk, so that an informed, albeit subjective, decision can be made. If another 2,000 sets of cash flows should be generated, then the resulting observations of IRR might have a slightly different report from the current one due to randomness of the cash flows. Nonetheless, several thousand observations will produce fairly stable results, whereas reports based on only a few hundred observations might vary considerably.

Random Numbers and Generators

Understanding the basics of random numbers and how the computer generates them makes Econsim easier to use. One way to describe possible observations of a random number is to specify its *density function* $f(x)$. The probability that X is between any two numbers a and b is the integral of $f(x)$ from a to b or the area under its density curve, as shown in Figure 1. An important property of a density function is that it shows the relative likelihood of occurrence of potential observations. For example, if $f(b)$ is twice the value of $f(a)$, then observations in the vicinity of b are twice as likely to occur as those in the vicinity of a. A density function that is spread out over a wide range has a large measure of dispersion known as its *standard deviation*, and compact density functions have small standard deviations.

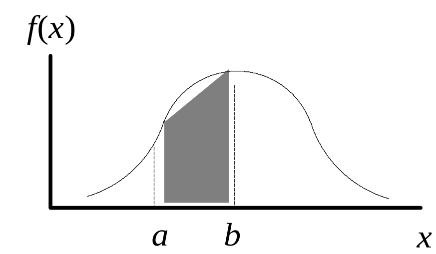

Figure 1. Density Function

Figure 2. Example of Economic Simulation

	A	B	C	D	E	F	G	H	I	J
1	**Economic Simulation**				Min	Avg	Max	StDev		
2	Measure Row:		8		0.27%	9.86%	20.65%	3.80%		
3	Measure Col:		2							
4	Number Obs:		2000							
5					Cell Max	Mid Pt	Rel Frq	Cum Frq		
6					2.54%	1.40%	0.01	0.01		
7					4.80%	3.67%	0.08	0.09		
8					7.07%	5.93%	0.16	0.24		
9					9.33%	8.20%	0.22	0.46		
10					11.60%	10.46%	0.20	0.67		
11					13.86%	12.73%	0.17	0.84		
12					16.13%	14.99%	0.11	0.95		
13					18.39%	17.26%	0.04	0.99		
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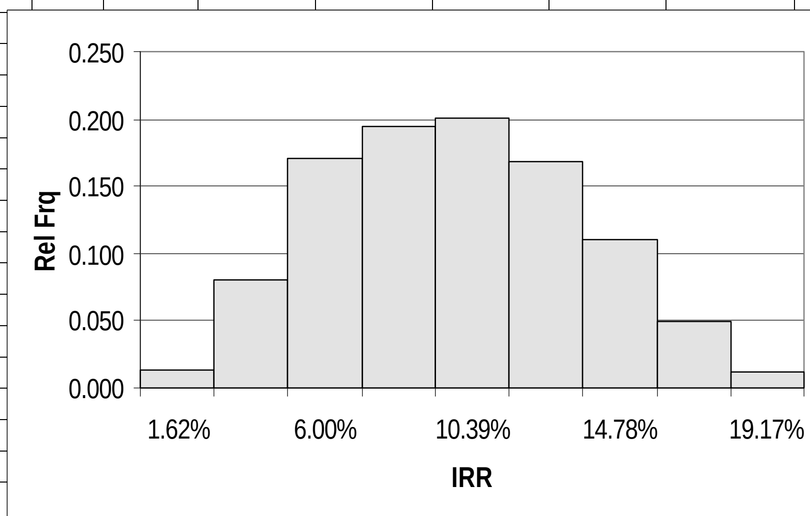

