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Enhancing Student Meaning-Making of Threshold Concepts via Computation: The Case of Mohr's Circle

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Introduction

Stress, strain, and the relationship between the two are foundational concepts within mechanics of materials. However, because these phenomena are complex and are often not directly observable, students often have trouble internalizing the concepts in consistently applicable ways [1]. Mohr's circle diagrams are often used as an important tool for visually representing the relationship between stresses and strains within a material. Indeed, Mohr's Circle has been identified as a "threshold concept" in engineering: a critical concept that integrates multiple important modes of thinking within a discipline [2, 3]. However, because these threshold concepts are often complex and difficult to learn, they require careful teaching approaches to ensure that students are able to combine ideas and navigate the complexity effectively. Computational tools are sometimes employed to help teach or illustrate the Mohr's circle technique through computer simulation, but these simulations often use a "configuring approach" to computational thinking, in which students alter input parameters of the system and the program outputs the resulting diagram [4, 5]. This study presents a method for simultaneously teaching Mohr's circle diagram concepts and computational literacy through a "programming approach" in which students are asked to construct, operate, and interpret results from a computational simulation [6]. The research question is: What are students' benefits and challenges when making meaning of Mohr's Circle diagrams following a "programming approach?"

Computational Literacy in Engineering Education

Despite the ever evolving nature of applied science and technology in the modern world, analytical problem-solving abilities have remained central to the engineering profession, and therefore central to engineering education [7]. Modern engineering workplaces now commonly use modeling and simulation practices (applied through computational tools and software) in the design and testing of products and systems [8, 9]. As such, employers have begun to seek out students with high training and ability to "understand engineering principles and computational principles that allow them to use computational tools to solve engineering problems by moving between physical systems and abstractions in software" [10]. Policymakers in the engineering education community have also begun to recommend that modeling and simulation skills be integrated more fully into the undergraduate engineering curriculum [11]. Furthermore, the Accreditation Board for Engineering and Technology (ABET) student outcomes (accreditation criterion three) have recently been updated to reflect the importance of students developing "an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice" [12].

In response to the increased demand for computational literacy in industry sectors, modeling and simulation practices are being implemented into course content by professors who commonly use these practices in their research [13, 14]. Situating these modeling experiences within disciplinary content often presents challenges, particularly when students come into the class with varying levels of computational experience. However, despite the sometimes limited ability of inexperienced learners to engage fully with computational tools and procedures, recent research of undergraduate students taking a semester-long computational science course has shown that more intensive exposure to modeling and simulation methods tends to improve students' self-efficacy beliefs about their ability to use computational tools and interpret simulation data [14, 15]. After a semester in a course dedicated to computational methods within Materials Science and Engineering, students reported positive perceptions regarding their abilities to perform various computing tasks in discipline-centered applications (i.e., solve a set of linear equations, represent an atomic molecular structure, solve initial and boundary value problems, implement a numerical model, etc.), as well as positive perceptions surrounding their acquisition of computational concepts and procedures for solving well-structured problems. This suggests that repeated exposure to modeling and simulation techniques at the undergraduate level is an important first step in preparing students for the computational expectations of many modern engineering workplaces.

Threshold Concepts in Engineering

A "threshold concept" is an idea or topic that tends to have transformative properties on the knowledge of the individual learning the topic. As described by Meyer and Land [2], threshold concepts "open up a new and previously inaccessible way of thinking about something." These concepts tend to present new ways of approaching or interpreting information within a subject, and their mastery is often necessary before the learner can effectively continue on to more advanced topics in the field [2]. Threshold concepts are distinguished from "core concepts" in that they are not simply important or fundamental concepts for the student; they also act as tools for integrating multiple types of knowledge and ways of thinking into one new and unified view of the relevant concepts. Threshold concepts have been described as having five primary qualities, in that they are likely to be:

- *Transformative:* Once understood, threshold concepts have a significant impact on the student's overall understanding of the concepts, and often result in a shift in perspective of the subject.
- *Irreversible:* Threshold concepts are unique in that once the transformative change of perception has occurred it is difficult for the learner to forget or "unlearn" the new outlook. In other words, it is difficult to return to the previous mode of thinking once the threshold concept has been understood. This effect is often demonstrated in professors sometimes having difficulty understanding exactly why their students are struggling with a concept that seems simple from their perspective.
- *Integrative:* Threshold concepts tend to "expose the previously hidden interrelatedness of something" [2]. They combine several other important concepts in a way that is unfamiliar to the student, and helps to connect and re-contextualize the ideas in a new light.
- *Bounding:* Though not necessarily always the case, threshold concepts tend to act as boundaries within their subject space, acting as guides for retooling curriculum in places where significant numbers of students are met with severe difficulties.

