Economics of Financed Projects

John H. Ristroph
University of Louisiana at Lafayette

The present worth of a financed project has three distinct components: the project itself, its financing, and the effect on the external funding remaining for other projects. Failure to recognize the last component results in a phenomenon known as leveraging in which the merit of a project is artificially inflated. This paper introduces the basic concepts of financing projects, and then it provides a succinct theoretical framework that allows this commonly occurring topic to be brought into the classroom, complete with examples.

Basics of Financing Projects

A company obtains investment capital from four primary sources: retained earnings, stock, banks, and bonds. Retained earnings are the proceeds from earlier, profitable projects. The remaining sources of funds are external to a company. Issuing stock sells ownership in a firm, and shareholders expect to participate in its profits. Bank loans are a familiar source of capital, and bonds are debt instruments sold directly to the public to obtain lower interest rates than are available from banks.

Shareholders invest because of the prospect of dividends and stock appreciation, and creditors must be repaid. For example, if current external sources collectively require a return of 7% per year, then the average cost of capital is 7%. If a company must pay 8% to obtain additional financing, then its marginal cost of capital (MCC) is 8%.

Factors other than the cost of capital can limit the use of external funding. One consideration is that financiers gain differing degrees of control over a company. Another is the need to find, train, and retain competent employees to staff financed projects.

Economics

Companies use external funding because it is profitable to do so as long as the MCC is less than the average marginal rate of return[1] (AMRR). For example, consider a company that plans to expand its current external funding by \$100,000 to finance several new projects. Its MCC is 8% over a 5-year period, so the yearly payments shown in Figure 1 are,

$$\$25{,}046 = 100{,}000\,(A|P, 8\%, 5)\,, \qquad (1)$$

The funding flows affect marginal projects with an AMRR of 15%, so the present worth (PW) of the funding is:

$$\$16{,}043 = 100{,}000 - 25{,}046\,(P|A, 15\%, 5) \qquad (2)$$

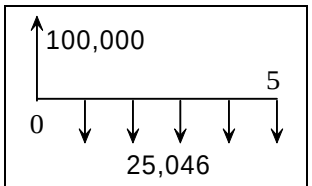


Figure 1. Funding for Additional Projects

[1] This is the average internal rate of return offered by marginal projects that are accepted when funding expands or rejected when funding contracts. The AMRR is used to model capital growth and to compute measures such as present worth [1]. Some textbooks refer to it as the minimum attractive rate of return.

Using external funding increases the company's total assets after 5 years by the future worth of the cash flows,

$$\$32{,}268 = 16{,}043\,(F|P,\ 15\%,\ 5)\ . \tag{3}$$

In general, consider the cash flows for the marginal funding shown in Figure 2. The amount financed b_0 is a cash inflow that is positive, and the payments $b_{j \geq 1}$ are outflows that are negative. The amount borrowed is repaid at the MCC f, so

$$0 = b_0 + b_1(1+f)^{-1} + \cdots + b_n(1+f)^{-n}\ . \tag{4}$$

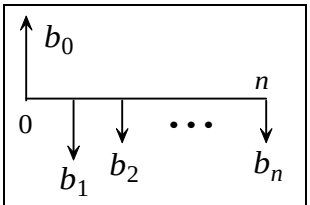


Figure 2. Marginal Funding

The signs of the cash flows cause all derivatives of the right-hand-side of equation (4) to be positive, so it is concave as shown in Figure 3, where the PW of the marginal financing is positive for values of the AMRR m greater than f:

$$PW_{MF} = b_0 + b_1(1+m)^{-1} + \cdots + b_n(1+m)^{-n} > 0\,, \quad m > f. \tag{5}$$

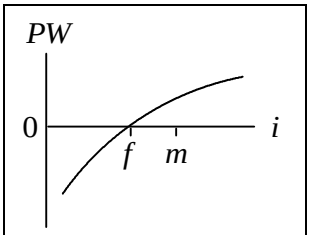


Figure 3. PW Curve

Viewpoint Problem

Financing projects can be economically beneficial, but it complicates analyses due to a phenomena known as leveraging. This section explains this problem and its resolution.

Leveraging

A leveraged project is one whose measures of merit, such as PW, are improved through financing. For example, Figure 4 shows the cash flows of project A before financing, project A's financing, project A after financing, and an alternative investment, project B. Project B's designers did not consider financing, so only the unfinanced project is shown.

