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Elaboration of a contextualized event for teaching eigenvalues and eigenvectors in the control and automation engineering course

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Elaboration of a Contextualized Event for teaching eigenvalues and eigenvectors in the Control and Automation Engineering program

Introduction

Research in Mathematics Education, for example, [1], [2], [3], [4], [5], [6] have explored the subject of Linear Algebra, due to its great application with other areas of Mathematics and the interdisciplinary of this theme with contents treated in specific courses of undergraduate program that do not aim to train mathematicians. Stewart, Larson e Zandieh [7] expose that researchers in this area have been concerned about the teaching and learning of Linear Algebra due to its abstraction level and difficulty reported by students to understand these contents, a fact that since the late 1980s has intensified the work in this area.

Dorier [1], as well as Prado [4] indicate that the difficulties faced by students are due to their immaturity at the time linear algebra is introduced, due to the amount of definitions present, as well as, the disconnection of this content with subjects covered previously.

Research shows that engineering students have reported difficulties in understanding mathematics and applying it in specific courses, mainly due to the lack of connection between the mathematical content in the initial years with specific Engineering themes covered during the courses [8]. According to Ferruzi and Almeida [9] engineers have been facing difficulties in applying the theory studied, in practical situations of their education, as well as making adaptations so that real problems can be modeled. Also, in line with this study, Karakok [10] drew attention some research that discussed the difficulties that students have in many linear algebra topics and which also indicated that this difficulty may be linked to different ways of thinking needed to conceptualize these topics. Karakok [10] also points out that there is currently a need to understand how students transfer the mathematical tools they learn to other courses that use linear algebra as a pre-requisite, that is, how to apply knowledge learned in one context to different ones.

According to this research, some studies mention the need to rethink and restructure the mathematics subjects taught in Engineering programs in such a way that there are connections between the areas. We can mention, for example: the link between the study of the semiconductor diode characteristic curve and real-valued functions of a real variable; and the link between the horizontal motion study of a two or more story gantry with the notions of eigenvalue and eigenvector. Ferruzzi and Almeida [9] have pointed out that we can no longer teach mathematics subjects in an isolated way, without connection with the specific courses.

“... it is considered important to conceive mathematics as a science that can be practiced, experiencing it in such a way that some problems that arise in the student's daily life can be developed in the classroom, enabling the exploration and construction of mathematical concepts through activities that are meaningful to the student, thus seeking to build their knowledge” [9].

Still, on the need to rethink and restructure the Mathematics subjects taught in Engineering programs it is worth noting that Prado [4], as well as Nomura and Bianchini [11], [12] discuss the importance of planning, executing and evaluating subjects that are linked to the interests and objectives of the courses in which these ones are being offered, reflecting on what skills we want students to acquire and how these are used in their careers.

Prado [4] also suggest that it is necessary to develop a more contextualized, consolidated and attractive course, applying multidisciplinary and transdisciplinary activities, using active methodologies, articulating practice and theory with the support of software, a fact that is also highlighted in the document that in Brazil guides the organization of engineering programs, the National Curriculum Regulations for Engineering Education (DCN[1]) [13].

Stewart, Larson, and Zandieh [7] emphasize the need of developing a curriculum based on contextualized applications and the development of materials that can be used by teachers. Considering these points, this article aims to show which were the subsidies adopted in [14] for the elaboration of a problem articulating mathematics with other areas of knowledge - a problem that, in Mathematics in the Context of Sciences Theory is called Contextualized Event - to teach eigenvalues and eigenvector in Control and Automation Engineering programs.

For Camarena [15], Contextualized Event are problems or projects with the purpose of integrating mathematical contents with the specific contents of the undergraduate program in which it is being performed.

It is important to emphasize that in this article the focus is not on contextualized event application results, nor on presenting data that would allow a comparison of the students'

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[1] We chose to use acronyms respecting the term names in their original languages

learning performance obtained through this eigenvalue and eigenvector approach with those resulting from other teaching strategies.

Philot [14] presents, in detail, the results of a pilot experience of contextualized event application with six second-year Engineering undergraduate students. The analysis of this initial implementation indicates the potential of this approach, what motivated us to make a future implementation considering a significant number of students to investigate statistically the possible advantages of this type of strategy over others.

