

# Evaluation of Inexpensive Hardware for Control Theory Application

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## Abstract

With the increasing class size, it is necessary to increase the laboratory size associated with that class. However, that creates problems like cost and space issues. If smaller and cheaper laboratory hardware is used these problems might be minimized by having hardware that students can purchase and use at home. However, inexpensive hardware may lead to poor matches between theoretical models and experimental data, which can confuse students at the introductory level. Care must be taken while using this kind of hardware. In this paper performance analysis of such an inexpensive set of hardware is discussed for introductory control theory class. The hardware is smaller in size and shows satisfactory experimental performance, which suggests that this can be a good fit for introductory control theory laboratory.

## Introduction

Studies show that introductory control theory classes have a growing need for laboratory exercise. But many difficulties like cost, class size and space limitations arise when laboratories are included [1, 2]. The New Earth Robotics Motor Lab (NERMLAB) can be a solution to these problems. It is potentially a cheaper replacement for expensive hardware which is used to teach introductory control theory. For example, a popular laboratory device such as the Quanser Qube [5] cost several thousand dollars while the NERMLAB could potentially be sold to the students for less than \$200. This might allow the students to purchase their own laboratory equipment to use outside of the formal lab environment where they can engage in experimental learnings at any time.

Experimental learning in laboratories helps students to connect the theoretical concepts obtained from the lectures with what happens in practice [4]. Unfortunately, the classroom size is growing continuously and that means the laboratory size should also be increased. But this results in an increased cost per student which motivates the desire for cheaper hardware [3]. To resolve this issue portable hardware can be introduced that uses cheaper motors, motor drivers, sensors etc. [2]. However, cheaper components can lead to large discrepancies between theoretical performance and the actual data produced with the hardware because of overwhelming non-linearities such as high static friction [3]. For an introductory class this can be very confusing to students NERMLAB aims to allow the students to perform experiments both in laboratory and home environment that validates the theory taught in class. This helps them to learn at their own pace while still achieving same learning objectives, which can be beneficial to them [2,4].

## Hardware Description:

The experiments in the following sections will be performed on the NERMLAB hardware. The NERMLAB consists of some key hardware pieces, such as, STM32 Nucleo development board, motor driver, and a Brushless DC (BLDC) motor (Figure 1). STM32 Nucleo houses a STM32F401RE Microprocessor Unit (MPU), which is a 32-bit processor with an 84 MHz clock speed and up to 512 Kbytes of flash memory. -Nucleo-IHM07M1 (a three-phase brushless DC motor driver) was selected as motor driver for the NERMLAB. The X-Nucleo has a nominal operating voltage of 8V-48 VDC with a 2.8 A peak current output. Ipower GBM2804H-100T Brushless Gimbal is used as the primary motor. An on-axis magnetic hall-effect position sensor is used to do position feedback. The encoder consists of 14-bit on-axis magnetic rotary position sensor chip AS5047D by AMS.

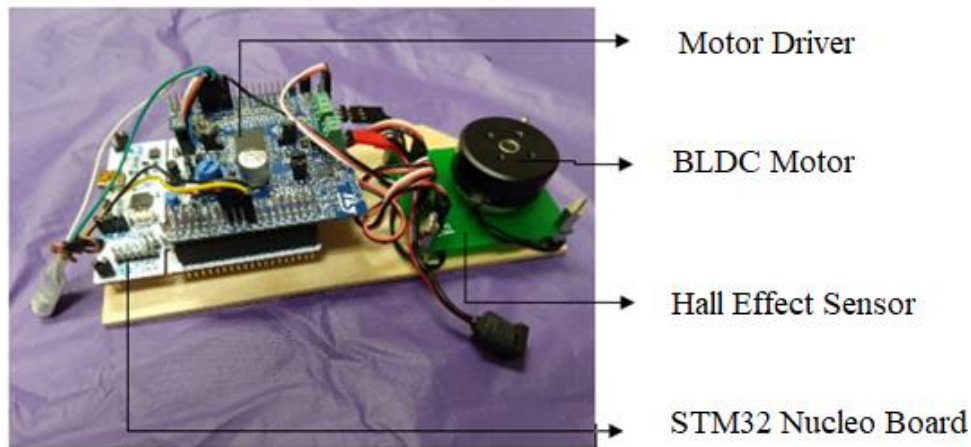


Figure 1: NERMLAB

To determine the motor's position, the number of pulses coming from the chip either from the leading or falling edge of the signal is counted. Using equation 1 the position in radians is calculated, where  $n_{resol}$  is the resolution of the encoder output and  $n_{count}$  is the current encoder count.

