Evaluating Design Alternatives – The Role of Simple Engineering Analysis and Estimation

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Introduction

The best design is achieved, in general, after exploring the widest possible design option space. Because there is a finite time in which to choose from these options, it is common practice to restrict the design space *a priori* or during the concept generation phase of the process. This on-the-fly exclusionary practice is acute among student engineers because in traditional coursework there is rarely a benefit to increasing the design space before solving a homework problem. In fact, the reverse is true since by narrowing the possible options for solution, an example problem pattern match becomes easier to find. Because design problems are open-ended, however, there is a benefit to a wide solution space. In many ways, design experience manifests itself as a filter that rapidly eliminates unpromising design alternatives. Estimation, order-of-magnitude analysis, and back of the envelope reasoning are also strategies that can be employed to eliminate design alternatives efficiently and methodically. Approximate methods are used to uncover the fundamentals of processes,[1] and can also be used to explore a larger design space, which increases the opportunities for finding a premium design solution.

Approximate and order-of-magnitude methods are a branch of qualitative reasoning that has been established as an important component in the engineering analysis of physical systems. The importance of approximate reasoning has led to many attempts to quantify outcomes from a fundamentally qualitative understanding so that commonsense information can be formalized as part of a decision process.[2-4] These decision-making research efforts notwithstanding, it is important to develop approximate reasoning skills among engineering students for the express purpose of increasing the efficiency with which they navigate design options.

The first task in applying engineering analysis to a real problem is to make the physical system similar to one that has a known solution. This process is generally referred to as modeling. Modeling involves the invocation of assumptions, and with enough assumptions, it is always possible to extract an analyzable model from the real system. Unfortunately, the assumptions may be so drastic as to make the model and its analysis useless or irrelevant. Consequently, the model must always be validated in some way. Often the model is validated through a test, or it can be validated by relaxing the assumptions (i.e., using a more complex model). There is a hierarchy of model complexity that begins with rough estimates and ends with detailed (sometimes exact) models that include all of the important physics. Reference 5 describes the value of rough estimation,

> “*An engineer is often called upon to make quick decisions that must be based on rough estimates, on simple and incomplete models of the situation. The ability to do this depends to a large degree on experience—on knowledge of similar situations. The young engineer cannot start too early to practice correct estimating. If later the estimates are checked with reliable calculations, then errors can be detected and the estimating technique can be improved steadily.”*

Therefore, any design class should include rough estimation. Rough estimation allows students to recognize and catalog orders-of-magnitude, behavioral responses, methods of analysis, and rules of thumb that will give them an instant feel for the range of solutions in which their more detailed analyses should fall.

Approximate or back of the envelope analyses in the physics community are often termed Fermi questions or Fermi problems. In such a problem, following the practice of Enrico Fermi, the solver must draw on a deep understanding of the world and everyday experience to make approximations or inspired guesses.[6] A classical example of a Fermi question is, "How many piano tuners are there in the city of Chicago?" Several websites are devoted to Fermi questions and problems,[7-9] but they focus on the physics perspective of this kind of reasoning. This paper basically describes the relatively straightforward extension of such methods to engineering design.

Approximate Reasoning for Engineering

There is no fundamental difference between the classical Fermi question and the estimation challenges that can help sort through design alternatives. There is a difference, however, in the practical outcome. A Fermi question is more concerned with process than with the answer. Estimates that will provide design alternatives, however, provide an order-of-magnitude result will be used to narrow the solution space. In both cases, some minimal set of basic information must be known. As an example, consider the Fermi question, “What is the circumference of the earth?” One reasoning method[9] is: there are 3 time zones from the east to the west coast of the U.S.; there are approximately 3000 miles from California to New York, so we expect 1000 miles per time zone. With 24 time zones around the world, the earth’s circumference is 24000 miles. As basic information, the estimator must know both the number of time zones from the east to the west coast and the distance across the country. The latter might be guessed from the time required to fly from east to west, but one must then know the speed of jet air travel. In all cases, therefore, foundation facts play an important part in the estimation process. Which facts shall we endeavor to carry with us? This will vary with engineering specialty, but basic values of gas density, liquid density, heat conductivity, heat capacity, and material strength are very powerful.