- *Troublesome:* The critical nature of threshold concepts often makes them "troublesome" for learners. Because threshold concepts typically act as a catalyst for shifting into a new perspective on a subject, the learning involved is frequently met with a certain amount of inherent difficulty.

Some examples of threshold concepts include ideas such as universal gravitation in Newtonian mechanics, opportunity cost in economics [2], equilibrium states and phase equilibria in chemical engineering [16], and atomic structure in physics and chemistry [17], among others. Students often face difficulty when learning these concepts, as they tend to require non-intuitive, unconventional, or "troublesome" ways of thinking about the subject [18].

The Case of Mohr's Circle

In civil, mechanical, and materials engineering, Mohr's circle has been identified as an important threshold concept when teaching mechanics of materials, particularly for teaching stress/strain relationships and stress transformation concepts [3]. The integrative nature of the Mohr's circle process requires students to combine knowledge from several different sources when performing the calculations needed to depict the circle, often providing the opportunity to represent or think about stress/strain relationships in a "new" or "completely different" way [3]. Due to the already "algorithmic" method of the calculations involved in stress transformation problems, Mohr's circle presents an ideal opportunity for teaching threshold concepts using computational methods.

Previous research into teaching Mohr's circle and stress transformation using computational tools and simulations have shown these methods to be useful in student learning. Lee et al. [19] found when implementing a simulation for demonstrating stress element analysis in real time that students reported feeling more comfortable with eigenvalue, eigenvector, and matrix algebra approaches to solving stress transformation problems. Scores on the final exam also increased noticeably after using the software to teach these concepts, which were featured heavily on the exam. Similar studies found that students using Mohr's circle simulation tools were able to complete and check their work faster than when using mathematical methods [4], and that using simulations that associate Mohr's circle with "real" stress behaviors through corresponding finite-element analysis diagrams tend to increase exam performance, engagement with assignments, and responsiveness to lecture materials [5].

While these approaches have proven useful in teaching the core concepts of stress transformation, much of the work focuses on implementing computational tools and exercises using a "configuring approach," in which students obtain results from a simulation program or software by modifying settings and input parameters using a graphical user interface. Little research exists addressing the issue of approaching the implementation of computational tools from a "programming approach," in which students are also responsible for coding portions of the simulation software themselves. The following study presents a new pedagogical approach for teaching Mohr's circle as a threshold concept in mechanics of materials courses, using tools designed to simultaneously support the learning of key disciplinary content and improve students' computational literacy.

Methods

This design-based research study investigates students' perceptions of a computational Mohr's circle activity used to facilitate learning of threshold concepts in civil engineering. Data was collected from student reports and analyzed using qualitative methods to create a profile of student experiences with and perceptions of the activity. A qualitative approach was used to investigate students' first-hand accounts of the learning experience to aid in the creation of improved scoring rubrics and data collection protocols for future implementations of the project activity.

Participants

The participants for this study consisted of a sample of 25 out of 98 civil engineering students enrolled in a 200-level mechanics of materials course at a large, Midwestern university. The 25 participants were the students who chose to participate in the Mohr's circle programming activity. The exercise was assigned as an optional extra credit assignment.

Data Collection Method

The Mohr's circle activity was used as the data collection method. Students who chose to participate in the Mohr's circle activity were required to evaluate the risk of damage to the concrete structure of a soccer stadium by investigating the loads applied to 16 support columns of a single seating section during a typical soccer game. Initial data was provided to the students in the form of strain values, gathered from strain gauges affixed to each of the 16 columns (see Appendix A). The students then had to perform the stress transformation process to find the total axial loading on each column and determine how many columns (if any) were in danger of failing. To do this, the students were asked to derive the necessary equations to convert the given strain values into stress values, and then write a portion of a MATLAB code to calculate the stress transformation results for each of the 16 columns using the derived equations. The program then prints the corresponding Mohr's circle for each of the columns, which the students can then use (in conjunction with their calculations) in their analysis to determine whether or not the stadium seating is safe for use. Students were also asked to provide possible methods of reinforcing the existing seating or to propose other methods of preventing the damage, as detailed in the steps of the project description, listed below.

1. Describe the problem that is being solved, and provide a justification for using Mohr's Circle as an appropriate approach to solve this problem.
2. Determine the necessary equations to convert the strain rate rosette to stress (in any system of coordinates). Identify key assumptions and limitations.
3. Determine the principal stress and principal planes with respect to the global system of coordinates *xy*, and plot the Mohr's Circle by providing to the special MATLAB function the center and radius of the circle.
4. Implement the equations in MATLAB and comment the code accordingly.
5. Complete the provided table (see Appendix A) by following the considerations detailed at the end of the table.
6. Analyze the results, assess the risk of damage and explain your rationale.

