# Effective Teaching of Determining Beam Deflections Using Moment-Area Theorems: Avoiding a Pitfall

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## Abstract

Several established methods for determining deflections of beams may be found in textbooks on mechanics of materials. The *method using moment-area theorems* is preferred by many engineers because it has the inherent advantage of graphical visualization in solving problems. This method employs two moment-area theorems, which are derived by integrating well-behaved functions in domains along beams. Unfortunately, most textbooks do not provide explicit warning that one **cannot** apply any moment-area theorem directly to the **entire** beam when the beam under loading has a *discontinuity* in its *slope*, such as that in a hinge-connected beam. This is a pitfall for unsuspecting beginners, who often reach erroneous results and are puzzled by them. This paper contributes definitive concepts and detailed explanations to expel ambiguities often encountered by students in applying the method to solve problems. The examples illustrate steps for use in effective teaching of the method and help beginners avoid tumbling into pitfalls. It is aimed at contributing to the better teaching and learning of mechanics of materials.

## I. Introduction

There are several established methods for determining deflections of beams in mechanics of materials. They include the following:[1-12] *(a)* method of double integration (*with* or *without* the use of singularity functions), *(b)* method of superposition, *(c)* method using moment-area theorems, *(d)* method using Castigliano's theorem, *(e)* conjugate beam method, and *(f)* method using model formulas. Naturally, there are advantages and disadvantages in using any of the above methods.

Many engineers favor to employ the *method using moment-area theorems* because it has the built-in advantage of graphical visualization during the drawing of diagrams of elastic weights (i.e., the bending moment divided by the flexural rigidity of the beam), as well as the drawing of tangential deviations associated with the deflected beams, in solving problems. There are two moment-area theorems. Both of them are derived by integrating well-behaved functions in domains along the beams. Unfortunately, most textbooks do not provide explicit warning that one **cannot** apply any moment-area theorem directly to the **entire** beam when the beam under loading has a *discontinuity* in its *slope*. If a beam is composed of two or more segments that are connected by hinges, then the beam has discontinuity in slope at the hinge connections when loads are applied. In such a case, the deflections must be analyzed by dividing the beam into segments, each of which must have *no* discontinuity in slope. Otherwise, erroneous results will be reached.

In this paper, attention is focused on the *method using moment-area theorems*. A **working knowledge** of sign conventions and some key terms, besides the two *moment-area theorems*, is a **prerequisite** for readers of this paper. For the benefit of a wider readership, a refresher on sign conventions, definitions of key terms, and the two theorems is included in this paper. Readers, who are familiar with the rudiments of this method, may *skip* the *refresher* in Sect. II of this paper.

## II. Sign conventions, key terms, and moment-area theorems

■ Positive directions of shear forces, moments, and applied loads.

In the analysis of beams, it is important to adhere to the adopted positive and negative signs for loads, shear forces, bending moments, slopes, and deflections of beams. The free-body diagram for a beam *AB* carrying loads is shown in Fig. 1. The positive directions of shear forces $\mathbf{V}_A$ and $\mathbf{V}_B$, moments $\mathbf{M}_A$ and $\mathbf{M}_B$, at ends *A* and *B* of the beam, the concentrated force **P** and concentrated moment **K**, as well as the distributed loads, are illustrated in this figure.

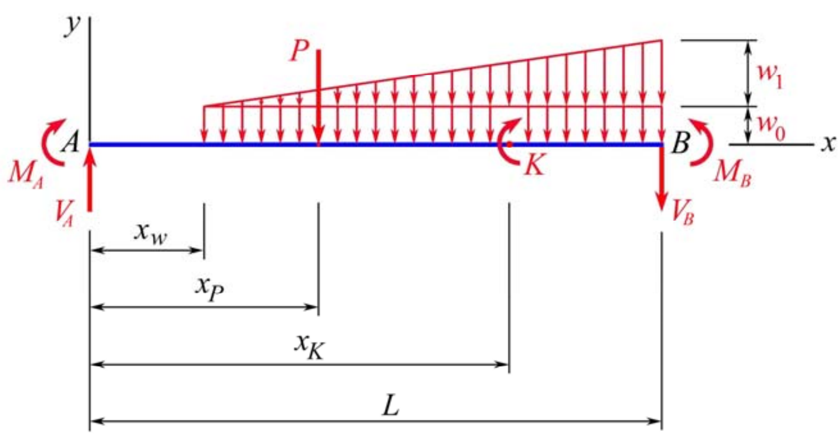


**Fig. 1.** Positive directions of shear forces, moments, and applied loads

In general, the sign conventions for shear forces, moments, and applied loads are as follows:

- A *shear force* is *positive* if it acts upward on the left (or downward on the right) face of the beam element (e.g., $\mathbf{V}_A$ at the left end *A*, and $\mathbf{V}_B$ at the right end *B* in Fig. 1).
- At ends of the beam, a *moment* is *positive* if it tends to cause compression in the top fiber of the beam (e.g., $\mathbf{M}_A$ at the left end *A*, and $\mathbf{M}_B$ at the right end *B* in Fig. 1).
- Not at ends of the beam, a *moment* is *positive* if it tends to cause compression in the top fiber of the beam just to the right of the position where it acts (e.g., the concentrated moment **K** in Fig. 1).
- A *concentrated force* or a *distributed force* applied to the beam is *positive* if it is directed downward (e.g., the concentrated force **P**, the uniformly distributed force with intensity $w_0$, and the linearly varying distributed force with highest intensity $w_1$ in Fig. 1).