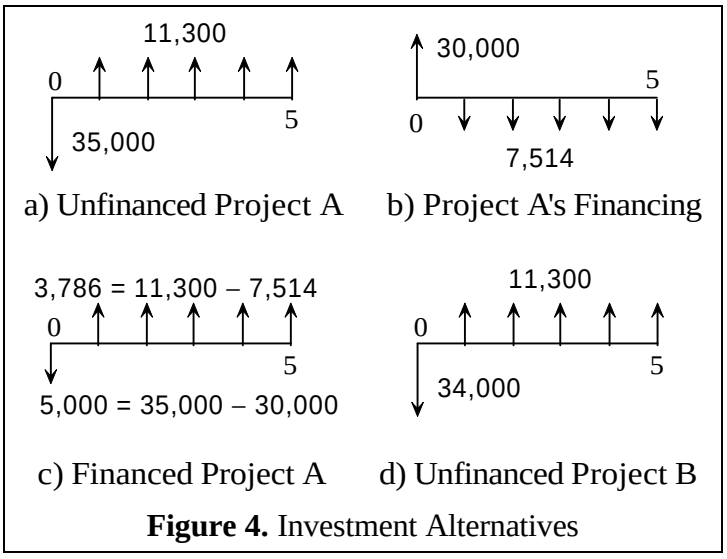


Figure 4. Investment Alternatives

If the AMRR is 15%, then the PW of project A before financing is

$$PW_A = \$2{,}879 = -35{,}000 + 11{,}300\,(P|A,\ 15\%,\ 5)\ . \tag{6}$$

Its internal rate of return, IRR_A, is 18.4%. The PW of the financing for project A is

$$PW_{FA} = \$4{,}813 = 30{,}000 - 7{,}514\,(P|A,\ 15\%,\ 5)\ , \tag{7}$$

and the PW of the financed project is

$$PW_{A\&FA} = \$7{,}691 = -5{,}000 + 3{,}786\,(P|A,\ 10\%,\ 5)\ . \tag{8}$$

The IRR of financed project, $IRR_{A\&FA}$, is 70.5%.

The measures of merit of the financed project are better than those of the unfinanced project. This leveraging occurs because the project's financing has a positive PW, so its cash flows

enhance those of the unfinanced project.

Now consider project B. It costs $1,000 less than project A and has the same returns, so its PW is $1,000 more than the PW of project A before considering financing:

$$PW_B = \$3{,}879 = -34{,}000 + 11{,}300\,(P|A, 15\%, 5) \tag{9}$$

The financed version of project A with $PW_{A\&FA}$ equal to $7,691 has a larger PW, but it consumes more of the available financing than project B. Which one should be selected?

System Viewpoint

Solving this problem requires using a system viewpoint that considers all impacts of a project, including the effect of its financing on the external funding that remains available for other projects. The use of external capital is limited by increases in its MCC and by non-financial considerations. Suppose the funding limit is $100,000, with yearly payments of $25,046 as shown in Figure 1, and that financing project A results in the remaining external funding shown in Figure 5.

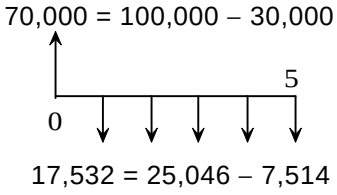


Figure 5. Remaining Funding After Financing A

The PW of the remaining funding given the selection of project A and its financing is

$$PW_{RF|A\&FA} = \$11{,}231 = 70{,}000 - 17{,}532\,(P|A, 15\%, 5)\,. \tag{10}$$

The system-level PW for financing project A is

$$PW_{System,A} = PW_{A\&FA} + PW_{RF|A\&FA}\,, \tag{11}$$

so

$$PW_{System,A} = \$18{,}922 = 7{,}691 + 11{,}231 \tag{12}$$

The PW of the marginal financing in Figure 1 before selecting any projects is $16,043. Suppose that this is not affected if the unfinanced alternative B is chosen. The system-level PW of B is its PW plus the PW of its remaining funding or

$$PW_{System,B} = \$19{,}922 = 3{,}879 + 16{,}043\,. \tag{13}$$

The system-level present worth of project B is larger than that of project A, so project B should be chosen.

General Financing

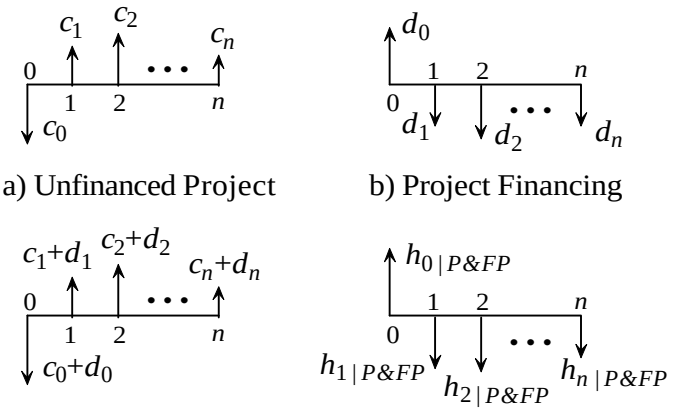
This section extends the foregoing example by generalizing the equations for computing system-level PWs with and without financing. Consider project P having the cash flows shown in Figure 6. The cash flow for year j before financing is c_j with a financing flow of d_j, so the cash flow of the project and its financing is $c_j + d_j$. Each cash flow of the remaining funding given the selection of the project P and its financing is $h_{j|P\&FP}$. The system-level cash flows of project P and its financing are

$$s_{j|P\&FP} = c_j + d_j + h_{j|P\&FP}\,. \tag{14}$$

so the system-level present worth of project P and its financing is

$$PW_{System,P\&FP} = s_{0|P\&FP} + s_{1|P\&FP}(1+m)^{-1} + \cdots + s_{n|P\&FP}(1+m)^{-n}\,. \tag{15}$$

