ASEE's 123rd Annual • Conference & Exposition • New Orleans, LA • June 26-29, 2016

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Paper ID #14719

End Fixture Design to Enhance Column Buckling Lesson

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End Fixture Design to Enhance Column Buckling Laboratory Experiment

Abstract

Column buckling is an important topic in strength of materials courses. This topic has been emphasized with a compression/buckling experiment using a Satec uni-axial testing machine to compressively load 1/2 inch diameter Polyvinyl Chloride (PVC) pipe columns to failure due to buckling. Several lengths of pipe have typically been used to demonstrate failures due to both normal compressive stresses and column buckling for a variety of slenderness ratios and end fixity conditions. In the past, the butt ends of the PVC pipes were placed against the moving loading platform and the stationary stage of the Satec machine to approximate an end fixity condition similar to that of a "fixed-end"; however, often buckling would occur in two stages, especially for taller PVC columns. First, the PVC columns would behave as if they were fixed at both bearing surfaces. Minor lateral deflection prior to the critical buckling load lead to eccentric loading conditions at the bearing surfaces as the neutral axis shifted. This eccentric load caused local deformation in the portion of the PVC loaded highest in compression at the surface, which accelerated the asymmetry and caused the opposite portion of the pipe to experience lower and lower compressive loads. At a critical point, the end fixity transitioned from an approximately cantilevered connection to something more closely related to a pinned connection when the butt end dramatically pivoted about a corner of the PVC tube resulting in a single point of contact remaining between the tube and the surface. Although this transition was interesting and provided an opportunity to discuss in more detail why the demonstration progressed the way that it had, the ambiguity of end fixity made it difficult for students to correlate the resulting load-deflection curves to those predicted by theory.

To better simulate loading conditions discussed in lecture, two sets of fixtures were designed and fabricated. One set (top and bottom) was rotationally fixed and provided support for the pipe ends to simulate a cantilevered/fixed end condition. The second set was designed to hold the pipe end securely but allowed rotation about one axis to better approximate a "pinned" joint. Both sets of fixtures worked very well in practice. The effective length factor calculated by the students as a result of using these fixtures was very comparable to predicted values.

Introduction

Buckling of axially loaded columns is an important topic for mechanical and civil engineering students. Compressive stresses can be below the yield strength of the material, but the column may still fail due to a lateral deflection, called buckling. This concept is obviously relevant to the design of building structures and is also critically important for machine design as well. Teaching buckling to engineering students through hands-on experiments helps to reinforce the concept but can be challenging to properly simulate.

The Strength of Materials course lectures at Pitt-Johnstown are supplemented by a laboratory activities intended to reinforce theory with hands-on experience with various modes of failure. In lecture, students are taught that the critical buckling load is dependent on four factors: the material's modulus of elasticity or Young's Modulus (E), the column's second area moment of inertia (I), the column length (L), and an effective length factor (K) that depends on the end conditions of the column. End conditions include pinned (restricted position), fixed (restricted position and angle), and others.

In the column buckling laboratory activity, students observe behavior predicted by theory and collect data for comparison between theoretically predicted and experimentally measured failure loads. A Satec uni-axial testing machine is used to compress ½ inch PVC pipe of various lengths to failure. PVC pipe was chosen because it is inexpensive and fairly easy to buckle (low critical loads) resulting in minimal wear-and-tear on the Satec machine. A 5" long piece (which failed due to compression and not buckling), a 20 inch piece, a 24 inch piece, a 30 inch piece and a 35 inch piece have typically been tested. The purpose of the shorter piece, which failed due to compression, was to provide a stress-strain curve for estimation of the material's elastic modulus. As longer PVC pipe specimens were tested, the force required to cause buckling decreased. This procedure worked reasonably well, but the inconsistent nature of PVC piping (not a truly straight column) obscured the concept that the experiment was designed to demonstrate, especially for longer pipe sections.

In the past, the pieces of pipe were pressed between the loading table and the crossbar of the Satec machine (see Figure 1). This end-to-surface connection condition is not described by the theory as presented in lecture and has no recommended effective length factor. Lateral motion is restricted by the friction between the ends of the pipe and the support surfaces and the angle (with respect to horizontal) of the column ends is restricted to some extent by the flat ends of the pipe contacting the rigid support surfaces. However, at higher loads, one side of the pipe can separate from the table and crossbar prior to the critical buckling load being reached. Without fixtures holding the ends of the pipe in place, the butt-to-surface end fixity condition *initially* most closely resembles a fixed/cantilevered condition, but, upon lifting, more closely resembles a pinned connection.

Figure 1: Pipe column buckling in the Satec testing machine showing original end condition.