7. Suggest possible conceptual ideas to prevent the damage. (Hint: Assume that the max. values of stress take place between columns.). Include this in the report.
8. Write the report following the same guidelines as previous lab reports.

To help scaffold the students' implementation of the equations into MATLAB, students were provided with the following code framework:

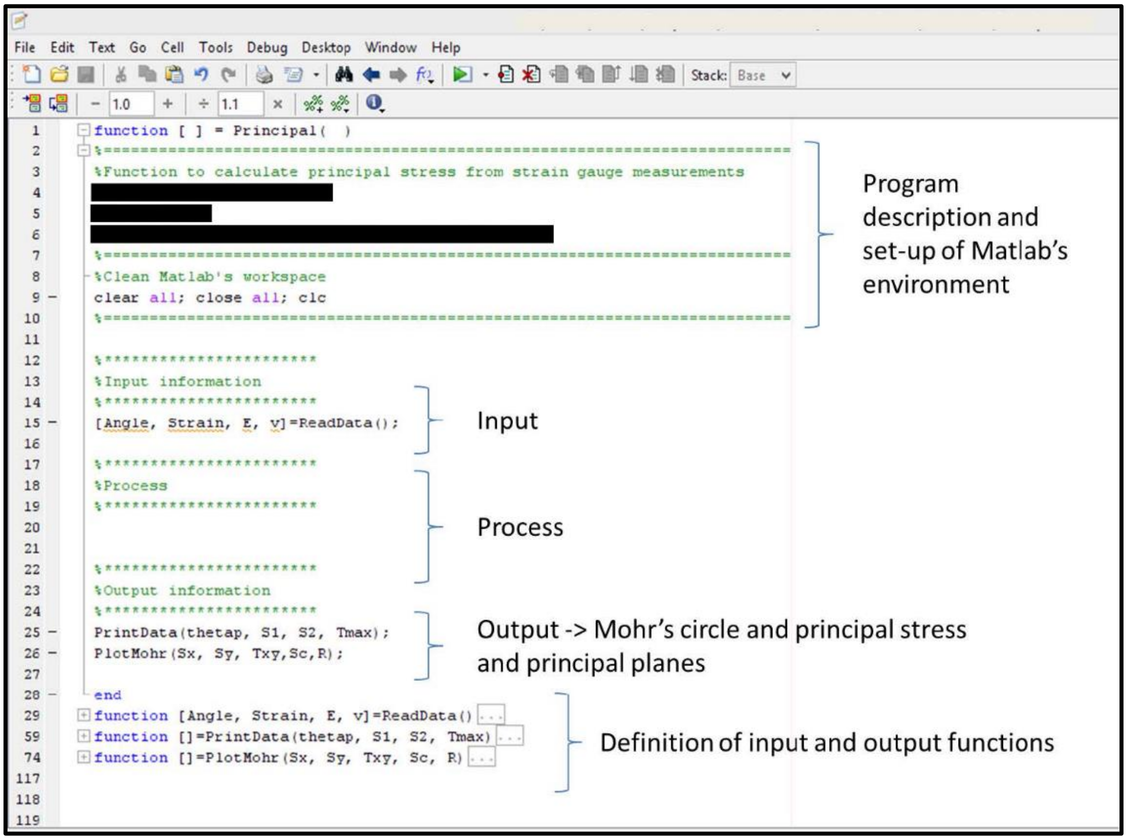

Figure 1: Screenshot of sample code provided to students during activity. The output functions "PrintData" and "PlotMohr" (for displaying results of the simulation in a table and plotting the Mohr's circle, respectively) were provided by the instructor. Course information and instructor names have been omitted from the program header for anonymity.

Students were also asked to provide a final reflection in their report once they had completed the assignment content. The following reflection prompts were included in the project description:

1. Considering the steps required in the project description, which step in the project was the most challenging for you?
2. Please explain why that particular step was challenging.
3. What strategies/resources did you use to overcome those challenges?
4. Considering the steps required in the project description, which step in the project helped you the most for your learning?
5. Please explain why that particular step was the most helpful for your learning.

Each participant completed the assignment and submitted an individual project report. Out of the 25 submissions, 24 of them contained a final reflection. These reflections, along with each students' discussion of his or her results, served as the primary source of data for the study.