## ■ Positive directions of slope and deflection.

The sign conventions for positive slope and positive deflection of a beam *AB* are shown in Fig. 2. The **slopes** $\theta_A$ and $\theta_B$ of the beam at points *A* and *B* are small angles (in radians) between the horizontal and the tangents drawn at *A* and *B*, respectively. A **slope** at a point of a beam is *positive* if the small angle is measured *counterclockwise* from the horizontal to the tangent drawn at that point of the beam. In general, we have the following:

- A *positive slope* is a counterclockwise angular displacement, such as $\theta_A$ and $\theta_B$ in Fig. 2.
- A *positive deflection* is an upward displacement, such as $y_A$ and $y_B$ in Fig. 2.

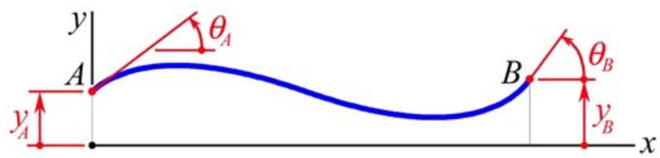


**Fig. 2.** Positive slopes and positive deflections of beam *AB*

## ■ Beam, elastic weight, elastic curve, change in slope, and tangential deviation.

In mechanics of materials, a **beam** is a horizontal structural member subjected to transverse loads, which act perpendicular to the axis of the beam. The **elastic weight** at a position on a beam is the bending moment *M* acting in the beam cross section divided by the flexural rigidity *EI* of the beam cross section, where *E* is the modulus of elasticity of the beam and *I* is the moment of inertia of the cross section of the beam about its centroidal axis of bending. Thus, the *elastic weight* on a beam is simply represented by the diagram showing $M/EI$ for the beam. As shown in Fig. 3, the **elastic curve** of a beam is the curve showing the deflected shape of the centerline of the beam under loading. In the study of slopes and deflections of beams, an *elastic curve* refers to a *deflected beam*, and these two terms are interchangeable.

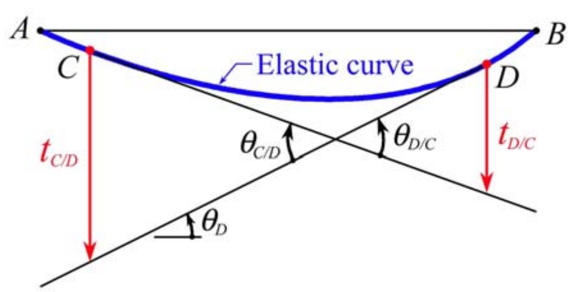


**Fig. 3.** Elastic curve, change in slope, and tangential deviations

Suppose that *C* and *D* are two points on a beam, as illustrated in Fig. 3, where *D* is to the right of *C*. The **change in slope** $\theta_{D/C}$ from *C* to *D* in the deflected beam is the *angular displacement* measured counterclockwise from the tangent drawn at *C* of the deflected beam to the tangent drawn at *D* of the deflected beam. Note that

$$\theta_{D/C} = \theta_D - \theta_C \tag{1}$$

$$\theta_{C/D} = \theta_C - \theta_D = -\,\theta_{D/C} \tag{2}$$

The **tangential deviation** $t_{D/C}$ of *D* with respect to *C* is the *vertical linear displacement* drawn from *D* of the deflected beam to the tangent drawn at *C* of the deflected beam, as shown in Fig. 3. Note carefully that the **tangential deviation** $t_{C/D}$ of *C* with respect to *D* is, on the other hand, the *vertical linear displacement* drawn from *C* of the deflected beam to the tangent drawn at *D* of the deflected beam.

■ Moment-area theorems.

The *method using moment-area theorems* for solving problems involving deflections of beams may be found in most textbooks on mechanics of materials.[1-8] This method is based on two major theorems, which are derived by integrating well-behaved functions. These two theorems may be stated as follows:

- **First moment-area theorem***:* If *C* and *D* are two points on a beam and *D* is to the right of *C*, then the change in slope $\theta_{D/C}$ from *C* to *D* of the deflected beam is equal to the elastic weight from *C* to *D* on the beam. We write

$$\theta_{D/C} = A_{CD} \tag{3}$$

where $A_{CD}$ denotes the *elastic weight* (i.e., area of the diagram showing *M/EI* for the beam) between *C* and *D* on the beam. Note that $A_{CD}$ (and hence $\theta_{D/C}$) is positive if the resulting area of the diagram showing *M/EI* for the beam between *C* and *D* on the beam is positive.