To demonstrate a feasibility analysis using back of the envelope methods, let us suppose that an inventor proposes to develop a personal hydroelectric generating station that charges batteries as people utilize domestic water (i.e., a small water turbine is placed in the water line between the main line (100 psi) and the house regulator (60 psi). How much power can I expect to recover if I use 100 cubic meters of water per month? Power is the pressure drop times volumetric flow rate. 40 psi pressure drop is approximately 2.5 atmospheres or 250 kPa. 100 cubic meters per month is 3.3 cubic meters per day. So with 86,400 seconds per day, the power produced would be approximately 10 W. These kinds of calculations can help make countless decisions. In order to do so, however, a subset of simple formulae and basic facts must be accessible. Consequently, as part of the development of estimation skills, students are asked to complete the following table and to consider ways to remember the values they find. A list of some rules of thumb follow the table, and students are encouraged to add their own rules to the list, as well as some of the key relations and equations that they find valuable in estimation (e.g., Bernoulli's equation).

Some Material Properties

	Density (kg/m^3)	thermal conductivity (W/mK)	Viscosity (m^2/s)	Heat capacity (J/kg)	Melting temperature (K)	tensile strength (Mpa)	Elasticity modulus	Thermal expansion
Air								
Water								
Steel								
Aluminum								
Concrete								
Plastic								

Notes: Viscosity is very sensitive to temperature; both concrete and plastic can have widely varying properities

- An apple weighs a Newton or a Newton burger is ¼ pound
- Too hot to hold comfortably – 140F or 60C (can't hold 60 C for 60 seconds)
- Hydrocarbon flame temperature – 2000K
- Air density @ STP – 1 kg/m^3
- Water density – 1000 kg/m^3
- Natural convection heat transfer coefficient = 5 W/m^2 K
- Typical commercial automobile engine power – 1 hp/cu in displacement
- 2000kcals/day = 100 W; human metabolic rate = 100 W – a light bulb
- 1 horse = 7.5 humans (i.e., 1 hp = 750 W)
- Mechanical spring energy density = 0.013 MJ/kg
- Battery (alkaline) energy density – 0.2MJ/kg
- Hydrocarbon fuel energy density = 3 MJ/kg (gasoline – 115,000 BTU/gal)
- Energy density – gasoline is 10 X batteries which are 10 X springs
- Average solar flux at earth surface – 200 W/m^2 – 2 light bulbs per square meter

Back of the Envelope – Example

As one example of estimation in an engineering context, the students are asked to evaluate the appropriateness of using a 9V battery to power a handheld wine opener. The literature indicates that the force required to pull a wine cork can be 200 N, and pulling over a distance of 5 cm gives a work required per cork of 10 J. Assuming that the customer would prefer to pull at least 50 corks per battery change means that we need to have at least 500 J of energy stored in our power source. What then is the approximate amount of energy stored in a 9V battery (alkaline, for example)? How should one start to solve such a problem? First, it should be clear that what is being asked for is energy content, so the appropriate units will be Joules (J), Watt-hours (W-hr), kilocalories (kcals), or the like. It is generally useful to ask the students if they have any physical sense of any of these units. What feel do they have for energy? Lifting things (potential energy), heating things (thermal energy), spring things (mechanical energy), electrical things (electric energy), moving things (kinetic energy). The key is to determine what in their experience gives them a comparative energy measure and how they can use this sense to estimate the energy content of a new object.

Solution Approach 1 -- The most common solution method students employ is to estimate the current load of a common device that might use the battery, estimate how long the device would stay on, and then compute the energy. This solution works as follows:

My radio is like a very small light bulb that I will estimate at 1 Watt. For a 9V battery this means a current draw of 0.1 Amp, or 100 mA. This number does not sound too bad (and if you look at those AC/DC converters for small electronics, they have a maximum output of about 0.5 A at 3 V or 1.5 W; so we are likely in the ballpark. So, now if I assume that the battery will last 8 hrs in my radio under this full power condition, I get 32 kJ/battery.

The trouble with this solution approach is, if I do not have any experience, I have had to make 2 wild guesses (the radio draw and the time the battery lasts). This is why it is important to have a large comparison base to estimate these values. Nevertheless, solution approach 1 yields 32kJ/battery

Solution approach 2 -- I will assume that the specific energy density (i.e., the energy/mass) of a battery is 1/20th that of liquid fossil fuels (e.g., gasoline). Why choose 1/20th? If it were 1/5th, then we would have electric vehicles all over the place, since there would be a fairly small penalty for using batteries. If it were 1/100th then there would be little chance that electric powered devices could ever have much impact.

