ASEE'S VIRTUAL CONFERENCE
At Home with Engineering Education JUNE 22 - 26, 2020 #ASEEVC
Paper ID #28924

Engineering Application Projects for Teaching Engineering Mathematics and Numerical Methods

Prof. Shengyong Zhang, Purdue University Northwest

Shengyong Zhang (syzhang@pnw.edu) is an Associate Professor of Mechanical Engineering at the Purdue University Northwest. He has teaching and research interest in the areas of computer modeling and simulation, vibration and acoustics, system dynamics, and automobile lightweight design.

Prof. Alain S Togbe, Purdue University Northwest

Dr. Alain Togbe is a professor of Mathematics at Purdue University Northwest. He received a master's degree and a Ph.D. from Universite Laval, Quebec, Canada. His regular research area is Number Theory. For many years, he is teaching the sequence of Calculus, Linear Algebra and Differential Equations to engineering students with a component of projects.

Engineering Application Projects for Teaching Engineering Mathematics and Numerical Methods

Shengyong Zhang[1] and Alain Togbe[2]

[1] Department of Mechanical and Civil Engineering, College of Engineering and Sciences, Purdue University Northwest
[2] Department of Mathematics, Statistics and Computer Science, College of Engineering and Sciences, Purdue University Northwest

Abstract

One ABET student outcome focuses on the ability to identify, formulate, and solve complex engineering problems by applying principles of engineering, science, and mathematics. Engineering Mathematics and Numerical Methods are two upper-level courses offered to mechanical engineering students at the former Purdue North Central. These courses are designed to provide students mathematical tools for other mechanical engineering core courses and equips students with modelling skills to formulate engineering problems into mathematical forms for solutions.

This paper documents an effort to integrate engineering vibration and heat transfer related projects into these two mathematical courses. These projects have been designed with an attempt to build a bridge between real engineering problems and the mathematical concepts and theorems learned in classes (Laplace transform, transfer functions, finite difference methods, for example). Student feedback on these projects is positive.

1 Introduction

Mechanical engineering students usually complete Calculus I and II, Multivariate Calculus, Linear Algebra and Differential Equations in four or five semesters. All these mathematical courses are required in the program curriculum. Primary contents that students learn from these courses include: (1) differentiation and integration, calculus of one variable and infinite series in Calculus I and II; (2) differential and integral calculus of functions of two or more variables and vector functions in Multivariate Calculus; (3) systems of linear equations, matrix algebra, vector spaces, determinants, eigenvalues and eigenvectors and diagonalization of matrices in Linear Algebra; and (4) first order differential equations, second order differential equations, linear and nonlinear systems of differential equations in the course of Differential Equations.

Engineering Mathematics is offered in the junior year and requires Multivariate Calculus, Linear Algebra and Differential Equations as the prerequisites. While the prerequisite courses in "calculus" and "differential equations" have fairly standard contents, Engineering Mathematics may vary considerably in course contents [1]. Due to the primary objective of this course is to provide students the necessary engineering mathematical knowledge for other core courses in mechanical engineering curriculum (e.g., applications of vector calculus in Fluid Mechanics, applications of eigenvalue problems in Vibration Analysis), the author consulted with other faculty members and selected the following main course contents: (1) the Laplace Transform, including its applications in solving differential equations with initial conditions, the two translation theorems, and convolution integrals; (2) the Fourier analysis, including Fourier series, Fourier integrals, and Fourier transforms; (3). vector calculus including divergence, curl, line integrals, surface integrals, Green's theorem and Stokes' theorem; and (4) partial differential equations (PDEs) with Neumann, Dirichlet, or combined boundary conditions. Much of these topics are traditionally admitted in the Engineering Mathematics course [2]. Furthermore, MATLAB and its applications are emphasized in this class for automatic processing of mathematical operations and visual plotting of mathematical solutions.