$$\theta_{rad} = 2\pi \frac{n_{count}}{n_{resol}} \quad (1)$$

Table 1 shows the components needed for NERMLAB and their prices:

Table 1: NERMLAB Cost

Component	Brand/Manufacturer	Cost
BLDC Motor	Ipower GBM2804H-100T	11.94 USD
Position Sensor	AS5047D AMS	4.21 USD
ST32 Nucleo	STMicroelectronics	10.12 USD
X-Nucleo-IHM07M1	STMicroelectronics	9.80 USD
Magnet	-	3.00 USD
Printed Circuit Board	-	30.00 USD
<b>Total Cost</b>		<b>69.07</b>

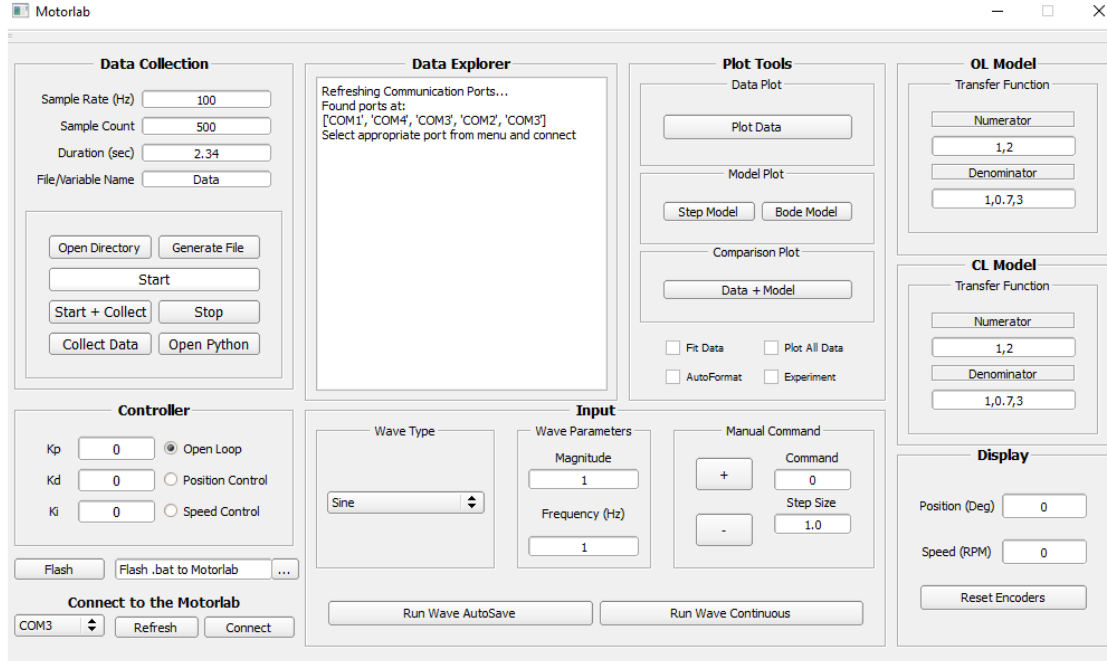


Figure 2: NERMLAB GUI

The user interacts with NERMLAB using a graphical user interface (GUI), shown in Figure 2, on a PC that communicates with the microcontroller over USB. The user inputs various parameters and commands the NERMLAB through the GUI.

## Nomenclature

Table 2 shows different parameters used and their descriptions.

Table 2: Nomenclature

Parameter	Description
V	Motor Voltage
$K_t$	Motor torque constant
$K_E$	Back Electromotive Force Constant
J	Lumped Mass Moment of Inertia
L	Motor Inductance
R	Motor Phase Resistance
b	Friction coefficient

## Modeling

NERMLAB uses voltage as input source. Figure 3 Shows the electro-mechanical model of NERMLAB. For NERMLAB the position model is developed as the transfer function stated in equation (2).

$$\frac{\theta(s)}{V(s)} = \frac{k_t}{(Ls + R)(Js^2 + bs) + K_E k_t s} \quad (2)$$