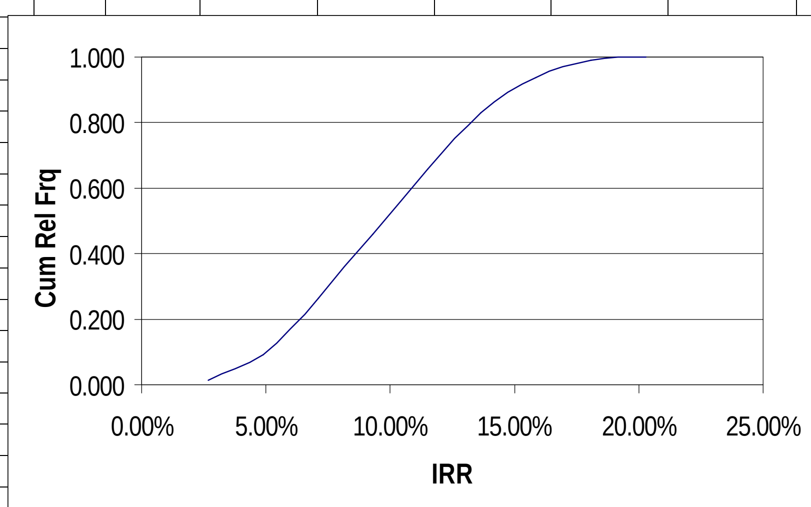

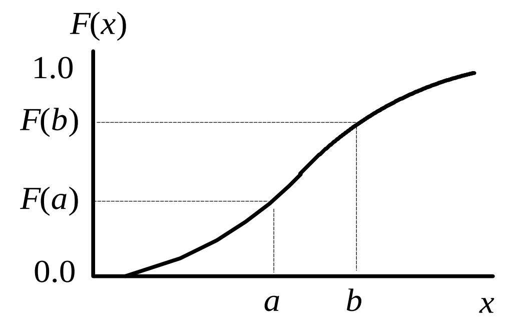

Figure 3. Distribution Function

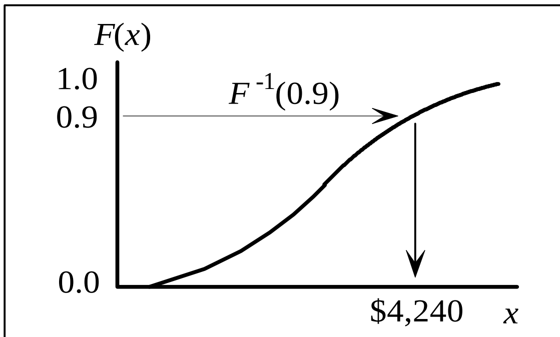

Figure 4. Inverse Distribution Function

The integral of the density function is the *distribution function*, $F(x)$. It equals the probability that a random variate X assumes a value less than or equal to x:

$$F(x) = P(X \le x) \tag{4}$$

In Figure 3, $F(b)$ percent of the time X is less than or equal to b, and $F(a)$ percent of the time X is less than or equal to a, so $F(b) - F(a)$ percent of the time X is between a and b.

Suppose that $F(4{,}240)$ is 0.9, so the probability is 90% that an observation will be less than or equal to $4,240. Then $4,240 is the 90th *percentile* of its statistical distribution. The *inverse distribution function*, $F^{-1}(p)$, computes the p^{th} percentile based on a percentage p between 0% and 100%. For example, Figure 4 shows that $F^{-1}(0.90)$ equals $4,240.

Generating Random Numbers

Computer routines for generating pseudo-random numbers all begin by multiplying fixed point integers in such a way that they overflow memory registers and form products that pass statistical tests for randomness. Two sequences of psuedo-random numbers will be identical if a user-supplied initial integral multiplier known as the *seed* is the same. However, any one sequence *appears* to be random since its values cannot be predicted without knowing the precise details of the generation routine. Scaling the pseudo-random integers produces *equally likely* "random" percentages p between 0 and 1.

The *inverse transform method* of generating random numbers uses random percentages p to compute observations x equal to $F^{-1}(p)$. The percentages are randomly chosen, so the observations are also, with probabilities of occurrence described by the distribution function $F(x)$. For example, consider a uniformly distributed random number for which any observation on some interval (c, d) is equally likely. Its distribution function is

$$F(x) = (x - c) / (d - c) \tag{5}$$

when x is between c and d. Let the percentage p equal $F(x)$ in equation (5) and solve the resulting expression for x to obtain

$$x = c + p(d - c) \tag{6}$$

The expression on the right-hand side of equation (6) is the inverse distribution function $F^{-1}(p)$.