Numerical Methods is offered in the senior year and requires Engineering Mathematics as its prerequisite. Unlike Engineering Mathematics, which is a mandatory course in the ME program curriculum, the Numerical Methods is an engineering elective. Various analytical methods learned in the Engineering Mathematics result in analytical solutions for some engineering problems. These solutions are often useful and provide excellent insight into the behavior of some systems and processes. However, analytical solutions can be derived for only a limited class of problems that can be approximated with linear models and problems that have simple geometry and low dimensionality. Numerical methods are techniques by which mathematical problems are formulated so that they can be solved with arithmetic operations (finite element method, FEM, for example). Numerical methods are extremely powerful tools for solving problems that are nonlinear and involve complex shapes and processes. With appropriate assumptions and interpretations, numerical methods can result in acceptable approximate solutions for many real engineering problems. The author consulted with other faculty members and selected the following main course contents: (1) curve fitting methods, including least-squares regression, interpolation and Fourier approximation; (2) numerical integration and differentiation; (3) numerical methods for solving ordinary differential equations (Runge-Kutta methods, for example); and (4) numerical methods for solving partial differential equations, including finite difference method (FDM) and finite element method (FEM) [3]. Again, MATLAB and its applications are emphasized in this class for automatic processing of mathematical operations and visual plotting of mathematical solutions.

This paper documents two course projects designed for the Engineering Mathematics class and the Numerical Methods class, respectively. These projects help students realize the real engineering applications of the mathematical concepts and theorems learned in classes and enhance their ability to identify, formulate, and solve complex engineering problems by applying principles of mathematics. Student feedback on these projects is positive.

2. Course Project for Engineering Mathematics

Quarter-car model is widely used in automotive engineering to simulate the dynamic behavior of vehicle suspension systems [4]. The main functions of a suspension is usually simulated by spring and damper components which provide the necessary ride isolation at each wheel. Figure 1 shows a simple quarter-car model. The sprung mass (M) represents the mass of the vehicle supported on the suspension and the unsprung mass (m) is defined as the total mass of the parts being connected to the wheel directly. The stiffness and damping coefficient of the suspension are denoted, respectively, by K_s and C_s. K_t denotes the stiffness of the tire whose damping effect is ignored.

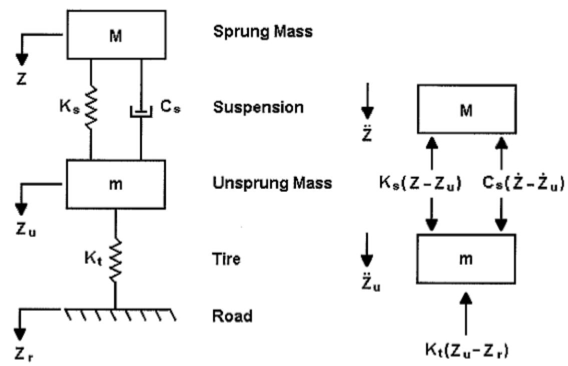

Figure 1 Configuration of a quarter-car model.

A systematic treatment of the vehicle as a dynamic system best starts with the basic properties of a vehicle on its suspension system – i.e., the motions of the body and axle. At low frequencies the body, which is considered to be the sprung mass portion of the vehicle, moves as an integral unit on the suspensions. This is rigid-body motion. The axles and associated wheel hardware, which form the unsprung masses, also move as rigid bodies and consequently impose excitation forces on the sprung mass.

The dynamic behavior of a vehicle can be characterized most meaningfully by considering the input-output relationships. In this project, the road excitation, Z_r, is chosen to be the input to the suspension system, and it will be transmitted to the occupants through the wheel/tire assembly, suspension system, frame, and cab. From the ride quality point of view, designers are mostly interested in the vibration of the sprung mass, Z, which is used as the system output. The ratio of output and input amplitudes represents a "gain" for the dynamic system. The term "transmissibility" is often used to denote the gain. Transmissibility is the nondimentional ratio of response amplitude to excitation amplitude for a system in steady-state forced vibration.