We can check the assumption of 1/20th in the following way. Thinking of gasoline, a 10 gallon tank will take us 300 miles (in a small car). Ten gallons is approximately 40 liters and 40 liters is approximately 40 kg of fuel (gasoline is 20% less dense than water). Therefore, we can go 300 miles on 40 kg or 150 miles on 20 kg.

For an electric vehicle, you can get at current best 150 miles on a full charge. The batteries are ½ the car's weight, which we will estimate to be about 1000 lbs or 500 kg. This means that we get 150 miles on 500 kg with batteries. Hence, the energy density of gasoline looks to be about 25 times that of a battery (not far from our guess of 20).

Now, from our rules of thumb, we know that hydrocarbons (i.e., gasoline) have energy density on the order of 3-5 kJ/g. We estimate that our battery mass is about 50 g (how would one estimate that?). This means we would have 200 kJ/battery if it somehow contained pure fossil fuel and, hence, we have 10 kJ/battery based on our 1/20th energy density estimate. So, solution approach 2 yields 10kJ/battery.

Solution approach 3 -- Let's now try a thermal argument. How much water can you boil using a 9V battery? That is, put a heating coil between the two battery terminals, dip the coil into a glass of water, and heat the water. Using the order of magnitude scaling methods common in Fermi problems, we ask ourselves, could we heat 1 cc of water with the battery? (certainly); 10cc? (likely); 100cc? (could be tricky); 1000cc? (most likely not). So we expect to heat successfully somewhere between 10 and 100 cc of water. How much energy is this? 70 cals/cc to heat water from room temperature to boiling. At around 4 J per calorie, this is 300 J/cc. So, our estimation is that we have at least 3kJ/battery, but not much more than 30kJ/battery. Take the average and you get 16.5 kJ/battery. Solution method 3 therefore gives 16.5 kJ/battery.

Solution approach 4 -- Radios are made for the outback and the bush country that do not use batteries. Instead, they have a hand crank that winds up a spring (like a windup clock – though these are not so common any longer). The spring then powers the radio. How long would I be willing to crank in order not to just have a bunch of batteries? I estimate that people will tolerate 2.5 minutes of cranking per battery equivalent (note, this does not necessarily mean 2.5 minutes continuously, but maybe 30 seconds, 5 times over the days I was using the radio). In any case, the rules of thumb say that the human body is about a 100 W lightbulb, and that with exertion it can go up to 500 W. Since I am only cranking, I will estimate a total exertion of 200 W, with 100 W of it going to the radio. Multiply this times the 2.5 minutes, and you get 100*150 or 15kJ/2.5 minutes. This would be the battery output. Solution approach 4 gives 15kJ/battery.

The actual energy content of a 9V alkaline battery is 17kJ/battery. None of our estimates is off by more than a factor of 2. Naturally, there are many more solution methods, and there is great value in stretching these methods in as many directions as possible. It is also interesting to note that an outcome of our efforts is a new rule of thumb. That is, since a 50 gram 9V battery stores 17 kJ, it has energy density of 340 kJ/kg. A good battery, therefore, can boil its own weight in water (i.e., 280 kJ/kg is required to raise water 70C). Furthermore, it appears that the 9V battery is a good potential power source for the portable wine opener. In fact, even the relatively low energy density of a mechanical spring looks to be sufficient.

Back of the Envelope Sample Questions

Practice and experience are important components of estimation. Below are a few additional questions that can be solved either individually or in a group. Also included are some rhetorical questions that can add to the discussion. Whenever possible, it is interesting to tie the numerical values to everyday physical objects and activities. There are certainly a wide range of such questions worth asking.

BOTE #1 – The hand cranked radio with a wound spring for energy storage is described above; it is for use far from supplies of domestic electricity or batteries. For decent sound performance (say a single 5 W speaker) how heavy would you expect the radio to be? Is this acceptable? Assume continuous 1 hour performance from a single winding is desired; 3600s*5W = 18 kJ; with spring energy density of 13kJ/kg (see BOTE); the spring alone (which is most of the radio weight) should weigh around 1.3 kg or about 3 lbs. Note that based on our prior estimates, this radio would only run for 1 hour on a 9V battery.