In order to understand better what a Contextualized Event is, we will bring some basic concepts of Mathematics in the Context of Sciences Theory.

Mathematics in the Context of Sciences Theory

The educational theory, Mathematics in the Context of the Sciences (TMCC), began in 1982, at the National Polytechnic Institute of Mexico, with the researcher Patricia Camarena Gallardo. This theory was developed to discuss the problem of teaching and learning of mathematics in engineering professions. The TMCC arose from the lack of studies in the higher education system that address the problem of why mathematics should be taught in Engineering programs, how to apply the contents taught in mathematics in the specific courses of Engineering programs, and even in the engineering profession. Later on, this reference was extended to other university careers, that is, to undergraduate programs in which this science is useful, but that do not have as an objective to train people who will have mathematics as a future area of professional activity.

According to Camarena [15], unlike most educational theories that focus on teaching and learning in Basic Education, this theory began at the university level, from questions that students made about the teaching of mathematics, more specifically in the Engineering program. The students asked questions such as: "Why do we study this content?", "Where do we apply what we are studying?", "How does this content help me?", and so on. According to Lima et al [16], based on [15], these questions started to raise reflections about: (a) the objective of the undergraduate program in which Mathematics is being taught; (b) how to motivate the student of this undergraduate program to learn Mathematics; (c) how to link this science with the future profession of the student; (d) how Mathematics can contribute to the integral formation of the student and, finally, (e) how this science can support the development of professional competences [16].

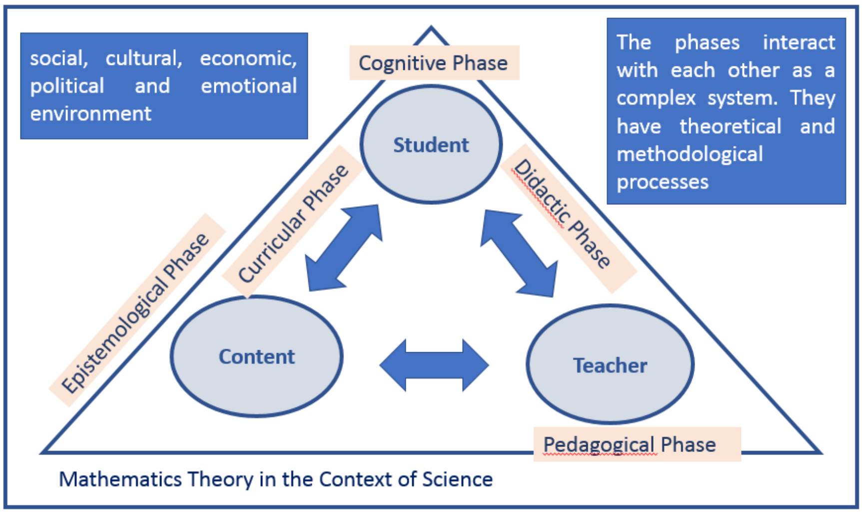
In TMCC, the concept of competence is understood as "cognitive mobilization of the attributes of a professional to face a problematic situation by making use of the integration of all their knowledge, skills, attitudes and values" [17].

TMCC has a social nature, since it is concerned with teaching Mathematics for life, which is useful for the scientific, technical and civil society in order to help the student to develop in society in a reasoned, critical, analytical and scientific way. Moreover Camarena [18] state that the philosophical educational assumption of TMCC is;

"... that, throughout their undergraduate program, the student should be able to make the transfer of knowledge of mathematics to the areas that require it and in this way the professional and labor skills are favored, because it is intended to contribute to the comprehensive training of the student and build a Mathematics for life" [18].

In TMCC, according to Camarena [19],[20], the teaching and learning environment is considered to consist of a complex system, as shown in Figure 1, which presents the content to be taught, the student and the teacher, as well as the interactions that occur between them, which encompass what Camarena [20] calls the five phases of the theory, which interact between each other in a non-linear manner.

Fig. 1. Didactic Triangle according TMCC



Source: adaptation [20]

The phases we refer to are called: curricular, didactic, epistemological, pedagogical, and cognitive. This system is also influenced by economic, cultural, emotional, social and political factors.