Sometimes it is convenient to compute the system-level PW based on its components, as in the previous section. The PW of the project before financing is



a) Unfinanced Project b) Project Financing

c) Financed Project d) Remaining Funding

Figure 6. General Financing Cash Flows

$$PW_P = c_0 + c_1(1+m)^{-1} + \cdots + c_n(1+m)^{-n}\,, \tag{16}$$

and the PW of the project's financing is

$$PW_{FP} = d_0 + d_1(1+m)^{-1} + \cdots + d_n(1+m)^{-n}\,. \tag{17}$$

The PW of the project and its financing can be computed using either

$$PW_{P\&FP} = c_0 + d_0 + (c_1 + d_1)(1+m)^{-1} + \cdots + (c_n + d_n)(1+m)^{-n} \tag{18}$$

or

$$PW_{P\&FP} = PW_P + PW_{FP}\,. \tag{19}$$

The PW of the remaining financing given the selection of project P and its financing is

$$PW_{RF\,|\,P\&FP} = h_{0\,|\,P\&FP} + h_{1\,|\,P\&FP}(1+m)^{-1} + \cdots + h_{n\,|\,P\&FP}(1+m)^{-n}\,, \tag{20}$$

so the system-level present worth of the project P and its financing can be written as

$$PW_{System,\,P\&FP} = PW_{P\&FP} + PW_{RF\,|\,P\&FA}\,. \tag{21}$$

Now consider a project without financing. Proceeding as before, let the cash flow for year j without financing be c_j. Each cash flow of the financing remaining given the selection of project P without financing is $h_{j\,|\,P}$. The system-level cash flows of project P without financing are

$$s_{j\,|\,P} = c_j + h_{j\,|\,P}\,, \tag{22}$$

so its system-level PW is

$$PW_{System,\,P} = s_{0\,|\,P} + s_{1\,|\,P}(1+m)^{-1} + \cdots + s_{n\,|\,P}(1+m)^{-n}\,. \tag{23}$$

One component of this system-level PW is the PW without financing, PW_P as given by equation (16). The other component is the PW of the remaining funding given the selection of project P without funding,

$$PW_{RF\,|\,P} = h_{0\,|\,P} + h_{1\,|\,P}(1+m)^{-1} + \cdots + h_{n\,|\,P}(1+m)^{-n}\,. \tag{24}$$

This allows writing the system-level present worth of the project P without financing as

$$PW_{System,\,P} = PW_P + PW_{RF\,|\,P}\,. \tag{25}$$

The following example illustrates comparing projects with general financing.

Example of General Financing

Consider a situation in which the initially available marginal funding is shown in Figure 1. A choice must be made between projects A and C, where project A remains financed as shown in Figure 4 and Figure 5. Project C is not financed, and Figure 7 shows its cash flows. It uses a new technology that financiers believe to be risky. If the company implements it, then financiers will make only \$90,000 available for other projects and increase the MCC from its current level of 8% to 9% over the 5 year repayment period, resulting in annual payments of

$$\$23{,}138 = 90{,}000\,(A|P,\ 9\%,\ 5)\ , \tag{26}$$

as shown in Figure 7. Project C's present worth without financing is

$$PW_C = \$6{,}874 = -30{,}000 + 11{,}000\,(P|A,\ 15\%,\ 5)\ . \tag{27}$$

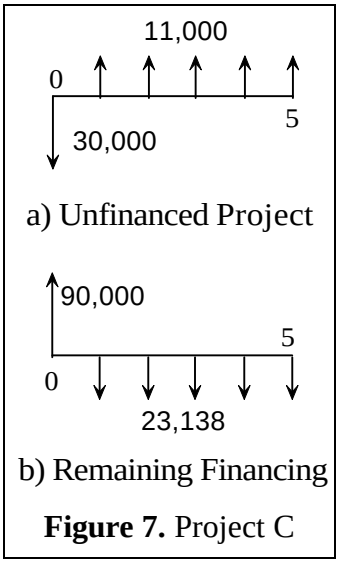


Figure 7. Project C

The PW of its remaining funding is

$$PW_{RF\,|\,C} = \$12{,}437 = 90{,}000 - 23{,}138\,(P|A,\ 15\%,\ 5)\ , \tag{28}$$

so its system-level PW is

$$PW_{System,C} = \$19{,}311 = 6{,}874 + 12{,}437\,. \tag{29}$$

The system-level present worth of project A remains \$18,922, so project C is preferred in spite of its impact on the financing remaining for other projects.