For example, if (c, d) is (200, 300) and a value of p equaling 0.70 is generated, then the observation is 270:

$$270 = 200 + 0.70(300 - 200) \tag{7}$$

All percentages are equally likely, so the probability is 70% that *p* will be less than or equal to 0.70. Hence, the probability that observations will be less than or equal to 270 is 70%, as desired. Also notice that the range of *p* from 0 to 1 implies that the range for *x* is from 200 to 300.

Econsim has functions that use the inverse transform method to generate random numbers from the following distributions: beta, gamma, lognormal, normal, and uniform. For example, entering =Uniform(Pct, Lower, Upper) into a cell returns the Pctth percentile of a uniformly distributed deviate between Lower and Upper. Relative frequency histograms of sample observations from a given density function vary statistically about the shape of the density function. Similarly, cumulative relative frequencies based on samples vary about the distribution function.

Sheet Density displays the functions and their arguments. Clicking on any of the functions provides help information, as does clicking on any of each function's parameters. A macro named Density plots the density functions. A *macro* is any Visual Basic for Applications subroutine without arguments that is stored in a workbook. They are executed by using the menu selections Tools | Macro | Macros to provide a list of all macro names and then double clicking the desired name. Econsim also provides command buttons that can be clicked to execute macros.

Simulation Logic of Econsim

If the cash flows in Table 1 should be known only to within 10%, then the entries in Table 2 would generate the desired random observations. The cash flows for year 0 vary by 10% about a mean cost of –$10,000, and thereafter the flows range 10% about mean revenues of $4,000. This example has cash flows that vary independently of each other, so 4 different random percentages, Pct1 through Pct4, are used to compute the random numbers. Cash flow models of any degree of complexity are allowed, as long as they can fit on the sheet CashFlow.

Table 2. Initial Cash Flow Model

	A	B
1	**Cash Flow Model**	
2		
3	Year	Cash Flow
4	0	=Uniform(Pct1, -11000, -9000)
5	1	=Uniform(Pct2, 3600, 4400)
6	2	=Uniform(Pct3, 3600, 4400)
7	3	=Uniform(Pct4, 3600, 4400)
8	IRR	=IRR(B4:B7, 0.10)

The economic measure in cell B8 can be anywhere on sheet CashFlow, so cells C2 and C3 on sheet Results inform Econsim of its location, as shown in Table 3. Cell C4 contains the number of observations, usually a few thousand. Then running a macro named Simulate computes new values of the random percentages, recalculates the cash flow model, and obtains new observations of the IRR in cell B8. Each new value of B8 is copied onto sheet Results and used to produce the report shown in Figure 2.

Table 3. Data for Macro

	A	B	C
1	**Economic Simulation**		
2	Measure Row:		8
3	Measure Col:		2
4	Number Obs:		2000

Modeling Dependent Cash Flows

Sometimes one or more cash flows depend on other ones. For example, consider the revenues in Table 2 and suppose that revenues in years 2 and 3 are expected to be within $200 of the revenue in year 1. Table 4 shows one way to model this situation. The use of different random percentages in cells B6 and B7 allows those cash flows to be anywhere in the interval centered on B5.