- **Second moment-area theorem***:* For *C* and *D* being two points on a beam, the tangential deviation $t_{D/C}$ of *D* with respect to *C* is equal to the first moment about *D* of the elastic weight between *C* and *D* on the beam. We write

$$t_{D/C} = (M_D)_{CD} \tag{4}$$

where $(M_D)_{CD}$ denotes the first moment about *D* of the elastic weight between *C* and *D* on the beam. To compute the tangential deviation $t_{C/D}$, we similarly write

$$t_{C/D} = (M_C)_{CD} \tag{5}$$

where $(M_C)_{CD}$ denotes the first moment about *C* of the elastic weight between *C* and *D* on the beam.

Note that many beginners encounter ambiguity or difficulty in computing $(M_D)_{CD}$ in Eq. (4) or $(M_C)_{CD}$ in Eq. (5). To overcome such ambiguity or difficulty, note the following:

- ▸ The elastic weight (i.e., *M/EI*) acts upward or downward when the bending moment *M* is positive or negative, respectively.
- ▸ The first moment $(M_D)_{CD}$ in Eq. (4) is equal to horizontal distance from *D* to the centroid of the area for the elastic weight between *C* and *D* multiplied by the area for the elastic weight between *C* and *D*.
- ▸ Contribution of first moment about point *D* by the elastic weight to the value of $(M_D)_{CD}$ in Eq. (4) is positive if the **direction** of such contributed moment about point *D* is **consistent** with the direction of moment about point *C* of an imaginary vertical force acting at point *D* and pointing in the same sense of direction as the sketched tangential deviation $t_{D/C}$.

➤ If the computed value of $(M_D)_{CD}$ in Eq. (4) is positive, then the actual direction of linear displacement for the tangential deviation $t_{D/C}$ as sketched is correct. Otherwise, the actual direction of $t_{D/C}$ is opposite to that assumed in the sketch.

➤ The first moment $(M_C)_{CD}$ in Eq. (5) is equal to horizontal distance from $C$ to the centroid of the area for the elastic weight between $C$ and $D$ multiplied by the area for the elastic weight between $C$ and $D$.

➤ Contribution of first moment about point $C$ by the elastic weight to the value of $(M_C)_{CD}$ in Eq. (5) is positive if the **direction** of such contributed moment about point $C$ is **consistent** with the direction of moment about point $D$ of an imaginary vertical force acting at point $C$ and pointing in the same sense of direction as the sketched tangential deviation $t_{C/D}$ .

➤ If the computed value of $(M_C)_{CD}$ in Eq. (5) is positive, then the actual direction of linear displacement for the tangential deviation $t_{C/D}$ as sketched is correct. Otherwise, the actual direction of $t_{C/D}$ is opposite to that assumed in the sketch.

■ A simple illustration of the method using moment-area theorems.

Example 1. By the *method using moment-area theorems*, determine the slope $\theta_A$ and deflection $y_A$ of the free end $A$ of a cantilever beam $AB$ with length $L$ and constant flexural rigidity $EI$, which is acted on by a concentrated force **P** at its free end $A$, as shown in Fig. 4.

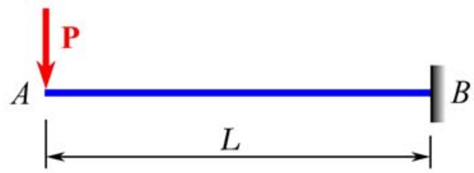


**Fig. 4.** Cantilever beam *AB*

Solution: In solving the problem in this example by the *method using moment-area theorems*, we first draw the diagram for the **elastic weight** (i.e., $M/EI$) on the beam $AB$, as shown in Fig. 5. Note that the bending moment $M$ in the beam is negative because it causes the top fiber of the beam in compression. Thus, the elastic weight acts downward on the beam in this figure, where the **resultant elastic weight** $A_{AB}$ acts through the *centroid* of the triangular area and its value is equal to the triangular area; i.e.,

