ASEE's 123rd Annual • Conference & Exposition • New Orleans, LA • June 26-29, 2016

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Paper ID #16957

Engineering Students Understand the Elastic Neutral Axis, but What About the Plastic Neutral Axis?

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Engineering Students Understand the Elastic Neutral Axis, but What about the Plastic Neutral Axis?

Abstract

Starting in engineering statics, undergraduate engineering students are taught how to find and calculate the elastic neutral axis (ENA) for a cross sectional shape by finding the centroid. The concept of the ENA is fundamental in terms of understanding and determining the flexural bending stresses and deflections of beams where plane strains remain plane. However, the plastic neutral axis (PNA) is seldom discussed in undergraduate engineering mechanics courses even in mechanics of materials. Perhaps this is due to the amount of course content in mechanics of materials. However, for students first learning about the PNA in a senior level structural steel course, bridging the gap back to fundamental mechanics is challenging since engineering mechanics courses are taken early on.

This paper presents a review of how to locate the ENA and PNA and their purpose in flexural behavior of beams. A new calculus based expression for calculating PNA is presented. Engineering examples are provided. All of this information was presented, discussed and assessed with two groups of senior civil engineering students in two structural steel design courses. The results of this assessment are also presented. Results indicate that the concept and purpose of the PNA needs to be introduced to students earlier in their engineering coursework. The work presented herein is of interest to civil and mechanical engineering students, engineering faculty, and professional practitioners who are analyzing and designing steel beams for various types of engineering projects where flexural strength and capacity are important.

Keywords: Elastic neutral axis, plastic neutral axis, plastic behavior, mechanics, student learning

Introduction

In undergraduate engineering education, linear elastic material behavior is discussed in detail[1-4]. This starts in engineering statics when students learn how to determine centroids of areas. Centroids represent the elastic neutral axis assuming the beam is prismatic. Linear elastic flexure of beams is further introduced and discussed in engineering statics when moment diagrams and moments of inertia are learned[5]. Later in a strength of materials course, linear elastic flexure is discussed in more detail when bending stresses and deflections of beams are covered. After this, engineering students typical take a course that covers topics in structural analysis. As part of this course, students learn more about linear elastic deformations of beams with more complicated loadings and boundary conditions. Beams may be determinant or indeterminant. Having taken two mechanics courses, including engineering statics and mechanics of materials, and one course with structural analysis topics, civil engineering students start taking design courses such as structural steel design and reinforced concrete design[6,7].

No real mention of plastic behavior is discussed with civil engineering students prior to taking a structural steel design course other than a brief mention of stress strain behavior in a strength of materials course[8,9]. However, a clear understanding of plastic behavior is critical in the design of every civil engineering structure. For example in flexural members, rebar in reinforced concrete structures is designed to yield in tension prior to crushing of the concrete in compression. As another example, beams in steel structures are ideally designed to carry their nominal plastic moment.

Since upper level engineering students do not have an adequate background in plastic behavior, engineering faculty typically resort to teaching these concepts on an as needed basis just to cover the necessary design equations without making a connection back to earlier engineering coursework such as in engineering mechanics[10]. To help make this connection to students in two separate senior level structural steel design courses, a new lecture that discusses the purpose as well as the similarities and differences between the ENA and PNA was presented. Homework assignments were required. Upon completion, both classes were assessed using a survey of questions or statements that required explanation. The results of the survey showed that making a lasting connection between engineering mechanics and plastic behavior is a real struggle since students are not exposed to this material earlier in the engineering curriculum. While civil engineering students can learn the material on an as needed basis to do the design calculations, most were unable to successfully connect these concepts back to fundamental mechanics.

Elastic Neutral Axis and Flexure in Beams

The following discussion presents the elastic neutral axis (ENA) for steel beams as taught in a strength of materials course and again later in a structural steel course. While this discussion herein is limited to steel beams, the concept can be extended to other types of beam materials such as reinforced concrete beams. The ENA of reinforced concrete beams following this concept will be presented in a future paper.

For a prismatic, simply supported beam with only vertical loads as show in Figure 1, the ENA corresponds to the y component of the centroid of the beam's cross sectional area. Along the ENA, there is zero axial strain. Therefore, no bending stresses are present along this axis. Above the ENA, the beam compresses reaching a maximum compression flexural stress along the top of the beam. Similarly below the ENA, the beam stretches in tension reaching a maximum tensile flexural stress along the bottom of the beam.