Additive Financing

This section examines an important special case of the general financing model, additive financing. A set of projects has additive financing if the cash flows of each project's financing plus the financing remaining for other projects equals the external funding prior to any project's selection:

$$\textit{Project Financing} + \textit{Remaining Financing} = \textit{Funding Before Selection}\ . \tag{30}$$

The importance of the additive model is that it only requires computing PWs before financing to identify the best project.

Suppose that Figure 2 shows the marginal financing available for new projects before any selections are made from a set with additive financing. Figure 6 shows the cash flows of any project from that set before financing, the project's financing, the financed project, and the funding remaining for other projects after financing the project. Rearranging equation (30) indicates that financing is additive only if

$$h_{j\,|\,P\&FP} = b_j - d_j\ ,\quad j = 1,\ \ldots\ ,\ n\,, \tag{31}$$

for all financed projects in the set. Similarly, financing is additive for projects without financing only if

$$h_{j\,|\,P} = b_j\,,\quad j = 1,\ \ldots\ ,\ n\,. \tag{32}$$

If a set of projects is fairly homogeneous with respect to size and risk, then additive financing seems reasonable. Additive financing also can occur if external funding is acquired in a manner such that its cash flows are stable regardless of which projects are selected, such as an unrestricted bond sale. Conversely, the general model presented in the preceding section should be used for situations where projects are heterogeneous with respect to size and risk and selections can affect external funding.

Present Worth Under Additive Financing

The reason why financing can be ignored for the additive model is that the system-level cash flows are the same whether a project is financed or not. The system-level cash flow has components from the project, any project financing, and the remaining funding given by equations (31) or (32). Under additive financing, the system-level cash flows of a financed project are

$$s_{j|P\&FP} = c_j + d_j + b_j - d_j. \tag{33}$$

Note that they equal the system-level cash flow of project without financing,

$$s_{j|P} = c_j + b_j\,. \tag{34}$$

In either case, the system-level PW of some project P is

$$PW_{System,P} = c_0 + b_0 + (c_1 + b_1)(1+m)^{-1} + \cdots + (c_n + b_n)(1+m)^{-n}\,. \tag{35}$$

The terms involving c_j equal PW_P, the PW of project P before financing; and the other terms equal PW_{MF}, the PW of the marginal financing before selecting any alternatives. Thus the system-level PW of any project from an additive set is

$$PW_{System,P} = PW_P + PW_{MF}, \tag{36}$$

regardless of financing. The term PW_{MF} is common to all projects, so it can be ignored when selecting projects. Thus selections from a set of projects with additive financing can be based solely on the PWs of the projects before financing, PW_P .

Example of Additive Financing

Consider projects A and B shown in Figure 4 and Figure 5. Note that the financing for project A is additive. Project B is not financed and its selection does not affect the originally available financing, so it is also additive. Thus the set consisting of projects A and B is additively financed, and they can be compared based on their PWs before financing.

The previously calculated values of PW_A and PW_B are \$2,879 and \$3,879, respectively, so project B is preferred, as before. Note that PW_B is \$1,000 larger than PW_A. The previously calculated value of $PW_{System,\,B}$ (\$19,922) is also \$1,000 larger than $PW_{System,\,A}$ (\$18,922), since both system-level PWs differ from the PWs before financing by PW_{MF}.

Summary and Conclusions

Financing projects is a common industrial practice because it is profitable as long as the MCC is less than the AMRR. This paper seeks to provide a succinct method for bringing financing into the classroom. The starting point of such analyses is to realize that the amount of funding available to a company is limited. Then a system-level viewpoint considers the productivity of a project before funding, the project's funding, and its impact on the funding remaining for other projects. Projects are selected on the basis of their system-level PWs.

An important special case occurs when the financing for a set of projects is additive. Under this condition, financing can be ignored, and selections are made among projects based on their PWs before financing. Additive financing is a reasonable assumption for projects having similar size and risk characteristics or projects using external funding with cash flows not affected by which projects are selected.

Bibliography

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Biography

Dr. John H. Ristroph is a Professor of Engineering and Technology Management and a registered Professional Engineer in Louisiana. His B.S. and M.S. are from LSU, and his Ph.D. is from VPI&SU, all in industrial engineering. He has taught engineering economics for over thirty years, and enjoys bringing his research into the classroom. His practical experience includes serving as Director of Policy and Planning for the Louisiana Department of Natural Resources where he performed numerous large-scale economic analyses.