The primary objective of this project is to study the sprung mass motions in response to harmonic excitations from road surface as a function of frequency. This is accomplished by building differential equations of motion for both sprung and unsprung masses and transforming these differential equations from time domain to frequency domain by using Laplace transform. Another objective is to study the effects of changes in the tire parameter (K_t) and suspension parameters (K_s and C_s) on the vehicle transmissibility.

Based on the free-body diagrams in Figure1, one may build equation of motion for each mass as:

$$M\ddot{z} = -K_s(z - z_u) - C_s(\dot{z} - \dot{z}_u) \quad (1)$$

$$m\ddot{z}_u = K_s(z - z_u) + C_s(\dot{z} - \dot{z}_u) - K_t(z_u - z_r). \quad (2)$$

Next, the Laplace transform is taken for each equation:

$$Z(s)(s^2M + sC_s + K_s) - Z_u(s)(sC_s + K_s) = 0 \tag{3}$$

$$Z_u(s)(s^2m + sC_s + K_s + K_t) - Z(s)(sC_s + K_s) = K_t Z_r(s) \tag{4}$$

These equations are rearranged to eliminate $Z_u(s)$ and then solved for the ratio $Z(s)/Z_r(s)$:

$$\frac{Z(s)}{Z_r(s)} = \frac{K_t(sC_s + K_s)}{s^4Mm + s^3C_s(M+m) + s^2(K_s(M+m) + MK_t) + sC_sK_t + K_sK_t}. \tag{5}$$

This ratio is the transfer function which characterizes the ratio output response to input excitation in s domain. Substituting s = jω results in the frequency response of the system:

$$\frac{Z(j\omega)}{Z_r(j\omega)} = \frac{K_sK_t + j\omega C_sK_t}{\omega^4Mm - \omega^2(K_s(M+m) + MK_t) + K_sK_t + j(-\omega^3C_s(M+m) + \omega C_sK_t)}. \tag{6}$$

The magnitude of this ratio gives the transmissibility which is a function of frequency:

$$\left|\frac{Z(j\omega)}{Z_r(j\omega)}\right| = \frac{K_s^2K_t^2 + \omega^2C_s^2K_t^2}{(\omega^4Mm - \omega^2(K_s(M+m) + MK_t) + K_sK_t)^2 + (-\omega^3C_s(M+m) + \omega C_sK_t)^2}. \tag{7}$$

Figure 2 shows the transmissibility in term of frequency. One can change design variables (K_t, K_s and C_s) in the above transmissibility expression to study the effects of design changes on vehicle responses. One group chose to change the ratio K_s/K_t to study the corresponding changes in vehicle responses, see Figure 3.

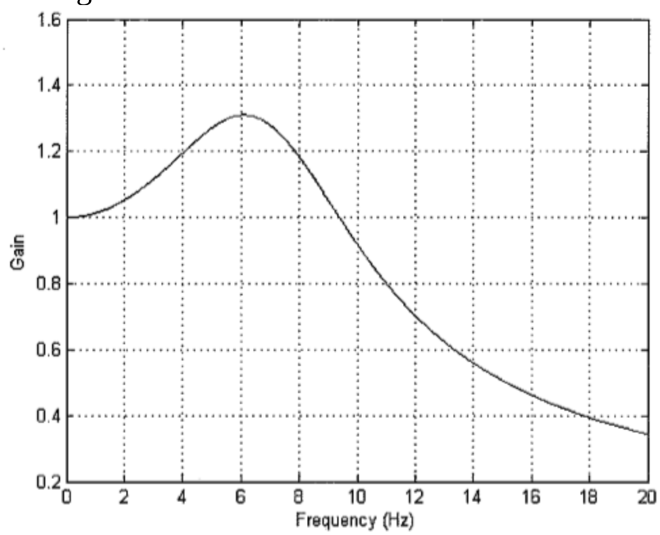

Figure 2 Nondimensional ratio of response amplitude to excitation amplitude as a function of frequency. M = 450 kg, m = 45 kg, K_s = 31250 N/m, K_t = 250000 N/m, and C_s = 3000 N*s/m.