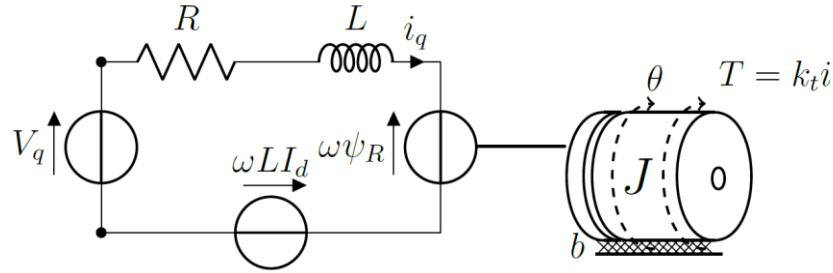


Figure 3: Electrical and Mechanical Diagram of NERMLAB

The dynamics involving inductance  $L$  are much larger than any other dynamics of the system. Thus,  $L$  can be ignored in equation (2). Which gives us the final position model for NERMLAB as equation (3).

$$\frac{\theta(s)}{V(s)} = \frac{k_t}{RJs^2 + (Rb + K_E k_t)s} \quad (3)$$

The speed model can be obtained from the position models, knowing the relationship between speed and position as shown in equation (4).

$$\theta(s) = \frac{\omega(s)}{s} \quad (4)$$

From equation (2) and (4) we get the speed model for NERMLAB as the transfer function stated in equation (5).

$$\frac{\omega(s)}{V(s)} = \frac{k_t}{RJs + (Rb + K_E k_t)} \quad (5)$$

A low-pass filter is used in NERMLAB when the speed control is used. The filter  $G_{hf}$  is a second order system having a cutoff frequency of 300 rad/s and damping ratio of 1/3 as shown in equation (6). The low-pass filter is used in the NERMLAB to reduce the high frequency noise when taking a derivative to convert angular position into speed.

$$G_{hf} = \frac{300^2}{s^2 + 212s + 300^2} \quad (6)$$

Students can participate in developing the models for NERMLAB as described above. But to perform experiments they can further simplify the model using lumped parameters. For example, equation (3) can be rewritten as equation (7) then different parameters can be lumped together.

$$\frac{\theta(s)}{V(s)} = \frac{\frac{k_t}{RJ}}{s^2 + \left(\frac{Rb + K_E k_t}{RJ}\right)s} \quad (7)$$

Now, considering,  $K = \frac{k_t}{RJ}$  and  $a = \frac{(Rb + K_E k_t)}{RJ}$  equation (7) gives the new lumped position model for NERMLAB as,

$$\frac{\theta(s)}{V(s)} = \frac{K}{s^2 + as} \quad (8)$$

And using equation (4) lumped speed model for NERMLAB can be determined from equation (8) as,

$$\frac{\omega(s)}{V(s)} = \frac{K}{s + a} \quad (9)$$

### Finding lumped parameters

To find the lumped parameters, equation (9) can be used. If we apply voltage to the motor and let it settle, in steady state the speed of the motor is the DC gain of the speed transfer function described in equation (9). DC gain of the speed model is,

$$K_{DC} = \frac{K}{a} \quad (10)$$

And thus, in steady state equation (9) becomes,

$$\frac{\omega_{ss}}{V_{ss}} = K_{DC} \quad (11)$$

In order to find the value of  $K_{DC}$  the motor was supplied with different voltages and the steady value of speed was recorded in Table 3. Data from Table 3 is then used to generate a voltage vs speed plot in figure 4. A straight line is fitted through these data. From the slope of the straight line  $K_{DC}$  can be calculated. Equation (12) shows the relationship between slope of the straight line and  $K_{DC}$ ,

$$slope = \frac{1}{K_{DC}} \quad (12)$$

Table 3: Friction coefficient estimate data

<b>Voltage (Volts)</b>	8	7	6	5	4	3	2	1
<b>Speed (rad/s)</b>	158	140	120	101	82	61	39	15

<b>Voltage (Volts)</b>	0	-1	-2	-3	-4	-5	-6	-7	-8
<b>Speed (rad/s)</b>	0	-15	-39	-61	-82	-101	-120	-140	-158