In an attempt to answer questions - such as: In which specific engineering situations do we use a certain mathematical content? and How will learning a certain mathematical content help in the education and later in the performance as an engineer? - the creators of the theory began to discuss with professors of mathematics courses, professors of specific courses of engineering programs and even professional engineers. In these conversations they found a difficulty in communication due to the specific language used in each training, often using different languages and different symbols for the same mathematical content.

From these concerns and the need to have a curriculum more appropriate to courses that use Mathematics as a tool, according to Camarena [15], the TMCC curricular phase emerged.

In this phase, a methodology was developed to design study programs, firstly of mathematics for engineering programs, called *Dipcing* (Diseño de Programas de estudio de matemáticas en carreras de ingeniería), and later, for other undergraduate programs in which mathematics is used, and this methodology was later extended to other basic sciences.

After the curricular phase was created and the *Dipcing* methodology was applied, they began to have a clear understanding about the use of Mathematics in Engineering, realizing where it is used, how it is used, among other issues, which ended up leading to other questions. For example: How to teach Mathematics content that is contextualized in an Engineering problem? How to work with students these interdisciplinarities? How to develop student skills in such a way that they are able to transfer their mathematical knowledge to their future areas of professional activity and to their social life? These questions gave rise to the didactic phase and the Didactic Model of Mathematics in Context (MoDiMaCo).

To contribute to the understanding of the interdisciplinarity of Mathematics with other areas of knowledge and why these courses are being studied comes the epistemological phase. Also, in this scenario, the concern of having a qualified teacher to work with this curriculum arises, with the interdisciplinarity of the Mathematics contents in the specific courses of the Engineering programs, in the professional life and daily life of the student, and from these concerns arises the pedagogical phase. Finally, to explain the cognitive functioning of learning with the use of contextualization, the cognitive phase emerges. In order to gain insights into all of the points and questions discussed in this section, research on these themes advanced and gave structure to the TMCC.

Then, in the next section, it is detailed, from what is recommended in the TMCC, what were the methodological subsidies used to build the Contextualized Event articulating Linear

Algebra, specifically the concepts of eigenvalue and eigenvector, with Control Theory through the context of the Covid-19 propagation.

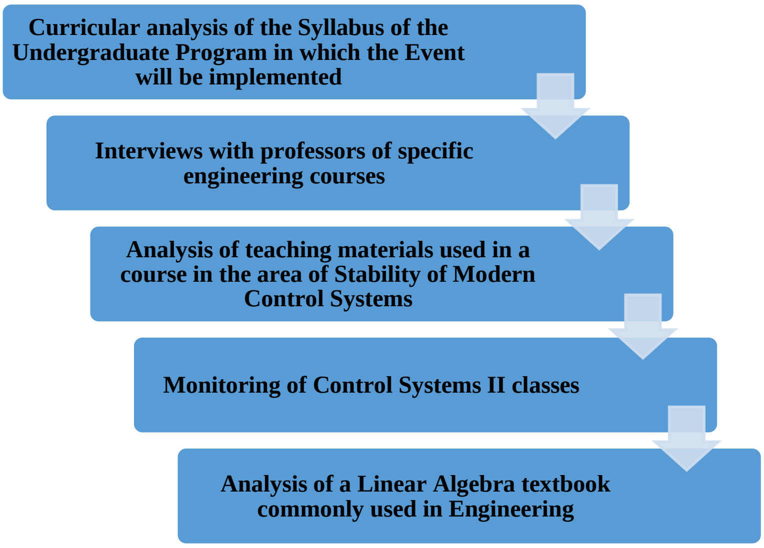
Methodological subsidies for the elaboration of a Contextualized Event

This section reports the procedures followed to elaborate the Contextualized Event presented in [14]. It is important to emphasize that these procedures are not, in any way, a script to be strictly followed. They are steps that, in the development of the first author's doctoral thesis, proved to be adequate considering the objectives pursued. Other professors and researchers, based on their specific objectives, may adapt or insert other steps. It is also worth mentioning that in this thesis, the author did not strictly follow all the steps recommended in the context of the TMCC for the elaboration of the problem.