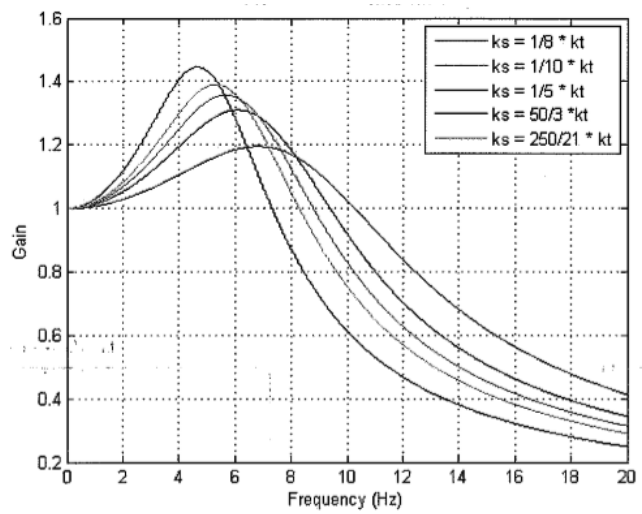

Figure 3 Changes in the vehicle transmissibility due to changes in ratio K_s/K_t .

3. Course Project for Numerical Methods

Finite difference methods solve partial differential equations (PDEs) by dividing the solution domain into a grid of discrete points or nodes, writing the PDE for each node, and replacing its derivatives by finite-divided difference. Linear, second-order PDEs in two variables are covered in this class and they can be classified into three categories: elliptic, parabolic and hyperbolic. Elliptic equations are typically used to characterize steady-state system, indicating the absence of a time derivative. These equations are typically employed to determine the steady-state distribution of an unknown in two spatial dimensions (temperature distribution on a heated plate with boundaries are held at different temperatures, for example). Parabolic equations are employed to characterize how an unknown varies in both space and time. This is manifested by the presence of both spatial and time derivatives in the equations (a long and thin rod that is insulated except of its ends). Hyperbolic equations also deal with an unknown varies in both space and time, but an important distinction is that the unknown is characterized by a second derivative with respect to time (a vibrating string, for example). The assigned course project is based on applications of parabolic equations in the heat flow along a rod.

A long, thin copper rod has a length of 7 in. and a diameter of 1 in. It is assumed that the heat flows only in the longitudinal direction and the curved surface has been insulted. The heat equation is

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial T^2(x,t)}{\partial x^2} - \frac{4h}{\rho c}(T(x,t) - T_\infty). \quad (8)$$

Where α = 3.08e-6 m^2/s, h = 11.5 $w/m^2 \cdot k$, ρ = 8.49 kg/m^3, c = 390 kJ/kg·k and $T_\infty = 0$.

Boundary conditions are specified as that the right boundary is insulted and the left boundary (x=0) is represented by

$$\frac{\partial T(L,t)}{\partial x} = 0$$

$$\frac{\partial T(0,t)}{\partial x} = -q\,. \quad (9)$$

Where k = 401 N/m·K and q = 20 w. Initial condition is specified as that the temperature over the rod $T(x,0)$ = 290°k.