Using calculus, the location, $\bar{y}$, from the bottom of the cross section to the ENA for a prismatic beam may be found from the following equation.

$$\bar{y} = \frac{\int_A y \, dA}{\int_A dA} \tag{1}$$

Once the location of the ENA is determined, the corresponding bending moment at the top or bottom of the beam may be found from the following equation,

$$M = f \, S_x \tag{2}$$

where f is the flexural stress at the top or bottom of the beam and S_x is the elastic section modulus with respect to the x axis as shown in Figure 2. The flexural stress, f, may be found based on the modulus of elasticity of the beam and the strain. When the top or bottom of the beam reaches yielding, the bending moment of equation (2) becomes,

$$M_y = f_y S_x \tag{3}$$

where f_y is the yield strength of the steel beam. The elastic section modulus, S_x, may be found from the following equation,

$$S_x = \frac{\int_A y^2 \, dA}{c} \tag{4}$$

where c is the distance from the ENA to the top or bottom of the beam as shown in Figure 1.

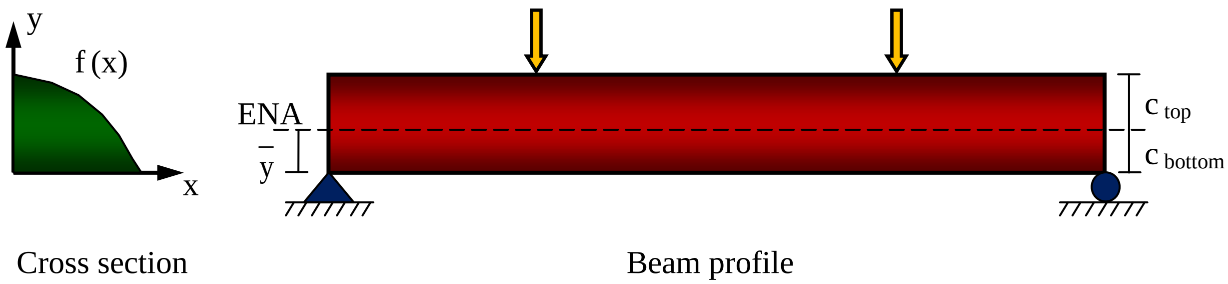


Figure 1. Beam showing ENA and cross section

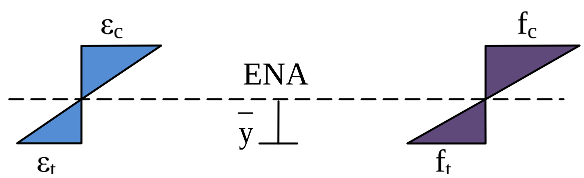


Figure 2. Linear elastic strains and flexural stresses

Plastic Neutral Axis and Flexure in Beams

Plastic behavior is important in the analysis and design of any civil engineering structure. Whether teaching civil engineering students about the nominal moment capacity of reinforced concrete beams or the capacity of beams made of structural steel, a fundamental understanding of yielding and plastic behavior is paramount. When calculating the moment capacity of reinforced concrete beams, the steel rebar is assumed to be yielding, meaning it is exhibiting plastic behavior prior to the concrete in compression crushing. In steel beams, the nominal moment capacity is ideally based on the entire cross section to be yielding. This is the fully plastic condition. Certain conditions in terms of beam stability must be met, but this will permit the steel beam to reach the maximum moment carrying capacity.

The following discussion presents the plastic neutral axis (PNA) for steel beams as taught in a structural steel course. While this discussion herein is limited to steel beams, the concept can be extended to other types of beam materials such as reinforced concrete beams. The PNA of reinforced concrete beams following this concept will be presented in a future paper.

For doubly symmetric cross sectional shaped beams, the location of the PNA is the same as the ENA. This is also true of beams that are symmetric about a single horizontal cross sectional axis such as a C channel as well as some nonsymmetrical cross sectional beams such as a Z purlin. For all other beams, the PNA is different than the ENA. The ENA is the axis corresponding to the center of area where as the PNA correspond to an axis such that half of the area is above the axis and half is below. In the fully plastic state, the entire cross sectional area is yielding. And for the beam to be in a state of equilibrium at this point, the PNA must correspond to this location since the force caused by yielding of the area in compression above the PNA must equal the forced caused by yielding of the area in tension below the PNA.

Using calculus, the location, $\bar{y}_p$, from the bottom of the cross section to the PNA for a prismatic beam may be found from the following equation. See Figure 3.

$$\int_0^{\bar{y}_p} f(x)\,dy = \frac{1}{2}\int_A dA \tag{5}$$

The plastic moment for a steel beam may be found from the following equation,

$$M_p = f_y Z_x \tag{6}$$

where f_y is the yield strength of the steel beam and Z_x is the plastic section modulus with respect to the x axis as shown in Figure 4. The plastic section modulus, Z_x, may be found from the following equation,

$$Z_x = \frac{a}{2}\int_A dA \tag{7}$$

where a is the distance between the centroidal areas of the top and bottom half areas as shown in Figure 3.

