AC 2008-1093: EFFECTIVE TEACHING OF PHOTONICS E&M THEORY USING COMSOL®

Bill Yang, Western Carolina University

Yeqin Huang, Western Carolina University

Robert Adams, Western Carolina University

James Zhang, Western Carolina University

Kenneth Burbank, Western Carolina University

Effective Teaching of Photonics E&M Theory Using COMSOL®

Abstract

Photonics and optical communications, after the exuberant growth and subsequent down turn in late 1990s and early 2000s, have entered a more mature and stable growth phase. As the technology of choice for long distance, high data rate, and high performance communication systems underlying the now ubiquitous Internet communications, photonics and optical communication professionals are and will continue to be in high demand. Because electromagnetic (E&M) theory is the foundation of photonics and optical communications, mastery of E&M theory is essential for those electrical engineering (EE) students who want to develop a career in this field.

Traditionally, rigorous analytic skills in advanced mathematics especially in subjects such as partial differential equations (PDE) and linear algebra are a must to the understanding and applications of E&M theory, as well as photonic device and waveguide designs. However, as practical designs grow in complexity, even the most sophisticated and advanced analytic techniques in these mathematical subject areas can quickly fall short of being a suitable practical design tool. Standard industry practices utilize comprehensive software simulation packages to address these design needs. It is therefore appropriate and advantageous for EE students to learn and more importantly visualize the E&M theory by combining the basic mathematical principles, e.g., the Maxwell equations and wave equations, with practical software tools that they are more likely to use in their professional life. This paper discusses the introduction of COMSOL®, a predominant industry PDE solver, to senior EE undergraduates as a learning tool of fundamental concepts in photonics such as transverse electrical (TE) modes and transverse magnetic (TM) modes in planar waveguide designs. This teaching method improves teaching effectiveness of E&M field and wave theory by helping the students better understand mathematical complexities through this readily available and reliable software tool. In addition to the theory, the students also gain the design capability using these industry standard software packages, and therefore bridging the gap between theory and practice. .

Introduction

The vector property of E&M fields is at the heart of optics and E&M wave theories. At the same time, it is also often a difficult knowledge point in an engineering curriculum. This in a major way is because the vector nature of the fields is abstract. First of all, an E&M field is not easily perceived directly. In addition, the vector relationship between the fields and their corresponding responses in a medium is even harder to 'see'. This non-intuitive nature of E&M fields has been consistently one of the main obstacles for generations of engineering students in their study of optics and E&M wave theories. On the other hand, optics and E&M wave theories are now becoming key components of core engineering knowledge as they have become increasingly important in modern industries and electrical engineering practices. Optical fiber networks, for example, are the backbone of the Internet and voice communications. Optical fiber communications relies heavily on photonic and opto-electronic technologies. The need of proficient workers and design engineers in this field is and will continue to grow. Photonics E&M theory is essential to those EE students who want to develop a successful career in this

field. Conventionally, teaching of E&M field and wave theory and photonics relies heavily, if not solely, on advanced mathematical analysis methods involving subjects such as partial differential equations (PDE), linear algebra, and vector calculus and analysis. Students may be asked to analytically solve wave equations for fiber and waveguide designs with specific boundary conditions. Since the boundary conditions that can entertain exact analytical solutions are only a few, the majority of practical designs remains untouchable for an undergraduate EE curriculum.

Following the traditional approach, an E&M field, represented typically by its electrical field (E field), is assumed to be linear polarized and the corresponding scalar PDEs are derived accordingly. In order to obtain the desired results such as eigenmodes and the effective index of a waveguide, proper boundary conditions have to be applied. Mathematically, the Neumann continuity condition is usually used and the scalar PDEs are solved by requiring the fields and their first derivatives to be continuous at the dielectric interfaces. This typical way of teaching has at least two shortcomings, one is conceptual and the other one is practical. The conceptual shortcoming comes from the use of the mathematical Neumann boundary condition. The Neumann boundary condition requires that the field and its first derivative to be continuous across the boundary. However, at dielectric interfaces, for example, while the tangential fields are continuous, the normal component of the E field is not. Since the first derivative of the magnetic field (H field) is directly proportional to the E field, for example, the Neumann boundary condition does not obviously hold. How this mathematical boundary condition can be related to the physical boundary condition that is required for E&M fields is often a neglected topic in the traditional way of teaching. This again is due primarily to the lack of ability for students to 'see' the vector relationship between E&M fields directly. On the application side, as practical designs grow in complexity, analytical techniques usually are far too complex and cumbersome to serve as suitable and practical design tools. Typical practices in the industry use commercial and/or proprietary software simulation packages to address the design needs. It is advantageous for EE students to be familiar with such simulation software packages as a highly desirable job skill. More importantly, the visualization of the E&M fields by the software's graphic user interfaces renders a direct experience and hands-on interaction for the students. An effective teaching method, therefore, combines the basic mathematical principles such as the Maxwell equations and wave equations with practical software tools that the students are more likely to use in their professional life.