The elaboration of Contextualized Events is inserted in the epistemological phase of the TMCC. After defining a certain mathematical content that will be worked in a contextualized way in an Engineering program, in this case the concepts of eigenvalue and eigenvector normally covered in a Linear Algebra course, it is necessary, according to the precepts of the mentioned reference, to understand in detail how such content is articulated with those of Engineering and how the mathematical knowledge to be built in the classroom should be transformed so that it can be applied in contexts beyond mathematics.

Originally, in the epistemological phase, from the methodological point of view, as highlighted by [21], [22], especially the analysis of texts, and the way in which this is carried out depends on what is being sought, as well as, consequently, the indicators to work. However, when preparing the Contextualized Event presented in [14], the author felt the need to also use other analysis mechanisms, such as conducting interviews and attending classes in specific Engineering courses, as will be schematically presented in detail in Figure 2.

Fig. 2. Methodological subsidies for the elaboration of the contextualized event



Source: the autors

Despite having presented, in Figure 2, the methodological subsidies in a linear and sequential way, it is important to be clear that some of these procedures can be executed simultaneously and that, even if one of them is apparently finished, when starting the next procedure there may be the need to resume and complement the previous ones.

Subsidy 1: curricular analysis of the Syllabus of the Undergraduate Program in which the Event will be implemented

The purpose of this procedure, originally planned among the methodological procedures of the epistemological phase of the TMCC, is to check, first of all, which Linear Algebra contents are explored in the Engineering major where the professor works and at which moment of the undergraduate program these are worked. For this purpose, we seek to identify in which period of the curriculum Linear Algebra is taught, the number of hours, which Mathematics courses are offered before and/or together with the Linear Algebra course, and which course use Linear Algebra as a requirement. Understanding the use of Linear Algebra concepts in these courses can help us understand the interdisciplinary nature of these courses with other Mathematics courses and the explicit or implicit connections between Mathematics and the specific Engineering courses.

The analysis of the Syllabus, however, allows only a partial view of the relationship between Linear Algebra and the other courses of the undergraduate programs, since the information is presented in an extremely synthetic and impersonal manner, which does not make it possible to perceive specificities about the connections between Linear Algebra and, in this particular case, the concepts of eigenvalue and eigenvector, and the specific situations of Engineering. Because of this limitation, it was felt the need to intensify the analysis by interviewing professors who teach specific engineering courses, as detailed below.

Subsidy 2: interviews with professors of specific engineering courses

Although in the epistemological phase of the TMCC interviews are not expected, being this a procedure inherent to the *Dipcing* methodology in the curricular phase, executed with engineers in order to explain how they use Mathematics in their daily professional life, Philot [14] developed this subsidy as an adaptation of what is foreseen by Camarena [15]. Since in her thesis the objective of Philot [14] was not to develop a Mathematics curriculum for Engineering, but to elaborate a Contextualized Event to teach a certain content of Linear Algebra, the researcher chose to interview the teachers responsible for the specific courses of the Engineering program with the main objective of understanding more deeply in which situations and in which way the eigenvalue and eigenvector contents are present in Engineering situations. Semi-structured reflective interviews [23] were conducted with 5 professors who taught classes for the specific courses of various Engineering majors, with the intention of, in the following methodological procedures, analyzing only the teaching plans and textbooks of the courses highlighted by these professors. In this way, it was possible to differentiate which subjects covered the eigenvalue and eigenvector contents more often, with more intensity, having a stronger importance.

With these interviews it was possible to notice that all teachers are somehow in contact with the contents of Linear Algebra and that most of them have a deep knowledge of them. They know where these concepts are being applied in the specific courses, but they don't approach more contextualized or closer examples and/or exercises to the professional life of the engineer neither as a subproblem of a larger real problem. Another aspect in common is that all of them claimed to use Linear Algebra content, and most use or know where the concepts of eigenvalue and eigenvector are used. These are used in courses from the third year of the Engineering program, such as Theory of Structures, Analytical Mechanics, Vibratory Mechanics, Control Systems I and II, as presented in Table 1.