Background

Using experimental data, students calculated the critical load using Euler's formula[1] provided in Equation 1.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \qquad Eqn\ (1)$$

Where:

P_{cr}	= Critical load that will cause the member to buckle.
E	= Young's modulus (Modulus of Elasticity)
I	= Moment of Inertia of the cross section.
L	= Length of the column
K	= Effective length factor[2]

Young's modulus, E, is determined through examination of the stress-strain curve produced from the data collected during compression of the shortest (5 inch) pipe. Using the initially linear portion of the stress-strain plot from that test, Young's modulus is calculated by Hooke's Law[3], Equation 2.

$$\sigma = E\epsilon \qquad Eqn\ (2)$$

Where: σ = Stress
ϵ = Strain
E = Young's Modulus (Modulus of Elasticity) (490,000 psi)[4]

The moment of inertia required in Equation 1 is a geometric property of the cross section of a column, in this case, the pipe,[5] and is determined using Equation 3.

$$I = \frac{\pi[(OD)^4 - (ID)^4]}{64} \qquad Eqn\ (3)$$

Finally, the effective length factor (K) is multiplied by the column length (L) to account for the end conditions. Table 1 shows K factors for commonly used end conditions. Students were required to compare the numbers from this table to the numbers resulting from their analysis of the Satec data using Equation 1.

Table 1: End conditions and equivalent length coefficient (K)

Condition	K
Both ends pinned	1.0
Both ends fixed	0.5
One end fixed and the other end free	2
One end fixed and the other end pinned	0.7

Problem Statement

The laboratory exercise requires students to predict the behavior of the columns based on what they learned in lecture. Since the end conditions used in the original test set up are not included in any discussion of theory or table of equivalent length factors, the students are confused and the educational benefit lessened. Data from the tests are used to calculate an actual K factor for the four lengths of pipe, and theory predicts that the K factor should be dependent only on end conditions, not on length. When students see that the actual K factor varies with length, their confidence in the theory is compromised.

Solution

The solution to the problem outlined is to secure the ends of the pipes so that they are fixed or pinned rather than just resting on the platform and crossbar. To that end, two sets of end fixtures were designed and fabricated. One set of fixtures held the end orthogonal to the surfaces and would not allow any rotation. Figure 2 shows the weldment and plastic insert for these two end fixtures. The fixture on the left was placed on the loading platform while the right fixture was bolted to the bottom of the stationary crossbar.

Figure 2: Non rotation end fixtures.

Using these fixtures to secure the ends, a 24 inch piece of PVC pipe was compressed in the Satec machine until it began to buckle (Figure 3). Student data obtained using this experimental set up yielded results that better matched theoretical values predicted using Equation 1.

A closer inspection of Figure 3 shows the relatively straight sections of pipe near the top and bottom fixtures. This shows the effect of a “secured” end condition resulting in K = 0.5. The inflections were thus located about a quarter of the length of the pipe from both ends.

Figure 3: PVC pipe buckling with fixed end fixtures.

Another fixture was designed to replicate pinned ends (see Figure 4). This fixture allowed the ends to rotate as if they were connected via pins to the Satec machine. A small amount of lubricant is applied to the fixture to allow for better rotation. According to theory, the K factor for pinned ends is 1, largely because the end-fixity does not allow any movement to be transferred from the connection to the load.

Two of these pinned fixtures were produced; one for the horizontal loading platform and one for the stationary crossbar. The axes of the pins were aligned to allow the pipe to bend in the same direction for both fixtures.

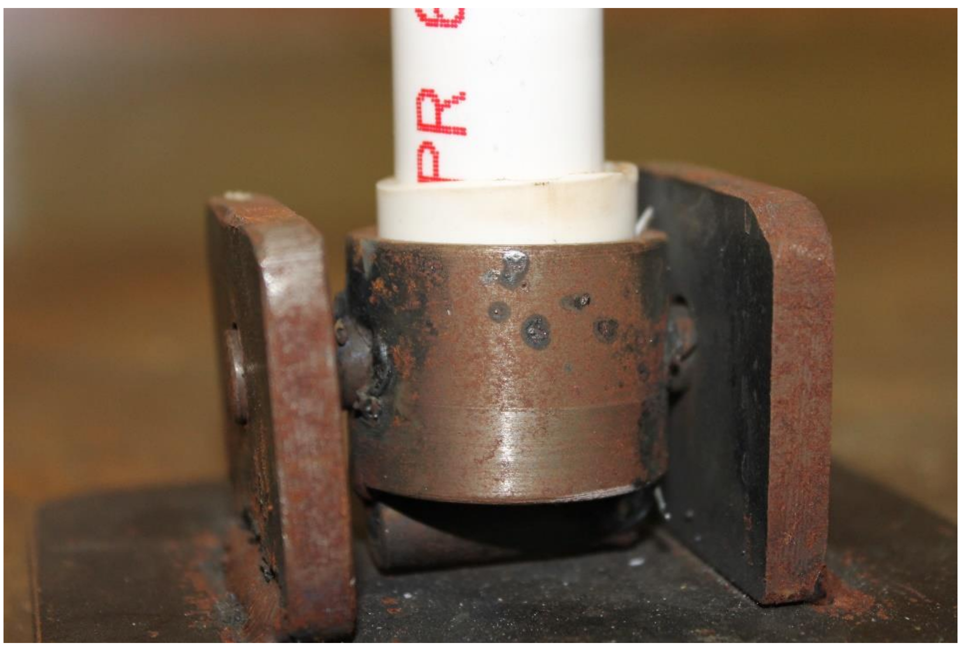

Figure 4: Bottom pinned end fixture. The PVC is constrained by a small piece of rod welded across the bottom of the cylindrical “cup”. That rod does never contacts the lower plate. The “cup” can freely rotate about a left-to-right axis through its center.