In this paper we discuss the use of COMSOL®, a predominant industry PDE solver, in an EE senior level photonics course. Fundamental concepts such as TE and TM modes in planar waveguide designs are directly demonstrated in both analytical vector relations and the more intuitive graphs and pictures. This method of teaching enhances the effectiveness of teaching E&M and wave theories to EE undergraduate students through computer interactions that students can have with the mathematical complexities. In addition to the benefit of 'seeing' the abstract relations, the students also acquire a practical design capability and therefore bridge the gap between theory and practice.

Wave Equations and Boundary Conditions

Wave equations derived from the Maxwell equations can take various field and/or potential representations. Typically in an undergraduate level course one uses either the electrical field (E field) or the magnetic field (H field) for solving waveguide eigenmode problems. The derivations of E field and H field wave equations are similar and the resulted wave equations are

$$\nabla^2\mathbf{E} + \frac{n^2\omega^2}{c^2}\mathbf{E} = 0 \qquad (1a)$$

$$\nabla^2\mathbf{H} + \frac{n^2\omega^2}{c^2}\mathbf{H} = 0 \qquad (1b)$$

where n is the index of refraction, ω is the frequency of light and c is the speed of light in vacuum. These are in general vector equations in three dimensional (3-D) space, but most textbooks use the linear polarization assumption to simplify them into scalar wave equations. For planar waveguides, usually a one-dimensional (1-D) model is used as illustrated in Figure 1.

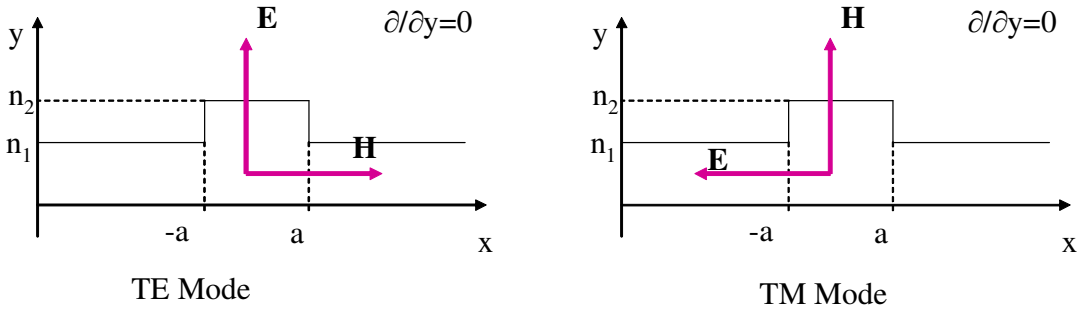

Figure 1: 1-D model of planar waveguide modes. Correspondingly there are two types of eigenmodes, namely, TE modes, whose electrical field is parallel to the dielectric interface, and TM modes, whose electrical field is normal to the dielectric interface.

The 1-D eigenmode problem is given by

$$\frac{d^2}{dx^2}E + \frac{n^2\omega^2}{c^2}E = \beta^2 E \qquad (2)$$

where β is the effective propagation constant of the eigenmodes.

Traditionally the teaching effort starting from here is to apply the above equation in each uniform domain of a structure and then apply the Neumann continuous boundary condition at each dielectric interface. This approach, however, can result in ambiguous explanations of the differences between TE and TM modes and easily cause confusion among students. This is especially true if a student wishes to correlate the mathematical Neumann continuous boundary condition here with the physical requirements of the dielectric interface. Recall that the Maxwell's E&M law requires that for a dielectric interface in lossless media the tangential components of both the E field and the H field are continuous while the normal component of the E field is not. The mathematical Neumann boundary condition, on the other hand, requires the field and its first derivative to be continuous.