The objective of the project is to build numerical models based on finite difference methods for predicting the changes of temperature along the rod with respect to time. The 7-inch-long rod is divided into n uniform segments, with nodes numbered from 0 on the left end to n on the right end. The first-order and second-order derivatives in the heat equation are approximated by finite divided differences respectively

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1}^{l} - 2T_i^l + T_{i-1}^l}{\Delta x^2}$$

$$\frac{\partial T}{\partial t} = \frac{T_i^{l+1} - T_i^l}{\Delta t}. \qquad (10)$$

Substituting into the heat equation gives the numerical model at each interior node:

$$T_i^{l+1} = T_i^l + \alpha \frac{\Delta t}{\Delta x^2}\left[T_{i+1}^l - \left(2 + \frac{4h\Delta x^2}{\rho c \alpha}\right)T_i^l + T_{i-1}^l\right], \quad \text{i =2, 3… n-1} \qquad (11)$$

Where the superscript (l) denotes the time step, while the subscript (i) the node number along the rod.

Derivative boundary conditions are specified in this project and can be incorporated into parabolic equations. Two imaginary nodes (i = -1 and i = n+1) are introduced outside the rod at each end. These exterior fictitious nodes serve as the vehicle for incorporating the derivative boundary conditions into the problem. This is done by representing the first-order derivatives by the finite divided difference at both ends:

$$T_{-1}^l = T_1^l + 2\Delta x\left(\frac{q}{k}\right)$$

$$T_{n+1}^l = T_{n-1}^l. \qquad (12)$$

Substituting into the above numerical model gives the temperature at each end respectively:

$$T_0^{l+1} = T_0^l + \alpha \frac{\Delta t}{\Delta x^2}\left[2T_1^l - \left(2 + \frac{4h\Delta x^2}{\rho c \alpha}\right)T_0^l + 2\Delta x\left(\frac{q}{k}\right)\right] \qquad (13)$$

$$T_n^{l+1} = T_n^l + \alpha \frac{\Delta t}{\Delta x^2}\left[2T_{n-1}^l - \left(2 + \frac{4h\Delta x^2}{\rho c \alpha}\right)T_n^l\right]. \qquad (14)$$

Figure 4 shows the distribution of the temperature along the rod at difference instants from 0 to 5 s at an interval of 1 s.

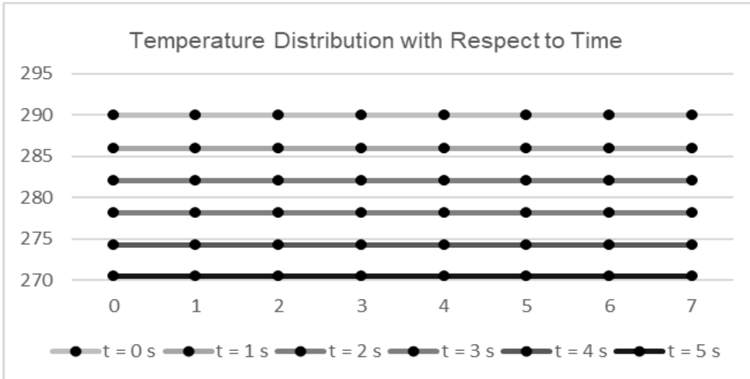

Figure 4 Temperature distribution along the rod at different instants at 1s intervals

4. Project Management

Project-based Learning (PBL) helps students develop skills in critical and creative thinking, written and oral communication, collaboration and problem solving. Especially, the assigned projects help students develop an ability to apply knowledge of mathematics to identify, formulate, and solve mechanical engineering problems.

Team projects have advantages over individual projects in that more and more engineering research problems require collaboration of investigators with different expertise. Another advantage is the improvement of students' skills for dealing with personality clashes and making decisions efficiently. Each team is formed voluntarily without any interference from the instructor and it is completely the students' responsibility to allocate workloads among the team members.