Data Analysis Method

In order to assess students' perceptions of and experiences with the MATLAB assignment, each discussion and reflection section was scanned into QSR International's NVivo software for analysis. The twenty-five students' reports were analyzed using thematic analysis [20]. Thematic analysis is "a method for identifying, analyzing, and reporting patterns (themes) within data" [20]. In this study, the thematic analysis was conducted through an inductive and iterative process of reading, identifying, and categorizing the common themes within participants' written reports. Initially, the reports were split randomly between two of the authors for independent analysis. One of the authors analyzed 13 of the students' reports, while the other author analyzed 12 reports. This step was to identify an initial set of codes and themes from the data. In a second step, these authors met to discuss and reach consensus on a final structure of codes and themes. Once the consensus was achieved, the authors created a codebook. Then, both authors reviewed all student reflections and, together, recoded these reflections, this time using the structure of codes and themes reflected in the codebook. Three main themes were identified: Challenges, Benefits, and Gains, which each had several subcategories. The "Challenges" theme groups codes that express the perceived difficulties students faced when working with the Mohr's circle activity. The "Benefits" theme describes the features or steps of the project activity and guidelines that students identified as the most helpful in their learning process. The "Gains" theme reflects the students' perceived learning outcomes or takeaways from the project.

Findings

During the thematic analysis, several subcategories emerged within each of the three main themes. This section describes the following subcategories within each theme:

Challenges	Beneficial Steps	Gains
Familiarizing with the computational framework	Converting strain into stress	Combining knowledge
Converting the mathematical model into an algorithmic representation	Implementing the equations	Exposure to computational methods
Troubleshooting	Deriving meaning from the graphical analysis	Practical applications
Converting strain into stress	Performing the structural analysis	Understanding stress transformation
Understanding the solution approach		Understanding stress/strain relationship

Theme 1: Challenges

As part of the final reflection section of the project, students had to describe their main challenges and difficulties in conducting the different steps of the project. Twenty-three out of the twenty-five students explicitly reported their struggles in their reflections. We coded these struggles and identified five main coding categories: *familiarizing with the computational* *framework*, *converting the mathematical model into an algorithmic representation, troubleshooting, converting strain into stress,* and *understanding the solution approach*.

Familiarizing with the computational framework: Students struggled with understanding how to approach the computational algorithm, or with adjusting to the computational process (i.e. struggling to adjust to MATLAB or how to approach the problem computationally). This challenge was reported by ten out of the twenty-three students who reported difficulties during the project. A typical example of this kind of challenge is,

> **"***The hardest part of the project was getting the Matlab code started. It took me quite a long time to understand the code that was already written, as well as what I needed to name my variables in order to display the results correctly. Once I figured this part of the code out, the rest of the project, whether it be debugging the code, or interpreting the results, all came fairly easy to me*."

Converting the mathematical model into an algorithmic representation: Students struggled with converting the derived mathematical equations into code during the implementation phase. Eight out of twenty-three students reported difficulties in converting the model equations into the Matlab code. One student reported his main challenge in the following way,

> "*Among the steps required in the project description part, I can figure out the strain and stress transformation relationship between them, but I was having some difficulties in converting them into Matlab language. They were required to be inputted as the matrix algebra approach. I thought it was the most challenging part in this project. I learned matrix algebra last semester, and I learned how to use Matlab last year, it was a long time ago, so I have to pick them up and restart from the very beginning*."

Troubleshooting: The student struggled while debugging his or her code, tracking units of measurement, or determining the correct sequence of inputs and outputs. Ten out twenty-three students faced difficulties in making the necessary adjustments until the code presented the right results.

> *"The hardest part about the code was trying to figure out the pre-set functions such as ReadData and other functions within the Matlab code. To try and overcome not knowing how the functions, I tried to hard code the solution. I tried to set a system of equations to solve for the three unknowns with the three equations. I believe that I got the system of equation to work however there is some error within the pre-set function which I do not know how to fix."*

Converting strain into stress: Students struggled to convert the given strain values into stress measurements ("typical" problems provide stress values to be converted into strain). Five out twenty-three students had difficulties in dealing with strain-stress relationships. As a student wrote,

> "*In regards to difficulty, the most challenging aspect of this activity was converting the strain values to the stress values. It was difficult to make the jump because the equations can be quite complicated especially when inputting them into a program that requires*

perfection. If the code works but your solution is wrong, it is hard to find where the mistake was made. In order to overcome the problem, I attempted to solve the problem by hand in order check with the solution for strain rosette #0 which was given to us in the manual. Once I knew that my process was correct, I slowly revised the code until it functioned correctly."