Table 1. Courses that employ the notions of eigenvalue and eigenvector and some examples of applications mentioned by the interviewees

Professor	Courses	Application
1	Control Systems II	He did not cite a specific application
	Data Science and Artificial Intelligence	*Machine Learning and Deep Learning*
2	Electrical Machines	Electric Machines (Capability curve)
	Introduction to Electric Power Systems	Electric circuits
		Stability of an electric system
3	He did not cite a course, but an application in the engineer's daily professional life	*Software* AHP (Business Administration tool)
	Theory of Structures	He did not cite a specific application
4	Analytical Mechanics	Moment of Inertia
	Analytical Mechanics	Triple Voltage State
		Vibrations
5	Control Systems II	System stability through state representation

Source: adapted from Philot [14]

Analyzing Table 1, one notices that as the main result of these interviews, it was inferred that some topics studied in specific courses of Engineering programs apply the content of eigenvalues and eigenvectors. The topics that were most highlighted were Moment of Inertia about any axis (Main Moments of Inertia), the study of Triple Space of Stresses, and also the part that studies the Theory of Structures, which is highlighted as an example of applications. Another frequently mentioned topic was the Control System, when approached by differential equation models in the form of state spaces. For the study of this theme, the concepts of eigenvalue and eigenvector are very often used to understand the dynamics of systems.

It should be noted that only with the analysis of the Syllabus and with these interviews, it was not possible to identify a specific Contextualized Event that could meet the objectives pursued by Philot [14], namely: to approach in a contextualized way the notions of eigenvalue and eigenvector in Engineering. However, the data obtained through these interviews, especially the references to specific courses, concepts and teaching materials that contain eigenvalue and eigenvector, were extremely important to guide the researcher to continue her research. From the results of these interviews it was identified that in order to elaborate the Contextualized Event aimed at, it is necessary to go deeper into the studies in the field of Stability of Modern Control Systems, which was initially done by analyzing teaching materials in this area, as presented in the following subsidy.

Subsidy 3: analysis of teaching materials used in a course in the area of Stability of Modern Control Systems

The procedure following the interviews was the analysis of some teaching materials (textbooks and workbooks) to better understand the contextualization of mathematics, more specifically the content of eigenvalues and eigenvectors, in Control Systems II. The analysis of teaching materials of specific courses that are part of Engineering programs' curriculum is also a methodological procedure foreseen in the TMCC, as pointed out in [21], and its purpose is to provide the teacher who will elaborate the Contextualized Event with an understanding of how the mathematical content is applied in specific situations, for what purpose and how it is mobilized, with which nature, which aspects related to it are more interesting to the students and which aspects are less interesting.

From the textbook written by teacher of Control Systems II and also from the book indicated as reference in the course, entitled Modern Control Engineering [24], it was realized that there are two ways to study the dynamics and stability of a system, one using the State Space (SS) approached in Modern Control Theory and the other using the Transfer Functions (FT) studied in Classical Control[2]. The notions of eigenvalues and eigenvectors are employed in studying the dynamics and stability of a system using State Space (SS), since the poles of a system in SS form are defined as the eigenvalues of the state matrix and the poles provide the entire dynamic behavior of the system and the eigenvectors, along with the eigenvalues, represent the dynamic modes of the system.

In general, the exercises in the textbook used do not present a context closer to the daily life of the professional who will be graduated. The chapters dedicated to discussions of modeling and its contexts are presented in a very general way and not directly related to the student's future profession or to the engineer's daily life problems.

Since the academic qualification and professional area of activity of the researcher who sought to elaborate the Contextualized Event is Mathematics and not Engineering, for a more effective and deeper understanding of what she had perceived through the analysis of the teaching materials, she needed to study, briefly, some concepts present in courses that precede and are prerequisites for Control System II and also decided to follow some classes

[2] Control Systems Theory can be divided into Classical Control and Modern Control. The main differences are regarding the approach in the frequency domain (complex) or time domain, and number of input and output variables. In general, Control Systems Theory (Classical and Modern) studies the dynamics of mathematical models represented by dynamic systems and how to control such systems.

of Control Systems II taught in the fifth year of the Control and Automation Engineering program of the institution of higher education where she works, as described below.

Subsidy 4: monitoring of Control Systems II classes

Again as a complement to what is proposed in TMCC, Philot [14] proposed as a methodological procedure for the elaboration of the Contextualized Event with which she would work, to observe, for a few months, the classes of the specific course that would contextualize the approach to Mathematics that would be proposed through the Event. From this action, it could be noticed that some fundamental concepts and interpretations for the understanding of Control Systems II were taught in the course Modeling and Analysis of Dynamic Systems (from the third year) and in the course Control Systems I (from the fourth year).