$$A_{AB} = -\frac{1}{2} \cdot L \cdot \frac{PL}{EI} = -\frac{PL^2}{2EI}$$

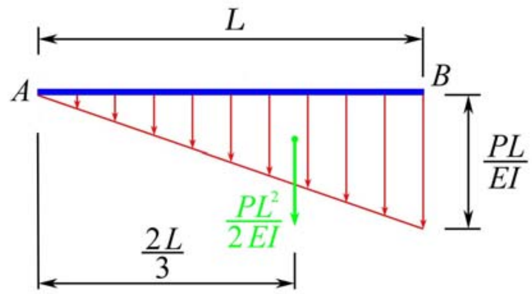


**Fig. 5.** Elastic weight on the beam *AB*

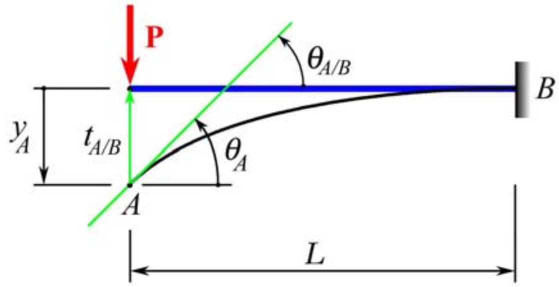


**Fig. 6.** Elastic curve of the beam under loading

Next, we sketch the elastic curve of the beam under loading as shown in Fig. 6, where the tangent drawn at *B* is a horizontal line because the beam is fixed at *B*. Therefore, the **slope** $\theta_A$ at the free end *A* becomes equal to the **change in slope** $\theta_{A/B}$. Since point *B* is to the right of point *A* in Fig. 6, we apply the **first moment-area theorem** to write

$$\theta_{B/A} = A_{AB} = -\frac{PL^2}{2EI}$$

$$\theta_A = \theta_{A/B} = -\theta_{B/A} = -\left(-\frac{PL^2}{2EI}\right)$$

$$\theta_A = \frac{PL^2}{2EI} \circlearrowleft$$

The tangential deviation $t_{A/B}$ of *A* with respect to *B* is directed upward from *A* to the tangent drawn at *B*, as shown in Fig. 6. Furthermore, we note that the direction of moment about point *B* of an imaginary vertical force acting at point *A* and pointing in the same upward direction as the sketched tangential deviation $t_{A/B}$ is **clockwise**. Therefore, contribution of first moment about point *A* by the elastic weight $A_{AB}$ to the value of $(M_A)_{AB}$ is positive if the **direction** of such contributed moment about point *A* is also **clockwise**. In this case, *both are clockwise*! Referring to Figs. 5 and 6 and applying the **second moment-area theorem**, we write

$$t_{A/B} = (M_A)_{AB} = \frac{2L}{3} \cdot \frac{PL^2}{2EI} = \frac{PL^3}{3EI}$$

$$y_A = -t_{A/B} = -\frac{PL^3}{3EI} \qquad y_A = \frac{PL^3}{3EI} \downarrow$$

### III. Analysis of a hinge-connected beam: a pitfall to avoid

The two moment-area theorems presented in Sect. II are derived in textbooks on mechanics of materials [1-8] by integrating well-behaved functions, whose domain lie along the beam. Nevertheless, most textbooks do not provide explicit warning that one **cannot** apply any moment-area theorem directly to the **entire** beam when the beam under loading has a *discontinuity* in its *slope*, such as that in a hinge-connected beam. In such a case, the deflections must be analyzed by dividing the beam into segments, each of which must have *no* discontinuity in slope. Otherwise, erroneous results will be reached. A hinge-connected beam is a pitfall for unsuspecting beginners, who often reach erroneous results and are puzzled by them.

Example 2. A beam *AE* with a hinge connector at *C* carries a concentrated force **P** at *D* and is supported as shown in Fig. 7, where the segments *AC* and *CE* have the same flexural rigidity *EI*. An unsuspecting beginner, who tries to apply the *method using moment-area theorems*, arrived at a set of wrong answers for *(a)* the reaction moment $\mathbf{M}_A$ and the vertical reaction force $\mathbf{A}_y$ at *A*, and *(b)* the vertical reaction force $\mathbf{B}_y$ at *B*. What may be the likely **improper way** taken by this person?

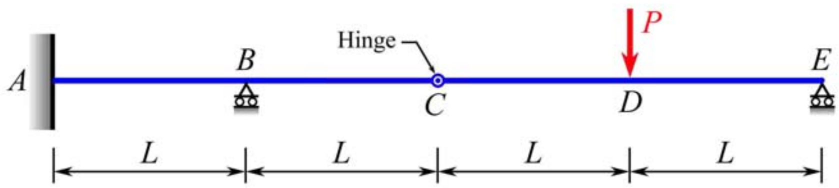


**Fig. 7.** Hinge-connected beam *AE* with a fixed end and two simple supports

Solution – improper way: Let us assume that this person has drawn a *correct* free-body diagram of the beam, as shown in Fig. 8, in the beginning of the solution.