Understanding the solution approach: The student initially struggled to understand how to approach the problem, or how to narrow down and choose between different approach options. Five out twenty-three students reported such difficulties, and typically the challenge was described as,

> *"The hardest step throughout the process is to conceptually understand how to solve the problem. Mainly, understanding that you're able to solve for σ_x and σ_y in terms of ε_a, ε_b, ε_c, and then simply plug in the angle to the equation written out in the experimental procedure part, to obtain values for $\sigma_{x'}$ and $\sigma_{y'}$, specific to my given angle."*

Theme 2: Beneficial Steps

Students were also asked to identify which parts of the project process that they felt was most helpful to their learning process. Twenty-one of the students explicitly identified these "benefits" in their reflections. We coded these helpful steps into four categories of codes. They are: *converting strain into stress, deriving meaning from the graphical analysis, implementing the equations, and performing the structural analysis*.

Converting strain into stress: Students found the process of converting strain into stress helpful to their learning, as the initial values for stress/strain transformation problems are often provided as stress. Eleven out of twenty-one students described this step as the most important in terms of learning. A typical example of students' comments is,

> *"The step in the project that helped me the most for learning was probably the step that was most challenging and that was converting the inputted strains to the normal stress and shear stress. This was most helpful in learning because most every other time strain and stress problems were completed, the procedure was going from the normal and shear stress to the principal values. In this case, the problem was to determine the normal and shear stress and strain. It allowed me for better understanding on how the equations of Mohr's circle and the generalized Hooke's law work together."*

Deriving meaning from the graphical analysis: The students found the plotting of multiple Mohr's circle diagrams useful, as it gave a strong visual method for analyzing the problem. Four out of twenty-one students described the importance of visualizing and analyzing the Mohr's circle as paramount for their learning process. As a student expressed,

> *"Consider the steps required in the problem description, the step of plotting Mohr's circle would be the most helpful for my learning. Because the cases of different rosette angles and strain values helped me to see the contribution of each strain gage rosette value and rosette angle to the maximum principal stress. That can improve my knowledge of Mohr's circle in my head."*

Implementing the equations: The student found the process of turning the mathematical model into a MATLAB solution to be helpful to his or her learning. Six out of twenty-one students identified step 4 in the project description as the most helpful for their learning. This step is exactly the one that asks students to "Implement the equations in MATLAB and comment the code accordingly." Below we present how one student described this step.

> *"Step 4 in the project description was the most helpful to me. By taking the equations we have used from earlier in the semester on homework, lab reports, and exams, I was able to simplify them by hand on a sheet of paper, taking into account the assumptions made such as no stress or strain in the perpendicular z direction or no change in temperature to cause thermal stress."*

Performing the structural analysis: One student found the fact that they were performing risk analysis on an actual structure (rather than an isolated or abstract example problem) to be helpful to his or her learning. This student wrote,

> *"Additionally, analyzing the structure was of benefit to me, because I feel like sometimes we are given problems and told to find a particular quantity, but we aren't actually given the chance to step back and see what is actually going on from a physical point of view."*

Theme 3: Gains

This theme emerged naturally when students tried to explain the most beneficial steps. Through elaborating on their reasons for their perceptions, they often ended up describing their perceived takeaways or learned outcomes. Twenty-one students reported their learning gains, and we coded these gains within five main categories. They are: *combining knowledge*, *exposure to computational methods*, *practical applications*, *understanding stress transformation*, and *understanding stress-strain relationship*.

Combining knowledge: Students thought that the exercise facilitated the synthesis of various knowledge from multiple sources. Five out of twenty-one students reported this as a learning gain. One student remarked,

> *"These phases were of most benefit to me because they reinforced what I have practiced in previous assignments and they allowed me to solve a problem using computational methods, which we don't get the opportunity to do very often."*

Another student wrote,

> *"Knowing that I can relate stresses and strain is very important in materials so one day if I am working on a project I will be able to tell if something is indeed failing. This step really brought the entire semester together for me."*

Exposure to computational methods: Students thought that the exercise provided useful exposure to computational approaches in engineering, or useful exposure to coding in tools like MATLAB. Nine out of twenty-one students explicitly mentioned this as a learning gain. One of these students said, *"I enjoyed the challenge of taking an important concept that we learned in class and applying it to a real-life example while using a computational tool like Matlab. Having to program the equations in Matlab required me to iterate, and keep trying to change the function until I got the correct results."*

Practical applications: Students felt that their learning was improved due to the "real-world" or "everyday" nature of the engineering problem in the assignment. Seven out of twenty-one students mentioned this in their reflections. One example is,

"These calculations, as well as the rest of the calculations involved with creating the Mohr circle, helped me further understand the real world application of strain gauges to solve civil engineering problems. It was helpful because it finally gave these general equations some purpose. I was able to see how they can be applied in the real world, which always help me when I am trying to learn and understand new concepts and theories."