Now suppose that the revenue in year 3 is expected to be within $200 of the revenue in year 2, not year 1. This change is effected by changing the contents of cell B7 to:

= Uniform(Pct4, B6-200, B6+200) (8)

Table 4. Dependent Model 1		
	A	B
1	Cash Flow Model	
2		
3	Year	Cash Flow
4	0	=Uniform(Pct1, -11000, -9000)
5	1	=Uniform(Pct2, 3600, 4400)
6	2	=Uniform(Pct3, B5-200, B5+200)
7	3	=Uniform(Pct4, B5-200, B5+200)
8	IRR	=IRR(B4:B7, 0.10)

Another way that cash flows can be linked is through their percentiles. For example, the revenue for year 1 is the Pct2th percentile of its distribution. Following revenues might be expected to be between (Pct2 – 0.1)th and (Pct2 + 0.1)th percentiles of their distributions, if the range from Pct2 – 0.1 to Pct2 + 0.1 is within the interval 0 to 1. Econsim has functions named PctLower and PctUpper that adjust a range to insure it is between 0 and 1. For example, the range from -0.07 to 0.13 becomes a range from 0 to 0.13, and the range 0.87 to 1.07 becomes 0.87 to 1.

PctLower and PctUpper can be used with any generator. For example, the statement

=Uniform(Pct3, PctLower(Pct2 – 0.1), PctUpper(Pct2 + 0.1)) (9)

returns the Pct3th percentile of a uniformly distributed deviate between *max*(Pct2 – 0.1, 0) and *min*(Pct2 + 0.1, 1). This result is suitable for use as a random percentage, and Econsim assigns such *dependent* random percentages the names Dep1, Dep2, and so forth, to distinguish them from the *independent* random percentages Pct1, Pct2, …

Suppose that expression (9) is named Dep1 and that Dep2 is given by:

=Uniform(Pct4, PctLower(Pct2 – 0.1), PctUpper(Pct2 + 0.1)) (10)

Then Table 5 shows modifications to Table 4 that link the percentiles in years 2 and 3 to the one in year 1. If the year 3 percentile (Dep2) should be dependent on the year 2 percentile (Dep1), instead of the year 1 percentile (Pct2), then define Dep2 as:

Table 5. Dependent Model 2		
	A	B
6	2	=Uniform(Dep1, 3600, 4400)
7	3	=Uniform(Dep2, 3600, 4400)

=Uniform(Pct4, PctLower(Dep1 – 0.1), PctUpper(Dep1 + 0.1)) (11)

No change in Table 5 would be necessary once Dep2 is redefined.

Managing Random Numbers

Values of the random percentages (e.g., Pct4 or Dep1) and other information used by the random number generators are stored on sheet RandNum. Table 6 shows that the seed number for the simulation is input into cell B2. A value of 0 sets the seed equal to the number of seconds after midnight which, in this case, equals 42734 as shown in cell B3. If a positive number is input into B2, then that number is used as the seed number. Inputting the same seed number into B2 allows a simulation to be repeated, if desired. Cells B4 and B5 display the maximum numbers of independent and dependent

Table 6. Generator Information		
	A	B
1	**Random Number Information**	
2	Seed Number:	0
3	Seed Number Used:	42734
4	Max Independent Random %:	4
5	Max Dependent Random %:	2

Table 7. Random Percentages			
	D	E	F
1	**#**	**Pct#**	**Dep#**
2	1	0.3021029	=Uniform(Pct3, PctLower(Pct2 – 0.1), PctUpper(Pct2 + 0.1))
3	2	0.0246418	=Uniform(Pct4, PctLower(Dep1 – 0.1), PctUpper(Dep1 + 0.1))
4	3	0.8882141	XXXXXXXX
5	4	0.1414303	XXXXXXXX

random percentages available for the current simulation. They must be entered using the macro SetupPercent. Table 7 shows the independent random percentages Pct1 through Pct4 in cells E2:E5 and expressions for Dep1 and Dep2 in cells F2:F3.

Classroom Use

Econsim provides a convenient tool for analyzing an important category of real-world problems. Most persons quickly grasp the basic concept of simulation with independent random percentages, but dependent modeling requires more effort. If only an introduction into simulation is desired, then workbooks containing ready-to-run examples can be distributed. Executing such workbooks is as easy as inputting Tools | Macro | Macros | Simulate. Run times are moderate, less than 30 seconds on a 200 MHz machine for 2,000 observations of the models in this paper.