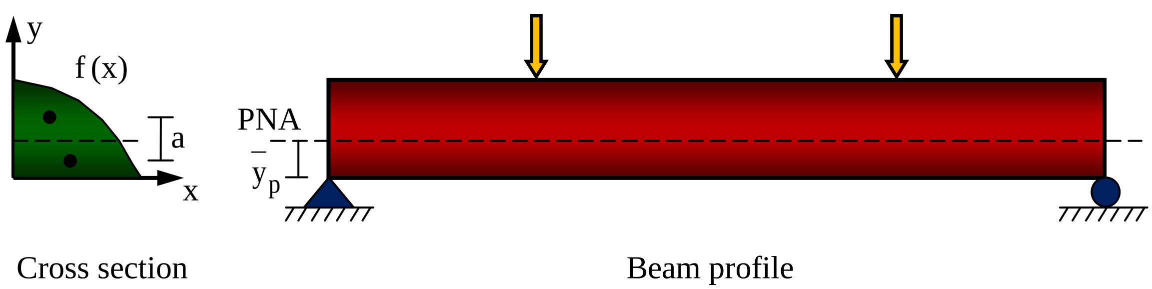


Figure 3. Beam showing PNA and cross section

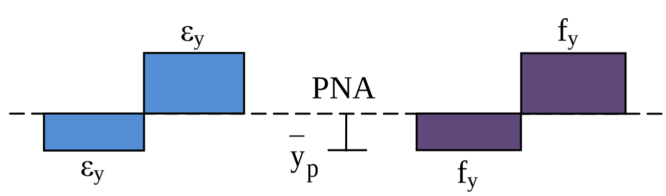
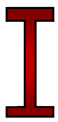
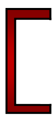
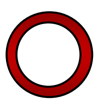
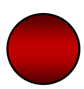
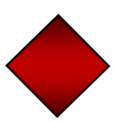
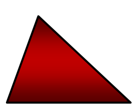


Figure 4. Fully plastic strains and flexural stresses

Bending shape factors have been computed for a wide variety of beam cross sectional shapes. Table 1 shows various beam shapes with their corresponding bending shape factor. Note, the bending shape factor is the ratio of the plastic section modulus, Z_x, over the elastic section modulus, S_x. This may also be thought of as the ratio of the plastic moment, M_p, over the moment at first yielding, M_y. For most commercially available beams, the bending shape factors range from 1.15 for W shaped beams to 1.2 for channel sections. For circular and rectangular tubes, the bending shapes factors increase slightly but only by about 4%. For solid cross sectional shaped beams, the bending shape factor can increase significantly to as high as about 2.3 or so. This corresponds to an increase of about 108% over commercially available W shaped beams. Beam sections that have more material area in the region near the centroid have higher bending shape factors since the bending strength in this area is not exhausted until the total cross section has fully reached the plastic state.

Table 1. Bending shape factors

Geometric shape	**Figure**	**Shape factor**
W beams		≈1.15
C channels		≈1.2
Rectangular tubes		≈1.2
Circular tubes		≈1.3
Rectangles		1.500
Circles		1.698
Rhombuses (square shaped)		2.000
Triangles		2.343

For all the figures shown in Table 1, the ENA and PNA are at the same location except for triangles. If the triangle has a height, h, then the distance from the top of the shape to the ENA is $2/3\,h$ (or $0.667\,h$) whereas the distance from the top to the PNA is $\sqrt{2}/2\,h$ (or $0.707\,h$) as shown in Figure 5.

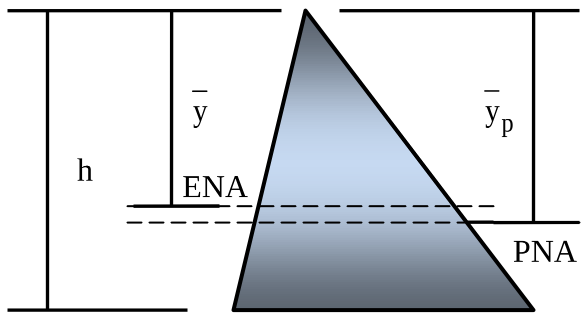


Figure 5. Triangle with the ENA and PNA

Teaching and Student Assessment

For decades, engineering faculty have taught the concept of yielding and plastic behavior in design courses such as reinforced concrete, structural steel design, and masonry design on an as needed basis so the students would learn how to apply the equations for design purposes. However, many of the concepts that are taught in design courses are built upon and initially learned earlier in their engineering curriculum such as in engineering mechanics courses. Engineering faculty can do better than just teach on an as needed basis. Students need to learn and understand more than just the equations and steps in the design process. Students need to see how the concepts in design connect with fundamental mechanics. This could be thought of as the big engineering puzzle, where students need to see and understand each puzzle piece but also understand how and why each piece connects with the rest of the puzzle.