BOTE #2 – An inventor visits you and suggests a floating coat hook. It would be a helium filled balloon with a hook below. How big would such a device be? Recall that buoyant force is the weight of fluid displaced. Assuming a coat weighs on the order of 1 kg, the device (even if weightless itself) needs to displace 1 kg of air (or 1 cubic meter). This requires an approximately 1.25 m diameter device.

BOTE #3 – The most powerful water kettle on the market is an 1800 W Russell Hobbes device. Most kettles are 1500 W units. Why are they limited? How long will it take the Russell Hobbes to boil a liter of water. Compare this value to the typical kettle rate. Is it worth an extra $30? Boiling water takes 70kcals/liter (from 30C to 100C boiling) or about 300 kJ/liter. At 1800 W, boiling takes at least 2.8 minutes; while in the 1500 W unit boiling takes 3.3 minutes. Half a minute difference for 1 liter or a full minute for 2 liters. Is that a lot? How does this compare to just boiling the water on the stove? A propane camping stove is about 2.5 kW, but not all of the heat gets to the water. How would you estimate how much? How powerful is a microwave oven? How long will it take to boil water?

BOTE #4 -- How much weight can you expect to hang from a single ¼” diameter bolt?

BOTE #5 -- An inventor proposes a car exhaust powered lift rather than the mechanical jack in your car. The idea is to place a balloon under the car frame and to then attach a hose to your exhaust pipe to blow it up. What size do think the balloon would be? Will it work?

BOTE #6 -- As power requirements increase for faster laptop computer processors, engineers are looking for ways to dissipate the heat without using a fan. One concept is to use the computer keyboard as the heat dissipation fin. How much heat do you estimate could be dissipated comfortably in this fashion? How does this amount compare to the processor heat generation rate?

BOTE #7 -- How long does it take for a blown up balloon to evacuate its contents of air?

BOTE #8 – How big a solar array is needed to power a garden path light? To demonstrate the power of intuitive estimation, we begin the solution to any problem with an order of magnitude guess that is voted on by the class. For example, in BOTE #8, would you expect the solar panel to be: (a) 1, (b) 10, (c) 100, (d) 1000, (e) 10000, (f) 100000 cm^2? A few in the class will think that the cell will be 10 square centimeters, most will go for 100, and some will think 1000 cm^2 is the correct order. This collective intuition (and collectivity is important in such things) would put the solar cell at around 10 by 10 cm. As it turns out, the solar flux at the earth's surface is about 200 W/m^2. Assuming that we want a 10 W light bulb to run for as many hours as we have daylight in which to collect solar energy, we need about 500 cm^2, or a 20 by 25 cm.

BOTE #9 – How much energy does a typical household use per day?

BOTE #10 – There is currently a great concern about global warming and greenhouse gases, of which carbon dioxide is one. How much carbon dioxide (mass) is created by an automobile trip to the grocery store? How does this compare with the amount of carbon dioxide created by a person walking to the store? Should we walk more? Gasoline is approximated by octane (C_8H_{18}) so that for every kg of gasoline burned we make approximately 3 kg of CO_2. Assume that a trip to the store is 6 miles, which takes 1 liter or about 1 kg of gasoline. We then make 3 kg of CO_2. A person walking 6 miles at 2 miles per hour takes 3 hours, or 180 minutes. Assuming a breathing rate of 10 breaths per minute, the walker processes 1800 lungfuls of air, or approximately 1800 liters. Assume that in each breath, 21% of the air is oxygen and ½ of the oxygen is converted to CO_2. 1800 breaths then process approximately 2.0 kg of air, converting 0.2 kg of oxygen, making approximately 0.3 kg of CO_2. Of this amount, only the respiration above normal resting breathing is due to the walking, so the actual CO_2 creation for the effort is far less. So long as you can carry the groceries, walking produces at least an order of magnitude less CO_2 for the same trip.

Summary

Rapid assessment of design alternatives requires a solid foundation in approximate engineering reasoning. Such reasoning is a natural part of any design course and should play a larger role in all of engineering education. The basis of approximate reasoning is in experience, so providing assessment experiences is a good first step towards developing a sense of scale that will eventually guide designs along the most promising paths without resorting to time consuming and expensive comprehensive test and modeling efforts.

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