One thing that, during the classes, caught Philot's attention [14] was the lack of more discussion about how to interpret the theory studied in more applied problems in the Control and Automation Engineering area, since this course is taught in the last year of the undergraduate program. It was also noted that the problems' modeling could be explored a little more, since most of the time it was previously provided.

The class monitoring was in 2020, a time when the world was facing the peak of COVID-19 pandemic. Using the world context of that time, the professor proposed to the students a work about the SIR Model - Susceptible (S), Infected (I) and Recovered (R), a mathematical model that studies the spread of infectious diseases as the case of COVID-19 and proposed the solution of several questions about this model involving concepts of the Control Systems II, course that uses as a tool matrix concepts, of linear systems and of eigenvalue and eigenvector. Besides mobilizing the mathematical concepts targeted by [14], a problem involving the transmission of COVID-19 was, at that moment, of great social relevance and, therefore, widely explored by different medias. It revealed itself potentially rich as a starting point for the elaboration of the Contextualized Event. Although, as already mentioned, it had already been defined that the context of the event would be Stability of Modern Control Systems, at this point, it was defined that the dynamics and stability of the SIR Model applied to COVID-19 would be studied. But how to combine the treatment of eigenvalues and eigenvectors in the textbook with this contextualized approach? This combination is only possible if the teacher who will conduct the Event also has a detailed knowledge of the approach given in the textbook. For this reason, the following subsidy is important.

Subsidy 5: analysis of a Linear Algebra textbook commonly used in Engineering

This procedure is accomplished in the epistemological phase of the TMCC with the intention of enabling the teacher who is elaborating the Contextualized Event to understand how the intended mathematical content (in this case, eigenvalue and eigenvector) is addressed in teaching materials. Among other aspects, it seeks to answer questions such as: definitions are presented? What kinds of examples are used? Are theorems mentioned? Are the theorems presented demonstrated? Are the applications of that content seen in other areas of knowledge? Are historical aspects related to that mathematical content?

In Philot's research [14] the book analyzed was Introduction to Linear Algebra with Application [25]. Especially in relation to applications, it was analyzed more specifically the section 8.6 entitled Differential Equations, in which the author discusses homogeneous linear systems and shows how to obtain the general solution of the problem and then exposes as an application a diffusion process. The approach to this problem is understandable from a mathematical point of view, but in Philot's [14] view, it is not enough in the sense of physical interpretations. Putting herself in the teacher's place, the researcher points out that she would probably feel insecure using only the information provided in the book. Another fact to highlight is that with this view it is not possible to know in which theory to go deeper to better understand that problem; and in which courses and undergraduate programs this type of situation is usually addressed. In conclusion, the didactic material under study deals with applications in a very superficial way and ends up serving only for the teacher to tell the student that this subject applies in certain situations, but they are not enough for a more articulated approach with the specific areas of applications in linear algebra classes.

After investigating the materials related to Control Theory and following the classes of this course, while analyzing the Linear Algebra book, specifically regarding the applications in differential equations, Philot [14] realized that the physical model addressed by the author, related to the diffusion of a fluid between two adjacent cells separated by a permeable membrane, is in the form of state space and, therefore, can be studied all its dynamics and stability by analyzing the dynamic modes (eigenvalue and eigenvector) of this system. However, as Philot [14] emphasizes, a teacher who has never studied Control Theory, probably would not know how to work this problem with students linking the mathematical approach to the Control one, nor to say how and in what way this subject is approached in a certain Engineering program. Therefore, for a mathematics teacher, it is important to work collaboratively with a teacher of the area, even if informally.

As a result of the implementation of the five subsidies explained above, the Contextualized Event presented below was elaborated.

The elaborated Contextualized Event

The Event elaborated from the previous steps is presented in Chart 1.

Chart 1. Contextualized Event as presented to the student.