In two senior level structural steel design courses, a new lecture that discusses the purpose as well as the similarities and differences between the ENA and PNA was presented in an attempt to connect the concepts plastic behavior with fundamental engineering mechanics. Finding $\bar{y}_p$ to determine the location of the PNA was formulated and presented using calculus based equations. Normally, this is presented to students in a simplified method using basic geometric figures. Furthermore equation (5) for finding the PNA is a new calculus based equation that is not presented in any textbook or paper that the author is aware of. While in concept equation (5) is not new, presenting it in this form is and should help students see the connection with fundamental mechanics where calculus based expressions are used frequently. The lecture also included a discussion of bending shape factors and how to determine the plastic moment capacity of beams with various cross sectional areas. Two homework assignments were given to both classes. The first was to find the ENA and PNA for nonsymmetrical cross sectional areas. The second was to calculate bending shape factors for various beam cross section shapes such as a rectangle, triangle, circle, rhombus, and rectangular and circular tubes. For comparison purposes, students also had to calculate bending shape factors for commercially available W beam shapes and channels sections. See Table 1.

After completing these assignments and finishing all of the lecturers related to nominal plastic moment capacity of steel beams in flexure, a survey of questions was given to assess student understanding and learning of the material. A total of twenty-two senior civil engineering students took the course in two different class sections. There were eleven students in each class section. All students completed the required survey. A total of nine questions and explanation statements were asked of each student. Table 2 shows each question and explanation statement asked and the number of students out of twenty-two who answered correctly as well as the percent of the number of students answering correctly.

Table 2. Student survey

Explanation statement or question	Students correct of 22	Percent correct
Explain what the ENA is.	16	72.7
What is the ENA used for?	12	54.5
Explain what the centroid is.	16	72.7
What is the centroid used for?	18	81.8
Explain what the PNA is?	12	54.5
What is the PNA used for?	14	63.6
Explain what the plastic centroid is?	6	27.3
What is the plastic centroid used for?	4	18.2
The difference between the ENA and PNA is?	4	18.2

The first four questions and explanation statements relate to the students understanding of the elastic neutral axis and centroids of areas. Senior students in a structural steel design course should have a fairly solid understanding of these topics since they have studied this material in several engineering courses. The percent correct for questions 1 through 4 in the survey ranged from a low of 54.5% to a high of 81.8%. The next four items in the survey related to student understanding of the PNA and plastic centroid. This material is relatively new to the engineering students. The percent correct for the next four questions, 5 through 8, in the survey ranged from a low of 18.2% to a high of 63.6%. Of these four items, two of them were answered incorrectly by over 72% of the students. Clearly, there is a significant difference between student understanding of the ENA and PNA. Only four out of twenty-two students or 18.2% responded correctly as to what the plastic centroid is used for and the difference between the ENA and PNA. While it should be expected that student understanding and learning of plastic behavior and PNA is lower than that of elastic behavior and ENA, the percent correct scores regarding plastic behavior and PNA were very low. Introducing this material to the students earlier in their engineering coursework such as in mechanics related courses would better prepare them for engineering design courses taken later.

Summary and Conclusions

For decades in undergraduate engineering education, faculty have taught the concept of yielding and plastic behavior in design courses such as reinforced concrete design and structural steel design on an as needed basis so the students learn how to apply the equations for design purposes. However, these concepts are built upon and initially learned earlier in their engineering curriculum such as in mechanics type courses. Engineering faculty can do better than just teach material on an as needed basis. Students need to learn and understand more than just how to apply the equations and steps in the design process. They need to understand and be able to explain how concepts like yielding and plastic behavior relate back to the fundamental concepts of mechanics.

In two senior level structural steel design courses, the concept of the plastic neutral axis (PNA) and plastic behavior was discussed in detail so that students could connect this material back to fundamental mechanics in terms of similarities and differences with the elastic neutral axis (ENA) and elastic behavior. Homework assignments on this material were required and after all discussion on flexural behavior was completed, a survey was given to the seniors to assess student learning and understanding. The results were not positive with less than 19% of students being able to correctly answer what is the plastic centroid used for and what is the difference between the ENA and PNA. While the study only represented a fairly small number of senior level undergraduate engineering students, twenty-two in all in two separate classes, this should have made this task easier to teach to the students. The concepts of the PNA and plastic behavior need to be introduced earlier in the engineering curriculum such as in a mechanics of materials course.

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