Grading is based upon project reports and oral presentations. Project reports are required to be in the standard format, including specifications on the font, margins, page numbers, paragraphs, and sections. Equations should be centered with sequential number in parenthesis. Figures and Tables must be captioned and sequentially numbered. Grading on reports is based on a rubric focusing on the introduction and statement of project objectives, complete and sufficiency analysis, and discussions of the project results. Oral presentations play an important role in developing communication skills. Team members cooperate in presentations, illustrating their research work, analysis details, and concluding remarks. At the end of each presentation, other teams may ask questions regarding the presenters' analysis and results, or they may challenge the presenters with questions from their own studies. Grading on oral presentations is based on a rubric focusing on the knowledgeable about the topic as evidenced by the depth and completeness of the presentation, logical sequence of the materials in the presentation, and answers and further explanations to audience questions.

5. Student Feedback

Post-semester evaluations of these mathematical courses were conducted in the second half of the next courses, including Fluid Mechanics, Vibration Analysis, System Modeling and Analysis, and Heat Transfer. These courses require a mathematical sophistication beyond calculus and ordinary differential equations. Eleven questions are designed in the survey, among which there are two questions related to the course projects:

1) Did the course improve your engineering problem solving ability? How can we make it better?
2) Did we cover enough (or too much) on the below material: Laplace transformation, partial differential equations, numerical methods and problem solving.

Responses to the first question show that more than 80% students benefit from the course projects which enhancing their ability in solving real engineering problems with mathematical tools and modellings. Many students suggested to integrate more realistic engineering problems in teaching mathematical classes.

Figure 5 shows the student responses to the second question. It seems that students realize that partial differential equations have so many applications in the fields of mechanical engineering they have learned in classes. Again, students suggest to have more realistic engineering problems based projects in their feedback to this question.

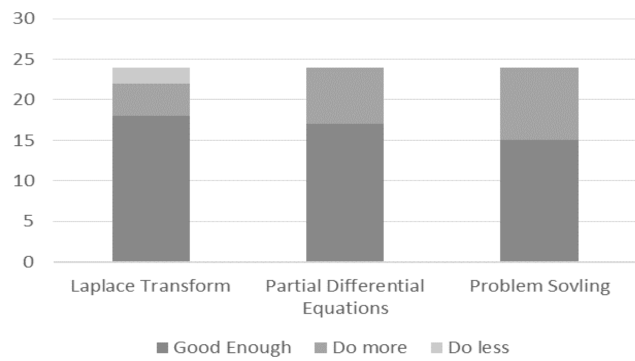

Figure 5. Student assessments of engineering projects on their learning

6. Conclusions

This paper documents an attempt to integrate engineering problems as course projects in teaching Engineering Mathematics and Numerical Methods. The first project is about the dynamic analysis of vehicle suspension systems. Students have enhanced their understandings of differential equations and Laplace transform through modelling the suspension behavior and transferring the analysis results from time domain to frequency domain. The second project is the application of parabolic equations and finite difference methods in simulating the heat transfer on a rod. Students have learned from this project that numerical methods are extremely powerful tools for solving complicated engineering problems that are often impossible to solve analytically.

References

1. Hennessey, M. (2015). Advanced Undergraduate Engineering Mathematics, 122nd ASEE Annual Conference & Exposition, Seattle, WA.
2. Zill, D. (2018), Advanced Engineering Mathematics, 6th edition, Jones and Bartlett.
3. Chapra, S. (2015). Numerical Methods for Engineers, 7th edition, McGraw-Hill Higher Education.
4. Gillespie, T. (1992). Fundamentals of Vehicle Dynamics, Society of Automotive Engineering, Inc.

Biographical Notes

Dr. Shengyong Zhang (syzhang@pnw.edu) is an Associate Professor of Mechanical Engineering at the Purdue University Northwest. He has teaching and research interest in the areas of computer modeling and simulation, vibration and acoustics, system dynamics, and automobile lightweight design.

Dr. Alain Togbe (atogbe@pnw.edu) is a Professor of Mathematics at Purdue University Northwest. He received a master's degree and a Ph.D. from Universite Laval, Quebec, Canada. His regular research area is Number Theory. For many years, he is teaching the sequence of Calculus, Linear Algebra and Differential Equations to engineering students with a component of projects.