In one typical traditional treatment, H_z is chosen as the field component to solve for TE modes and E_z is chosen as the field component to solve for TM modes. However, in this case the first derivative of H_z or E_z (i.e. $\partial H_z/\partial$x and $\partial E_z/\partial$x) does not by itself correspond to any field components. In fact ($\partial H_z/\partial$x-$\partial H_x/\partial$z) corresponds to E_y and ($\partial E_z/\partial$x-$\partial E_x/\partial$z) corresponds to H_y. The continuity of $\partial H_z/\partial$x and $\partial E_z/\partial$x by itself at the dielectric interface is either an assumption or an approximation that can not be traced back to the physical boundary conditions. Another popular treatment chooses E_y as the field component to solve for TE modes and E_x as the field component to solve for TM modes. In this case for TE modes, since $\partial E_y/\partial x = j\omega\mu H_z$, E_y and $\partial E_y/\partial$x are indeed both continuous across the dielectric interfaces. However, for TM modes, $\partial E_x/\partial$x does not correspond to any field components and its continuity across the dielectric interfaces is again questionable. Even when H_y is correctly chosen as the field component to solve for TM modes, since $\partial H_y/\partial x = -j\omega\varepsilon E_z$, $\partial H_y/\partial$x is actually not continuous across the dielectric interfaces. In any of these cases of ambiguous treatments, a diligent student can easily get lost when trying to match the mathematical Neumann condition with the physical boundary conditions. A traditional way of teaching that correctly differentiates TE and TM modes starts with Eq.(2) and applies different boundary conditions for TE and TM modes and ends up with different sets of analytic relations for solving TE and TM eigenvalues β[1]. This approach quickly becomes intractable when several uniform regions are involved as it is usually the case in practical waveguide designs, notably for example for double-hetero-junction lasers[2]. For such multi-layer structure waveguide designs, commercial available PDE solver packages are commonly used in practice. COMSOL® is one of such packages that have gained increasing popularity in recent years. To a great benefit of engineering students, this software package also has a well designed graphical user interface with versatile graphical representations of the fields such as field contour plots and arrow plots. In the following sections we will demonstrate two simple yet important practical waveguide models that also help to visualize the E&M fields.

Optical Fiber modes: Visualization of E&M fields

The first model we introduced to students is a step-index optical fiber. This model is particularly simple in its geometry but is indeed very useful practically. The analytic approach for optical fiber model involves solving the 2-D eigenvalue problem in a cylindrical coordinate system and results in fiber mode profiles expressed in special functions. This approach is very abstract and the required mathematics is difficult for undergraduate EE students. Using COMSOL®, however, the vector wave equation is directly solved numerically by the finite element method (FEM) embedded in the software and the results can be exhibited by various field plots with easy and clear visualization and interaction.

The vector wave equation is expressed in its general format,

$$\nabla \times \left(\frac{1}{n^2} \nabla \times \mathbf{H} \right) = \frac{\omega^2}{c^2} \mathbf{H} \qquad (3)$$

Figure 2 shows the step-index optical fiber geometry and two of its degenerated fundamental modes with different polarization. The arrow plots of the E field visually show the polarization direction of each fundamental mode. To build this model it only involves several simple steps

of selecting the appropriate 2-D model (vector wave equation eigenvalue problem), drawing the fiber cross-section in its real length unit (um here) and input of the corresponding core and clad