Understanding stress transformation: Students felt that they gained deeper understanding of stress transformation concepts and/or methods, or acquired a deeper understanding of why Mohr's circle is a useful tool for analyzing stress/strain problems. Nine out of twenty-one students reported this. A typical response was provided by one student who said,

"Ironically, the step I struggled the most helped me learn the most as well. After this project I truly understand better the purpose of a Mohr's circle and how [strain gauge] values, such as ε_a, ε_b, ε_c can easily relate to principle strains and planes, based off assumptions."

Understanding stress/strain relationship: Students felt that they acquired a deeper understating of the fundamental relationships between stress and strain. Five out of twenty-one students mentioned this in their reflections. As one student said,

"It really helped me link major parts of the mechanics of material together and build a foundation in learning stress-strain relationship."

Discussion

We start our discussion by analyzing how the challenges reported by students relate to the nature of our programming approach. Three out of the five main challenges are associated with students' struggles with the programming side of the project, while the other two are associated with understanding the subject matter that is being taught and developing problem-solving skills.

Familiarizing with the computational framework, converting the mathematical model into an algorithmic representation, and troubleshooting were described by students as the most challenging steps toward the solution of the problem. This finding was already expected by the research team, since preliminary work has identified that students face difficulties when learning programming, such as struggles with getting familiar with programming structure and tools, designing applied solutions, and fixing bugs [21]. Preliminary work has also identified that engaging students in a programming approach to modeling and simulation may pose programming challenges [6]. We believe, however, that when students engage in a programming approach they follow a *constructive* approach to learning, and when students follow a configuring approach they engage in an *active* approach to learning. Cognitive scientists have argued that student learning is more effective through constructive activities than through simply active activities [22]. Furthermore, following a programming approach can also result in increased self-efficacy and future learning benefits [23]. In a “programming approach,” students are provided with opportunities to work hard to figure out possible solutions, identify multiple ways of implementing them, and develop analytical and problem solving skills. This process would not necessarily occur in a “configuring approach,” where students only manipulate different inputs and observe the results passively. From an instructional perspective, our results also corroborate the works of Perkins [24] and Hansen [25], who suggest that students learn more if they have opportunities to work on the “hard” parts, or with troublesome concepts in a very applied way.

The process of converting strain into stress challenged the students because, in traditional solid mechanics classes, students are typically only required to compute stress values from given configurations and boundary conditions, but have few or no opportunities to relate stress and strain in an interconnected way. Traditional approaches may lead to students developing primarily rote understandings of stress-strain problems without truly understanding the underlying mechanisms. In dealing with this challenge, students had to explore the stress-strain relationship first by deriving and solving equations manually, and then by trying to implement their equations in MATLAB. This approach also facilitates working on the hard and difficult parts, resulting in deeper learning gains than a standard problem set. The quote below from one of the participants reinforces the importance of working on “hard” parts to students learning.

> *“The step that helped me the most in my learning would actually be the step above that gave me the most trouble. The step on determining the necessary equations to convert the strain gauge rosette into stress showed me how everything is related in this course and how everything has its place. Knowing that I can relate stresses and strains is very important in materials so one day if I am working on a project I will be able to tell if something is indeed failing. This step really brought the entire semester together for me.”*

Understanding the solution approach was also reported as a challenging step. This may be a consequence of the nature of our approach, which requires the students to work on an applied problem with few directions. As we discussed above, our approach differs from the traditional structured problems usually found in solid mechanics books, where the aim is often the calculation of stress states from well-defined given conditions. Students rarely need to make explicit relations between stresses and strains to solve problems. The quote below helps us to make sense of students’ perceptions of this challenge and how it supported students’ development of conceptual understanding.

> *“The step in the project that helped the most for learning was probably the step that was the most challenging, and that was converting the inputted strains to the normal and shear stress. It was most helpful in learning because most every other time strain and stress problems were completed, the procedure was going from the normal and shear*

stress or strain to the principal values. In this case, the problem was to determine the normal and shear stress and strain. It allowed for better understanding of how the equations of Mohr's circle and the generalized Hooke's law work together."