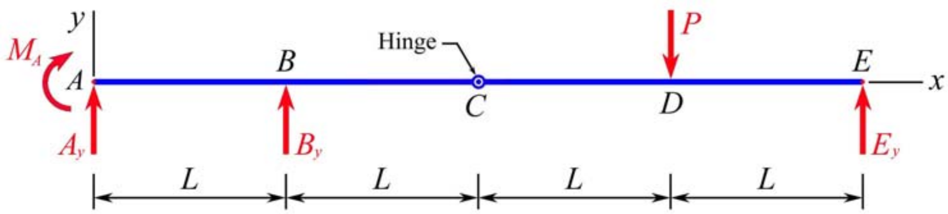


**Fig. 8.** Free-body diagram for the hinge-connected beam *AE*

Next, let us also assume that, based on Fig. 8, this person has drawn a *correct* diagram, by parts, for the elastic weight on the beam, as shown in Fig. 9.

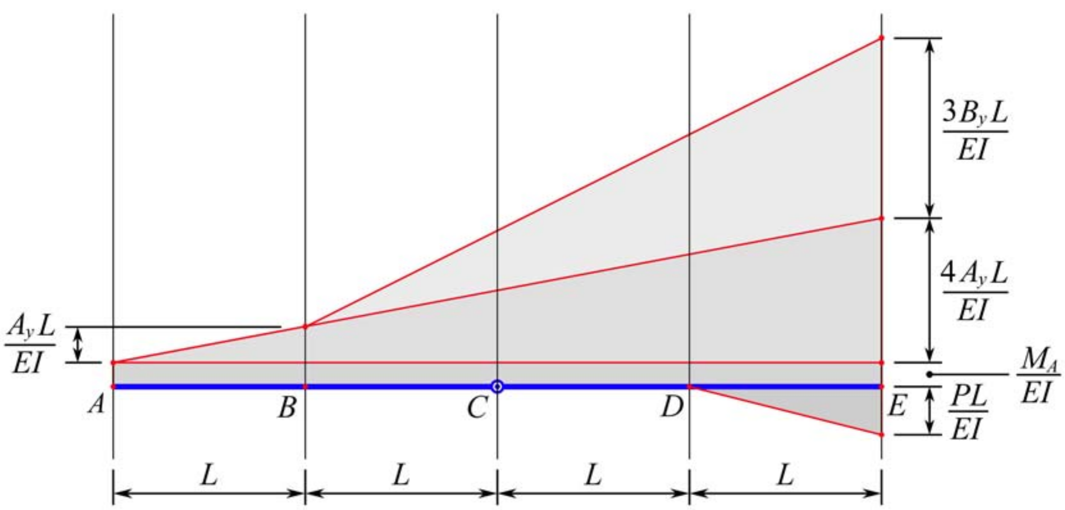


**Fig. 9.** Elastic weight diagram, by parts, for the beam *AE*

Furthermore, let us assume that this person has also draw an elastic curve for the deflected beam, which satisfies the boundary conditions, as shown in Fig. 10.

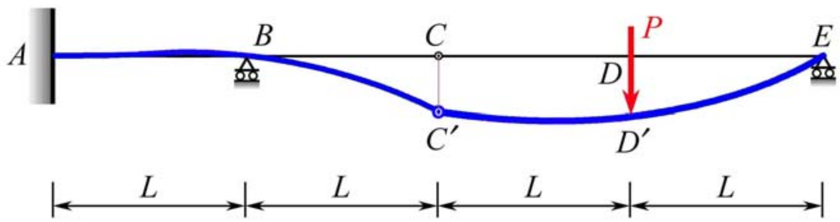


**Fig. 10.** Elastic curve for the beam under loading

In applying the *method using moment-area theorems*, this person correctly notes that the tangential deviation $t_{B/A}$ of *B* with respect to *A* is zero. Referring to Figs. 9 and 10, this person writes

$t_{B/A} = (M_B)_{AB} = 0$:

$$\frac{L}{2}\cdot\frac{M_A L}{EI} + \frac{L}{3}\cdot\frac{1}{2}\cdot L\cdot\frac{A_y L}{EI} = 0 \qquad (a)$$

Due to lack of adequate warning regarding a beam with discontinuity in slope, this person is likely of the impression or opinion that, "the tangential deviation $t_{E/A}$ of *E* with respect to *A* is also zero." Referring to Figs. 9 and 10, this person writes

$t_{E/A} = (M_E)_{AE} = 0$:

$$2L\cdot\frac{4M_A L}{EI} + \frac{4L}{3}\cdot\frac{1}{2}\cdot 4L\cdot\frac{4A_y L}{EI} + L\cdot\frac{1}{2}\cdot 3L\cdot\frac{3B_y L}{EI} - \frac{L}{3}\cdot\frac{1}{2}\cdot L\cdot\frac{PL}{EI} = 0 \qquad (b)$$

Equilibrium of the entire beam in Fig. 8 gives

$$+\circlearrowleft \Sigma M_E = 0: \qquad -M_A - 4LA_y - 3LB_y + LP = 0 \qquad (c)$$

Solution of the above *three* simultaneous equations in (*a*) through (*c*) yields

$$M_A = \frac{8PL}{243} \qquad A_y = -\frac{8P}{81} \qquad B_y = \frac{331P}{729}$$