With the pipe placed in the pinned fixtures, the Satec machine compressed the column. As shown in Figure 5, the pipe buckling shows a curved outline with no inflection points. This corresponds to an effective length factor of K = 1.

Figure 5: Example of pinned end fixity buckling of the PVC pipe.

To verify the effectiveness of these buckling fixtures, the K factor was calculated for the 24” pipe length. Without a fixture, the end conditions were always assumed to be fixed (or K=0.5). Table 2 shows the comparison of experiments conducted on the same machine, using the same size PVC pipe, both with the fixtures and without.

Table 2: The effective length factor, K, shows the effectiveness of the end fixtures.

End Conditions	No Fixtures	With Fixtures	Theoretical
Both Ends Fixed	0.958	0.675	0.5
Both Ends Pinned	N/A	0.721	1.0

The fixtures increased the accuracy of the experiments numerically, but for the students, the visual aspects were more beneficial as the shape of the buckling columns mirrored the theoretical shapes as expected.

Although second order buckling was beyond the scope of this laboratory activity, this phenomena was briefly investigated for demonstration purposes. Figure 6 shows before loading setup and the after buckling occurred. The column was braced with duct tape and required very little support to resist first order buckling. The pinned ends were aligned to aid in demonstration of this phenomena. The second picture clearly shows how second order buckling affects the PVC column. Higher order buckling is scheduled for future development.

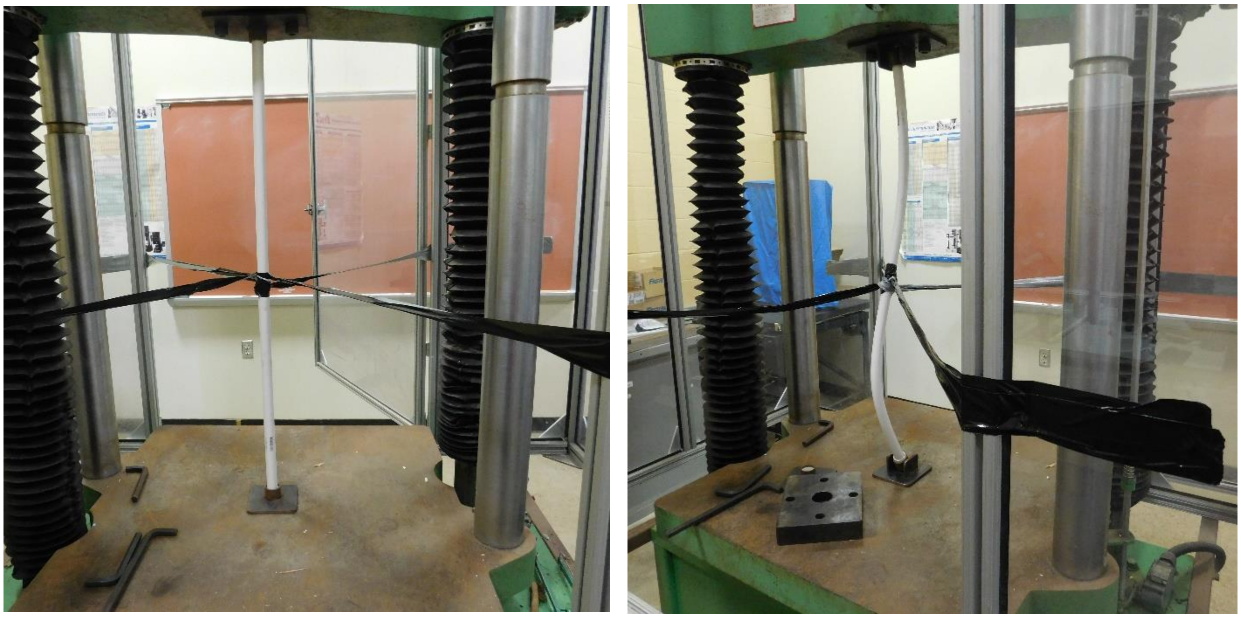

Figure 6: Second order buckling showing before loading and after the column buckled.

Conclusions

The Strength of Materials instructor has developed a simple laboratory procedure to teach engineering students the concept of column buckling. To enhance the activity, and thus enhance the learning experience, two sets of fixtures were designed and built to hold the ends of PVC pipes during compression loading in a Satec testing machine. The fixtures provide end fixity that more truly represents the ideal cantilevered and pinned end connections for a column subjected to compressive forces.

Column buckling is thoroughly studied in the Strength of Materials class (lecture). By utilizing this simple buckling experiment in a supplementary laboratory class, many teaching elements can be demonstrated and discussed with the students. One of these elements is higher order buckling. A demonstration of second order buckling is shown, but higher order buckling can also be discussed.

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