Threshold concepts usually make the teaching and learning processes very difficult for both faculty and students [26]. To tackle individual differences in students' learning processes, research indicates that faculty must diversify learning activities and bring in real-world experiences to help students struggling with threshold concepts [26, 27]. In our approach, the different steps ask students to interact with a real-world problem from different perspectives. By doing this, we create opportunities for students to interact with a difficult core concept in ways that are not common in traditional approaches. The results described as beneficial steps reflect the positive influence of these variations on students' learning. Indeed, the four main themes that emerged from students' reports as beneficial steps—converting strain into stress, deriving meaning from the graphical analysis, implementing equations, and performing structural analysis—indicate that students benefit from each experience in different ways. Therefore, to increase the overall learning gains, it is essential to create a varied set of experiences within the same threshold problem. In our study, some students learned more when trying to analyze and perform conversions between strain and stress. Other students felt that graphical analysis was most beneficial, while others learned better by implementing the equations, or conducting the structural analysis.

Furthermore, students' perceptions of their learning gains also varied as a consequence of the individual differences among the students. In general, students' perceptions of this project were highly positive, and indicate that the presented approach creates varied opportunities. First, it allows students to combine knowledge and apply knowledge to real-world problems. This approach resonates with the ideas of Perkins, who argued that to make learning more meaningful to students, educators must "play the whole game" by letting students work on applied problems linked with professional situations [24]. Second, it allows students to integrate modeling with the computational tools [28]. Aside from increasing students' motivation, the integration of modeling with computational tools promotes the development of professional skills highly recognized by industry [28]. Third, this approach seems to foster the development of higher levels of conceptual understanding, as indicated by students' perceptions of understanding stress transformation, and of understanding stress/strain relationships, the two core concepts embedded in the Mohr's circle study.

These results are promising, and indicate that the students who chose to participate in the activity found it to be beneficial to their learning. None of the 24 students who included a reflection reported a negative experience with the assignment, despite many of them facing challenges during their learning. However, it is important to emphasize that these results are based on the students' self-reporting of their perceptions of the project. Because this was the first implementation of the Mohr's circle MATLAB activity in the course, it was unclear how students would react to being asked to perform a programming activity in a class that traditionally does not require knowledge of MATLAB. Therefore, the assignment was offered as optional extra credit, and the grading rubric focused largely on successful completion of the assignment and structuring of the project report, rather than on assessing deep understanding of the concepts. This makes investigation into connections between students' experiences and their performance on the project difficult, as there were no true "low performers" from the sample group. However, our results do provide a framework for data collection in future studies in two ways: (1) by providing insight into areas for improvement of the modeling task and (2) by providing evidence that while students did face challenges during the assignment, the addition of MATLAB coding requirements did not create excessive cognitive demands. While our data does not make long-term connections to the participants' overall performance in the class, the specific types of challenges and benefits described by the students will aid in the development of stronger evaluation tools for assessing the modeling task's impact on performance in the next iteration of the study.

Conclusions and Future Work

This study has addressed a new method for teaching Mohr's circle as a threshold concept in mechanics of materials classes through the use of programming-oriented computational tools. Our results show that the computational approach provided in the assignment offered a *transformative* and *integrative* scaffolding approach for helping students work through the *troublesome* knowledge involved in learning Mohr's circle. Our approach encourages students to spend time "working on the hard parts" of the stress/strain relationship in solid materials by approaching the problem in a new and unfamiliar way. Although students often struggle at first, the students report in their reflections that the project was very helpful in helping to transform their knowledge of stress/strain relationship and stress transformation concepts.

This study, however, only features data from the first implementation of the computational Mohr's circle project. Some suggested improvements can be made to the activity for the next implementation that would improve students' perceptions of the learning experience and provide deeper insight into precisely where and how students struggle with the activity. One recurring challenge reported by students was that of troubleshooting or debugging their code. Many of the students had minimal exposure to computational methods and coding in MATLAB, and several of them explicitly remarked in their reflections that the debugging process was challenging or sometimes frustrating. While troubleshooting is an important computational skill, spending excessive time debugging technical errors may increase the risk of cognitive overload in students who lack previous programming experience. The primary focus of the activity is for students to spend their time converting their derived mathematical model into a computational algorithm, and too much time spent searching for solutions to small technical errors may ultimately detract from that goal. Stronger scaffolding for the debugging process, such as online tutorials for common debugging techniques or lists of fixes for frequently encountered errors may help keep students on track with the disciplinary content.

One method for assisting students during this process would be requiring students to add in-code comments to their computational algorithms for solving the problem. Currently, the project document does not make any requirements of students regarding the documentation of their code. Recent work on modeling and simulation in materials engineering courses, however, has shown a connection between coding expertise and the detail of comments included in the code [29].