Consistent with the defined sign conventions, this unsuspecting beginner is led to report

$$\mathbf{M}_A = \frac{8PL}{243}\circlearrowleft \qquad \mathbf{A}_y = \frac{8P}{81}\downarrow \qquad \mathbf{B}_y = \frac{331P}{729}\uparrow$$

According to these seeming "answers," which satisfy Eq. (*c*), the moment at *C* in Fig. 8 would be

$$M_C = M_A + 2LA_y + LB_y = \frac{8PL}{243} + 2L\left(-\frac{8P}{81}\right) + L\left(\frac{331P}{729}\right) = \frac{211PL}{729} \neq 0$$

Since the **moment at a hinge must be zero** (i.e., $M_C = 0$), the above answers must be wrong!

Example 3. A beam *AE* with a hinge connector at *C* carries a concentrated force **P** at *D* and is supported as shown in Fig. 7, where the segments *AC* and *CE* have the same flexural rigidity *EI*. Show the **proper way** to apply the *method using moment-area theorems* to determine for this beam *(a)* the reaction moment $\mathbf{M}_A$ and the vertical reaction force $\mathbf{A}_y$ at *A*, *(b)* the vertical reaction force $\mathbf{B}_y$ at *B*, *(c)* the deflection $y_C$ of the hinge at *C*, *(d)* the slopes $\theta_{CL}$ and $\theta_{CR}$ just to the left and just to the right of the hinge at *C*, respectively, and *(e)* the slope $\theta_D$ and the deflection $y_D$ at *D*.

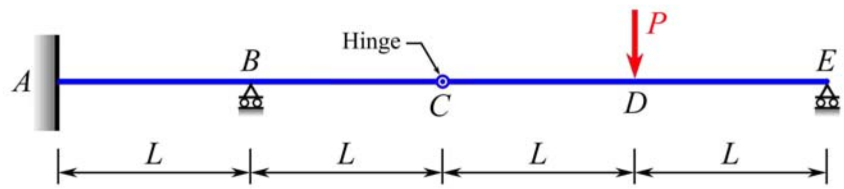


**Fig. 7.** Hinge-connected beam *AE* with a fixed end and two simple supports (repeated)

Solution – proper way: We first draw a *correct* free-body diagram of the beam, as shown in Fig. 8, in the beginning of the solution.

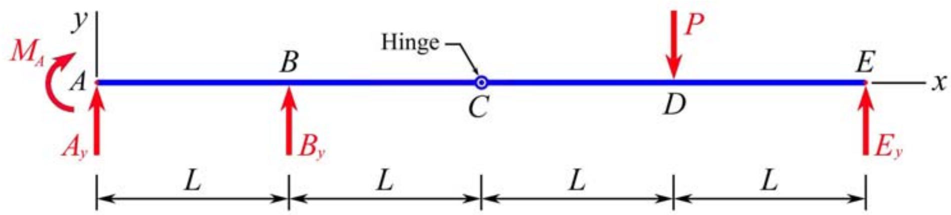


**Fig. 8.** Free-body diagram for the hinge-connected beam *AE* (repeated)

Based on Fig. 8, we draw a *correct* diagram, by parts, for the elastic weight on the beam, as shown in Fig. 9.

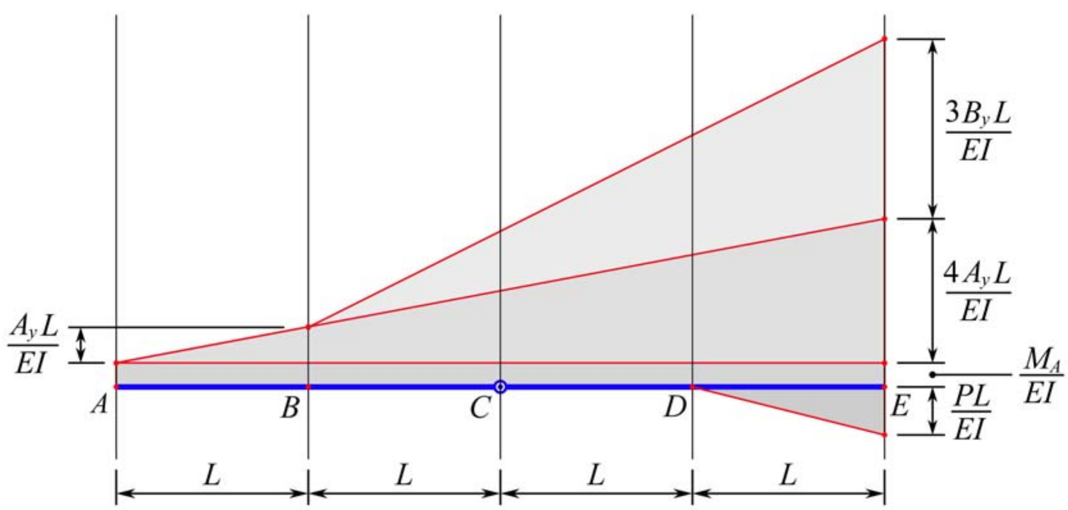


**Fig. 9.** Elastic weight diagram, by parts, for the beam *AE* (repeated)

Furthermore, we draw an elastic curve for the deflected beam, which satisfies the boundary conditions, as shown in Fig. 10.