Problem: The pandemic caused by COVID-19 has drastically changed life as we knew it, affecting the entire world population. According to data from the World Health Organization by October 8, 2021 we had 236,599,025 confirmed cases with a total of 4,831,486 deaths (https://covid19.who.int/ - accessed 10/10/2021). We could observe throughout the years 2020 and 2021 that the pandemic presented different characteristics in each country, depending, for example, on the measures taken to combat it. In some countries, states and cities were free to determine their own measures, and thus the pandemic within a country presented different characteristics, while in other countries this decision making was for the country as a whole. We heard experts and news reports estimating when the peak number of infected would occur for different localities, that if certain measures were not taken the number of infected would reach a certain value, what we could do to "flatten" the curve of infected and not overload the hospitals, that the number of infected could grow exponentially if lockdowns were not taken, that if governments did not decree lockdown we would have a large number of deaths. In summary, governmental decision-making was based on epidemiological models (mathematical models) that study the spread of infectious diseases. After this brief introduction, the goal of these meetings is to study the dynamics and stability of an infectious disease spread model, for the case of the disease COVID-19, for a population of 500 people over a period of 100 days of analysis. Consider the following initial conditions $\beta = 1e - 3\ (10^{-3})$, $r = 1e - 1\ (10^{-1})$, $S(0) = 499$, $I(0) = 1$ and $R(0) = 0$. It is known that these values do not

represent an epidemic, however these were chosen for better understanding and comprehension of the model.

Source: Philot [14]

The elaboration of the Contextualized Event is only the first step for an approach linking Mathematics and Engineering. It is necessary to organize the implementation of the event based on the teaching objectives.

For this, as Camarena [15], [20] emphasizes, it is fundamental to: 1) understand what is the objective and what is intended to be achieved with Event; 2) identify the variables and constants of the Event; 3) identify the concepts and themes involved in the Event; 4) determine the relationships between the mathematical contents and the contents of the contexts involved; 5) build the mathematical model for solving the Event; 6) solve the mathematical model proposed (thinking of all solutions); 7) determine the Mathematical solution for the Event; 8) interpret the Mathematical solution in the context of the Event; 9) and finally, depending on the purpose of the Event, present the contents of Mathematics in a decontextualized way.

Another aspect to be mentioned is that this same elaborate Event, with didactic organizations contemplating different questions, can be used to teach other mathematical contents besides eigenvalue and eigenvector.

In this article, as the purpose is to explain only the procedures employed for the elaboration of the Event and not to present the didactic organization for its implementation, it is recommended, in case of interest, to consult [14].

Final considerations

The process experienced by Philot [14] to elaborate the Contextualized Event exposed in her thesis, allowed not only the creation of a situation to teach Mathematics in a way related to Engineering, but also provided the opportunity to rethink the moment in which the essential contents of Linear Algebra should be presented to future engineers.

Perhaps, if some topics of this area were taught starting in the third year of the undergraduate program, for example, and not in the first year, as it was shown, from subsidy 1, it would be possible to work problems closer to the professional life of the engineer, making the course less abstract. Moreover, students would have more mathematical maturity to effectively learn these concepts, besides making it easier for the teacher of the specific courses that address these topics, since the notions of eigenvalue and eigenvector would be more present for the student.

The focus of this article is not the implementation of the Event, since, so far, this has been used only in a pilot experiment used to collect data in the investigation by Philot [14]. This is a research with a qualitative approach and, therefore, the results obtained in terms of learning cannot be widely generalized based on statistical criteria. Despite this, some specific comments about the perception of the six students who participated in the pilot experience can be highlighted.

They felt more motivated and noticed the importance of learning concepts studied in the initial years of the Engineering programs, and that they were able to notice the relevance of mathematical contents in contextualized problems in their area of education approaching an everyday subject.

It is hoped that Contextualized Events structured in this way will help students to connect mathematical knowledge acquired in other contexts of their education or even their professional life, enabling them to have a holistic education and a more meaningful learning, resulting from a better understanding of where and how the mathematical content being taught is used.

Thus, works such as the one developed can serve as reflection not only for the basic courses of Engineering programs, but also for the specific courses, which should be reconsidered based on the current context, the needs of companies and what is recommended by the documents that guide the proposition of Engineering programs. Finally, Contextualized Events can also be used as instruments in the continuing education of Mathematics teachers who work in Engineering.

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