When explicitly prompted to comment their code during a computational materials science activity, novice students tended to leave much longer and more detailed comments about how the computational model was operating, while more experienced students tended to leave shorter, more concise descriptions about what the code itself was doing. This suggests that less experienced students may use commenting as a way of mitigating the cognitive load caused by lack of experience with computational approaches. Additionally, in-code comments can provide useful information regarding the student's level of understanding of the computational model, and we recommend that students be required to comment their code in future versions of the project description and rubric.

Further improvements can be made to the activity regarding the reflection prompts included in the handout. One limitation of this study is that the report reflections varied significantly in both detail and structure from student to student. This is likely due largely to the broad nature of the discussion prompts. These prompts were useful in allowing an open-ended space for coding themes to emerge, but it also allowed a number of the students to provide very brief or simple responses. We recommend that future studies implement a more detailed reflection that breaks the existing reflection prompts down into more focused sub-prompts to help guide the students' reflections. Furthermore, additional prompts included throughout the activity would provide even deeper insight into the student's struggles and learning gains throughout the process, as opposed to a single reflection after the project has been completed. The themes of challenges, beneficial steps, and learning gains developed in this paper can be used to guide the development of such reflection prompts.

Finally, revisions to the grading rubric used to score the projects would help tie future results more closely to student performance. The current rubric focused primarily on the correctness and completeness of the calculation results, as well as on the structuring of the project document that was submitted by the students. As such, there was not much overall variation in student performance, and it was difficult to relate students' remarks to their scores on the project, as there were very few actual "low performers." A revised grading rubric for more closely assessing students' planning of the solution, as well as their discussion, validation, and interpretation of their results would improve the richness of data from future implementations of the project.

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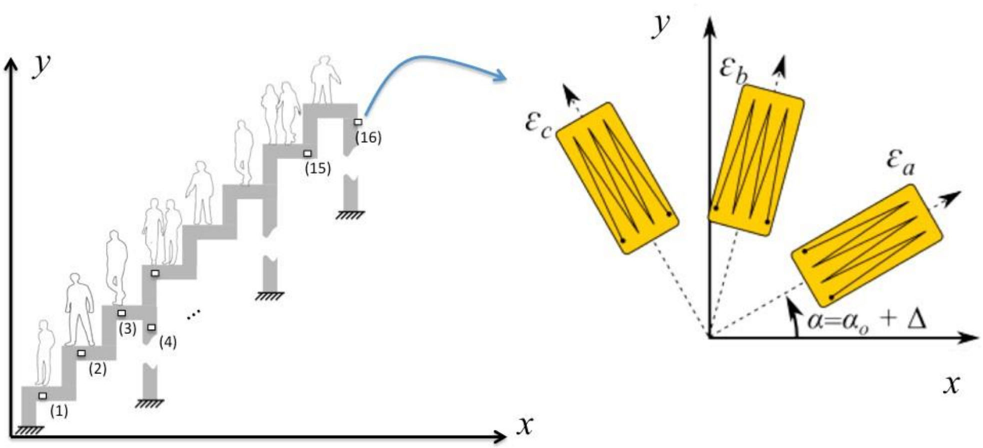
Appendix



(a) (b)

Fig. A.1: (a) Schematics of the concrete stands and placement of the strain rosettes (b) Details of each rosette. Note: The angle α (given by the axis of strain gage *a* and the global axis *x*) is calculated from α_0 (provided in the table) and Δ which is given by the last two digits of the student's ID number.

#	ε_a $[\mu\varepsilon]$	ε_b $[\mu\varepsilon]$	ε_c $[\mu\varepsilon]$	α_0	Δ	α	σ_1 [MPa]	σ_2 [MPa]	θ_{p1}	*Does it break?*
0	**129.99**	**-67.246**	**-49.990**	**0°**	**0°**	**0°**	**5**	**-2**	**-25°**	**YES**
1	80	80	80	23°						
2	40	20	40	-34°						
3	-463.33	-617.064	-43.333	120°						
4	32.516	-38.976	-133.85	3°						
5	80.695	-21.842	25.972	90°						
6	163.146	-259.503	-296.48	-34°						
7	134.205	-53.464	-184.87	-78°						
8	-250	-286.602	-150	66°						
9	152.60	46.213	-181.93	150°						
10	53.33	-106.66	-266.66	-33°						
11	37.466	-18.33	37.466	10°						
12	-58.372	-99.72	-234.961	-99°						
13	160.01	144.845	21.322	129°						
14	16	160	16	0°						
15	36.77	30.854	-36.77	22°						
16	86.096	-67.709	-139.43	-170°						

Fig A.2: The table of given values provided in the project description. The offset angle of each strain gauge (Δ) was randomly determined by the last digits of each participant's student ID number.