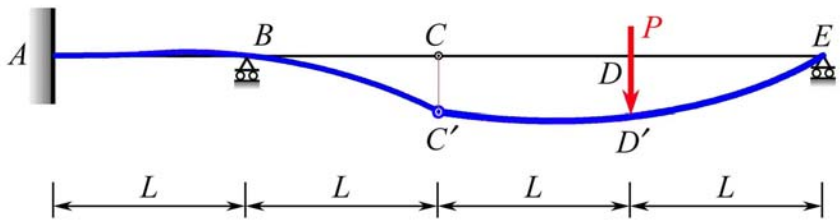


**Fig. 10.** Elastic curve for the beam under loading (repeated)

In applying the *method using moment-area theorems*, we note that the tangential deviation $t_{B/A}$ of $B$ with respect to $A$ is zero. Referring to Figs. 9 and 10, we write

$t_{B/A} = (M_B)_{AB} = 0:$

$$\frac{L}{2}\cdot\frac{M_A L}{EI} + \frac{L}{3}\cdot\frac{1}{2}\cdot L\cdot\frac{A_y L}{EI} = 0 \qquad (a)$$

This beam is statically indeterminate to the *first* degree. Because of the discontinuity in slope at the hinge connection $C$, this beam needs to be divided into two segments $AC$ and $CE$ for analysis in the solution, where no discontinuity in slope exists in either segment.

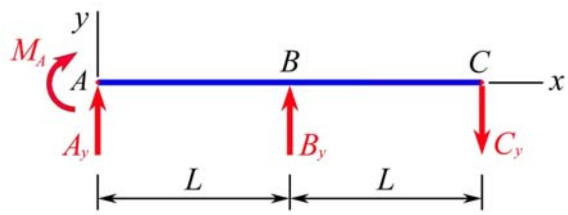


**Fig. 11.** Free-body diagram for segment *AC*

Referring to the free-body diagram for the segment $AC$ in Fig. 11, we write

$+\circlearrowleft \Sigma M_C = 0:$ $$-M_A - 2LA_y - LB_y = 0 \qquad (b)$$

$+\uparrow \Sigma F_y = 0:$ $$A_y + B_y - C_y = 0 \qquad (c)$$

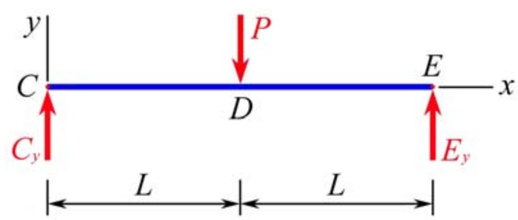


**Fig. 12.** Free-body diagram for segment *CE*

Referring to the free-body diagram for the segment $CE$ in Fig. 12, we write

$+\circlearrowleft \Sigma M_E = 0:$ $$-2LC_y + LP = 0 \qquad (d)$$

Solution of the above *four* simultaneous equations in ($a$) through ($d$) yields

$$M_A = \frac{PL}{4} \qquad A_y = -\frac{3P}{4} \qquad B_y = \frac{5P}{4} \qquad C_y = \frac{P}{2}$$

Consistent with the defined sign conventions, we report that

$$\mathbf{M}_A = \frac{PL}{4} \circlearrowright \qquad \mathbf{A}_y = \frac{3P}{4} \downarrow \qquad \mathbf{B}_y = \frac{5P}{4} \uparrow$$

(These answers are obtained in a *proper way* and are **different** from those obtained earlier for $\mathbf{M}_A$, $\mathbf{A}_y$, and $\mathbf{B}_y$ in an *improper way* by an unsuspecting beginner in Example 1.)

There is discontinuity in slope at hinge connection *C*. We note that $\theta_A = 0$ at the fixed end *A* and that $\theta_{CL}$ and $\theta_{CR}$ are the slopes just to the left and just to the right of the hinge at *C*, respectively. Therefore, we may refer Fig. 9 and utilize the above obtained solutions in applying the *moment-area theorems*. We write

$$A_{AC} = \theta_{CL/A} = \theta_{CL} - \theta_A = \theta_{CL} - 0 = \theta_{CL}$$

$$\therefore \quad \theta_{CL} = 2L \cdot \frac{M_A}{EI} + \frac{1}{2} \cdot 2L \cdot \frac{2A_y L}{EI} + \frac{1}{2} \cdot L \cdot \frac{B_y L}{EI} = -\frac{3PL^2}{8EI} \qquad \theta_{CL} = \frac{3PL^2}{8EI} \circlearrowright$$