One interesting exercise is to let the seed number equal the seconds after midnight and run the simulation three times with 200 observations, and then three times with 2,000 observations. The results for 200 observations are not nearly as stable as for 2,000 observations. In general, setting the number of observations to a high value reduces inter-run variation. A knowledge of statistics is required to make more definitive statements about model variability, but a few thousand observations should be enough to produce fairly stable results.

Another basic exercise is to provide two workbooks with highly similar examples. One workbook might model cash flows using the uniform distribution, and the other one might use a normal distribution. The objective of this exercise is to see how the choice of cash flow distributions affects the variability of the final results. In general, cash flow distributions with smaller standard deviations produce less variable final results than distributions with larger standard deviations. Thus it can worthwhile to make informed guesses about the cash flow distributions instead of always simply using a uniform distribution.

It is important to recognize that simulation does not provide *the* answer, since each person's reaction to risk is different. This can be illustrated in the classroom by seeing how many are willing to wager a nickel on a flip of a coin. Then steadily increase the stakes to $1, $10, and more. The expected value of each wager is the same, but there usually are progressively fewer takers as the stakes increase. Similarly, companies can be risk tolerant or risk adverse depending upon their perception of the stakes. Homework exercises should include requiring a discussion of whether a simulated project seems worthwhile and *why*.

Developing skill in simulation modeling is a learn-by-doing proposition. Table 8 shows the general steps in using Econsim, and various help messages are displayed automatically as the user moves about a workbook to perform these steps. A good starting point is a simple problem without random variation, perhaps a previously worked example or homework problem. Then

Table 8. Steps in Using Econsim

1. Set the maximum number of independent and dependent random percentages via the macro SetupPercent.
2. Input a seed number on sheet RandNum, or use the number of seconds after midnight.
3. Examine the density functions on sheet Density to identify which ones provide the desired patterns of variation.
4. Input the expressions for any dependent random percentages on sheet RandNum.
5. Create the cash flow model on sheet CashFlow.
6. Enter the location of the economic measure and the number of observations on sheet Results.
7. Run the simulation using the macro Simulate.

progressively add random elements, much as was done in this paper. Finer points of simulation are best understood by experiencing them in a step-by-step manner.

Summary

Simulation is an important tool for economic analyses. Econsim provides a convenient, inexpensive way to teach economic simulation in as little as one lecture hour, as shown in Table 9. More advanced treatments for students comfortable with spreadsheets can use two or three lecture hours to develop progressively more advanced models that include complex after tax cash flows or dependent cash flows. Delightfully curious individuals can consult references in the bibliography to learn more about simulation [1, 2] or about some of the inner workings of Econsim, as presented for an earlier version of the system [3].

Table 9. Lesson Plan for Brief Coverage

1. Display the uniform and normal density functions, and explain how they describe the relative chances of observing different cash flows.
2. Use Figure 3 to present distribution functions.
3. Summarize random number generation by showing how the inverse distribution function in Figure 4 maps a randomly chosen percentage onto its percentile.
4. Observe out that Econsim provides random percentages, and they are assigned the names Pct1, Pct2, and so forth.
5. Explain the simple model shown in Table 2 and its output in Figure 2. Also show a similar model using the normal distribution.
6. Distribute the workbooks containing the simple model via a web site, and assign as homework exercises involving varying the number of observations and comparing the final variability due to different cash flow models.
7. Each student can document his or her learning with a two or three paragraph report having sample printouts of sheet Results appended to it.

Bibliography

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Biography

Dr. John H. Ristroph is a Professor of Engineering and Technology Management and a registered Professional Engineer in Louisiana. His B.S. and M.S. are from LSU, and his Ph.D. is from VPI&SU, all in industrial engineering. He has taught engineering economics and various computer applications for over twenty-six years. The material in this paper is used as a handout in both undergraduate and graduate classes.

Acknowledgement

The author recognizes the many helpful suggestions of Dr. Alan W. Rice that improved the presentation of this material. Any lingering maladies are the responsibility of the author.