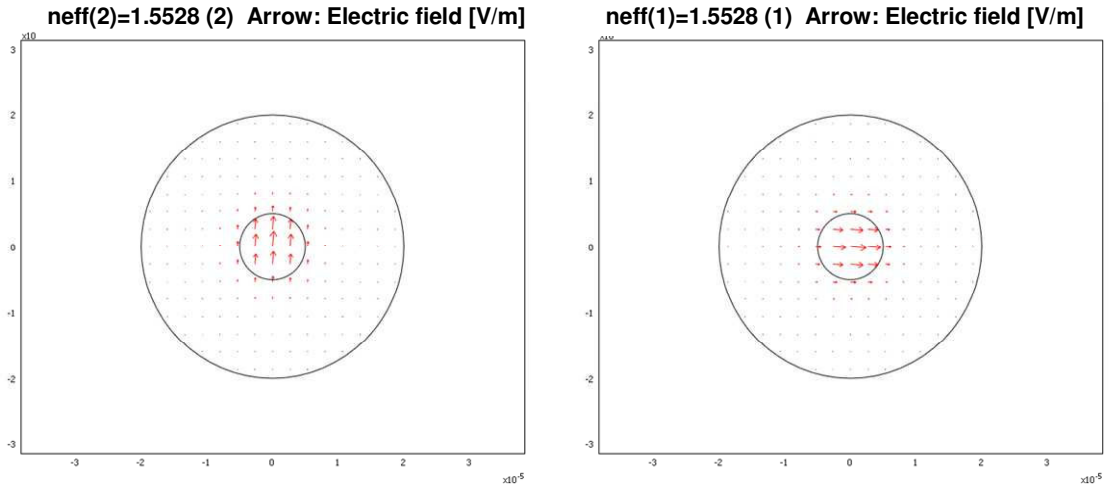

Figure 2: 2-D model of step-index optical fiber showing two of its polarization degenerated fundamental modes. The arrow plots visually show the E field direction (polarization) and the degenerated effective index.

indices. Once the GUI is familiarized, it only takes our EE seniors half an hour to build and solve this model. The teaching results are very satisfying as our students are inspired by solving this complicated real world practical problem and visualizing the results, or 'seeing' the fields, for themselves. The students immediately report that the graphical representation helped them to understand the linear polarized modes in optical fiber. Indeed, for advanced students including graduate students, one can also visualize the z-component of the fields which will clearly indicate that the linear polarization is only an approximation of the actual case.

The boundary condition for the circular dielectric interface for COMSOL® 2-D vector wave equation (Eq.3) is also in the vector format

$$\mathbf{n}\times\left(\mathbf{H}_1-\mathbf{H}_2\right)=0 \quad \text{and} \quad \mathbf{n}\times\left(\mathbf{E}_1-\mathbf{E}_2\right)=0 \qquad (4)$$

which states that both tangential E field and tangential H field are continuous. This is the appropriate physical boundary condition and is sufficient to uniquely determine the solution to the vector wave equation Eq. (3).

Planar Waveguide Modes: Boundary Conditions

One of the objectives of introducing the COMSOL® photonics simulation package to our EE seniors is to equip them with a practical tool to design planar waveguides. Planar waveguides are used intensively in various integrated photonics devices as well as semiconductor laser designs[2]. The proper PDE model here is a 1-D PDE eigenvalue problem. It is a standard practice to design planar waveguides, which are indeed a 2-D problem, by the method of effective index in two steps, first in the vertical (y) direction and then in the in-plane (x) direction[3]. The 1-D eigenvalue problem PDE equation that is applicable in COMSOL® reads

$$\frac{d}{dx}\cdot\left(-c\frac{d}{dx}u\right)+au=-e_a\lambda^2 u \tag{5a}$$

for low loss 1-D waveguide models. The corresponding Neumann continuous boundary condition for Eq. (5) reads

$$\left(c\frac{du}{dx}\right)_1-\left(c\frac{du}{dx}\right)_2=0 \tag{5b}$$

This boundary condition automatically takes into account both TE and TM cases provided that now Eq. (2) only applies to TE modes and TM wave equations take a slightly different format as

$$\frac{d}{dx}\left(\frac{1}{n^2}\frac{d}{\partial x}H\right)+\frac{\omega^2}{c^2}H=\frac{1}{n^2}\beta^2 H \tag{6}$$

In each uniform region, Eq. (2) and Eq. (6) are exactly the same. Referring to Figure 1, both the E field of TE modes in Eq.(2) and the H field of TM modes in Eq.(6) are the *y*-component of the fields. Given the 1-D waveguide model, the index of refraction only varies along one spatial dimension, chosen here to be *x* direction. Since the medium is uniform in *y* direction, the derivatives respect to *y* vanishes. For TE modes, the *z* component of the E field is also zero. From the vector relationship between E field and H field, this implies that the only non-zero E field component is the *y* component, to which Eq. (2) applies. Similarly, the only non-zero H field component is also the *y* component, to which Eq. (6) applies. Since all *y* components of the fields are tangential to the interface, they are all continuous as required physically. Their first derivatives, however, need to conform to Eq. (5b) for the Neumann boundary condition to be valid. For TE modes Eq. (5b) yields $\left(\frac{dE}{dx}\right)_1=\left(\frac{dE}{dx}\right)_2$ where $\left(\frac{dE}{dx}\right)_1$ is the first derivative of the E field respective to *x* evaluated at the boundary on the side of subdomain one and $\left(\frac{dE}{dx}\right)_1$ is the first derivative of the E field respective to *x* evaluated at the boundary on the side of subdomain two. Since *dE/dx* is proportional to the *z* component of H field, H_z, which is tangential to the