$$t_{C/A} = (M_C)_{AC} = -L \cdot 2L \cdot \frac{M_A}{EI} - \frac{2L}{3} \cdot \frac{1}{2} \cdot 2L \cdot \frac{2A_y L}{EI} - \frac{L}{3} \cdot \frac{1}{2} \cdot L \cdot \frac{B_y L}{EI} = \frac{7PL^3}{24}$$

$$\therefore \quad y_C = -t_{C/A} = -\frac{7PL^3}{24} \qquad y_C = \frac{7PL^3}{24EI} \downarrow$$

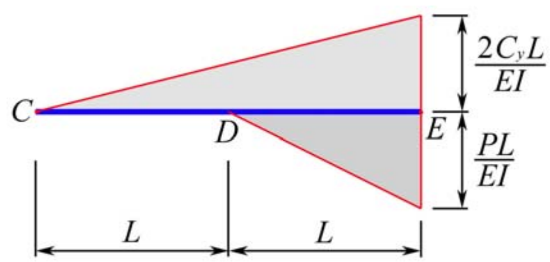


**Fig. 13.** Equivalent elastic weight diagram, by parts, for segment *CE*

Referring to Figs. 10 and 13, we apply the *second moment-area theorem* to write

$$t_{E/C} = (M_E)_{CE} = \frac{2L}{3} \cdot \frac{1}{2} \cdot 2L \cdot \frac{2C_y L}{EI} - \frac{L}{3} \cdot \frac{1}{2} \cdot L \cdot \frac{PL}{EI} = \frac{PL^3}{2EI}$$

$$\theta_{CR} = -\frac{t_{E/C} - t_{C/A}}{2L} = -\frac{5PL^2}{48EI} \qquad \theta_{CR} = \frac{5PL^2}{48EI} \circlearrowright$$

$$\theta_{D/CR} = \theta_D - \theta_{CR} = W_{CD} = \frac{1}{2} \cdot L \cdot \frac{C_y L}{EI} = \frac{PL^2}{4EI}$$

$$\therefore \quad \theta_D = \frac{PL^2}{4EI} + \theta_{CR} = \frac{PL^2}{4EI} - \frac{5PL^2}{48EI} = \frac{7PL^2}{48EI} \qquad \theta_D = \frac{7PL^2}{48EI} \circlearrowleft$$

$$t_{D/C} = (M_D)_{CD} = \frac{L}{3} \cdot \frac{1}{2} \cdot L \cdot \frac{C_y L}{EI} = \frac{PL^3}{12EI}$$

$$y_D = -t_{C/A} + L\,\theta_{CR} + t_{D/C} = -\frac{7PL^3}{24} + L \cdot \left( -\frac{5PL^2}{48EI} \right) + \frac{PL^3}{12EI} = -\frac{5PL^3}{16EI}$$

$$y_D = \frac{5PL^3}{16EI} \downarrow$$

The foregoing results and answers are obtained by the *method using moment-area theorems* via a **proper way**. These answers have been *assessed* and *verified* to be in agreement with the answers that were independently obtained for a problem involving the same beam but being solved using an entirely different method – the conjugate beam method.[10]

## IV. Conclusion

There are advantages and disadvantages in using any of the several established methods for analyzing deflections of beams. This paper contributes definitive concepts and detailed explanations to expel ambiguities often encountered by students in applying the *method using moment-area theorems* to solve problems. Furthermore, it points out a **caveat** to avoid a common unsuspected pitfall when applying this method to solve problems involving slopes and deflections, as well as statically indeterminate reactions at supports, of beams. The paper is *not* written to advocate this particular method over other established methods.

For the benefit of a wider readership, the paper goes over the sign conventions for beams and the rudiments of the *method using moment-area theorems* for analyzing beams. Most textbooks for mechanics of materials or mechanical design do not adequately warn readers about the *limitations* of the moment-area theorems and the *pitfall* in the case of hinge-connected beams, where discontinuity in slope of the beam exists. Beginning students tend to be of the impression that the moment-area theorems can be applied to the entire beam without the need to divide it into segments for analysis *in all cases*. Such an impression is a correct one if the beam is a single piece of elastic body that has a constant flexural rigidity, but it is a misconception for the analysis of a hinge-connected beam. Thus, a hinge-connected beam is a pitfall into which unsuspecting persons often tumble.

The paper includes simple and full fledge illustrative examples to demonstrate both **proper** and **improper** ways in using moment-area theorems to solve problems involving either simple or statically indeterminate beams. In general, deflections and any statically indeterminate reactions associated with a hinge-connected beam must be analyzed by dividing the beam into segments, as required, where each segment must have no discontinuity in slope. Otherwise, erroneous results will be reached.

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