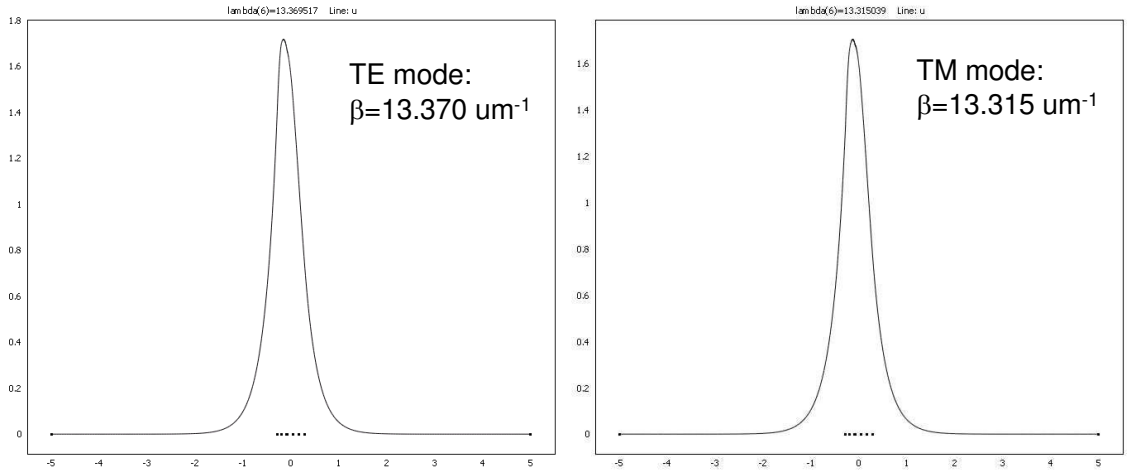

Figure 3: 1-D model of a multi-layer (5 different compositions, III-V semiconductors lattice matched to InP) planar waveguide model. Fundamental mode profiles and effective propagation constants for TE and TM modes respectively. (Student's course project work)

interface, the Neumann boundary condition Eq. (5b) correctly prescribes the physical boundary condition requirement for the E field of TE modes. For TM modes, Eq. (5b) yields $\left(\frac{1}{n^2}\frac{dH}{dx}\right)_1 = \left(\frac{1}{n^2}\frac{dH}{dx}\right)_2$. Now that *dH/dx* is proportional to $\varepsilon_r E_z$, and the *z* component of the E field, E_z, is continuous across the interface, therefore, Eq. (5b) again correctly prescribes the physical boundary condition, recognizing that $\varepsilon_r = n^2$. Figure 3 shows a result of a multi-layer core planar waveguide model obtained by one of our EE seniors for their course project. By applying the COMSOL® software with careful considerations of boundary conditions, students expressed that they understand and appreciate much better the vector relationship among E&M fields. One student commented in his course evaluation that the best aspect of the course is "doing the COMSOL project and being able to learn the new software and how that software actually relates to what we were doing in the class."

Discussions

Commercial software package COMSOL® has been introduced in an EE senior course of laser electronics. It has been a great help to the instructor to help students to 'see' the E&M fields and better understand the vector properties of E&M field like the polarization state of light. The approach is effective also because it teaches EE seniors a practical tool for photonic and RF waveguide design. The application of software to a course project, which in this case is to design an active semiconductor waveguide for diode lasers, deepens students' understanding of the related important topics such as TE and TM modes in planar waveguides. The results are satisfactory and the feedback from students is positive. Combining with this department's previous efforts and experience using other popular commercial software packages such as MATLAB® in the classroom, we believe that this effective approach is highly appropriate and advantageous for EE students to learn and, more importantly in the case of photonic E&M theory, to visualize the E&M theory by combining the basic mathematical principles, e.g., the Maxwell equations and wave equations, with practical software tools that they are more likely to use in their professional life.

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