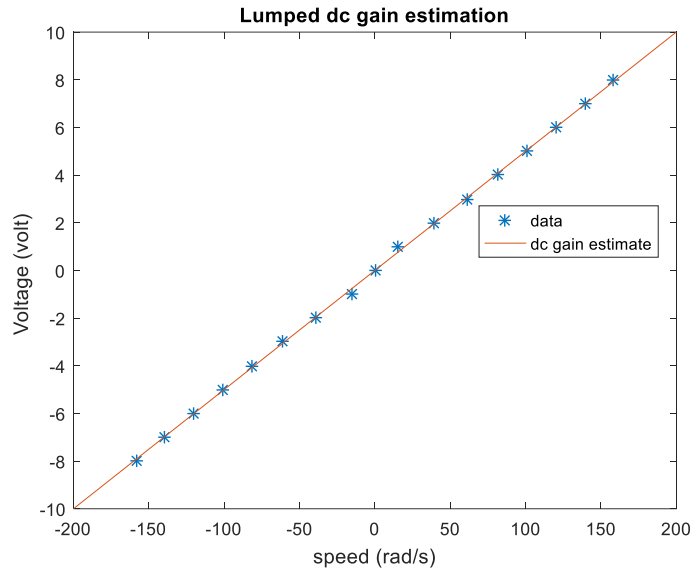


Figure 4: Determining lumped dc gain,  $K_{DC}$

The slope of the straight line found to be 0.05. Thus, using equation (14)  $K_{DC}$  is calculated as 20.

From equation (9) the time constant  $\tau$  of the speed model can be written as,

$$\tau = \frac{1}{a} \quad (13)$$

Thus, by finding the time constant from speed response value of  $a$  can be calculated using equation (14). To do so, the motor was supplied with 4 volts and 8 volts in open loop. Step responses for these voltages are shown in Figure 5 and 6. From the step response of these voltages the time constants were found to be 0.013 and 0.014. Taking the average of these values time constant  $\tau$  found to be 0.0135. Using equation (13), the value of  $a$  is calculated as 74.074.

Now, using equation (10) value of  $K$  found as  $1481.48 \frac{N.kg.m^3}{A.\Omega}$ .

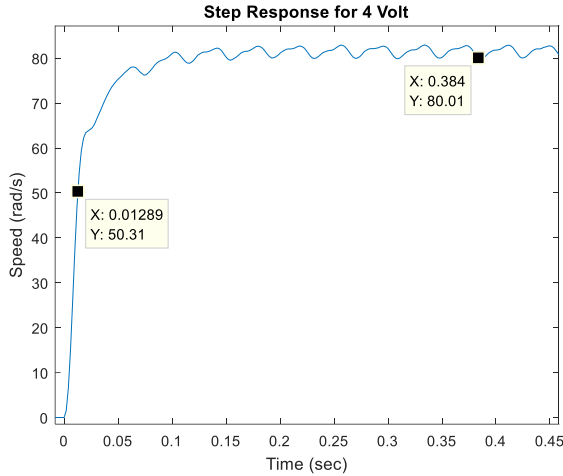


Figure 5: Speed step response for 4 Volt.

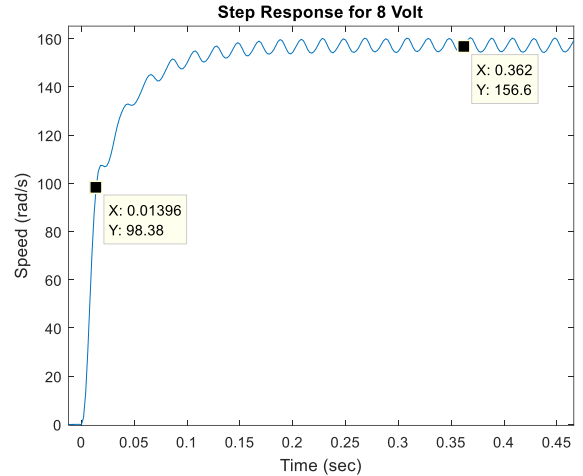


Figure 6: Speed step response for 8 Volt.

The ripples that can be seen in Figure 5 and 6 in steady state is potentially caused by misalignment of the rotary magnet. The waves in steady state have a repetitive pattern and the frequency of the waves is proportional to the speed.

### Position Control Experiment

The lumped parameter open loop transfer function  $G_m$  for the position model in NERMLAB is stated in equation (8). Considering  $G_c$  as a controller which is a simple proportional gain,  $K_p$ . Thus, the open loop transfer function for the system becomes,

$$G = G_c \cdot G_m = \frac{\theta(s)}{V(s)} = \frac{K_p K}{s^2 + as} \quad (14)$$

Assuming unity feedback, the closed loop transfer function for position control in NERMLAB can be obtained as,

$$T = \frac{\theta(s)}{\theta_c(s)} = \frac{G}{1+G} = \frac{K_p K}{s^2 + as + K_p K} \quad (15)$$

Three different proportional gains are used in this experiment. For each gain a different magnitude of square wave is used. This is done in a way so that the motor driver doesn't get saturated. These data are listed in table 4. For each gain experimental and theoretical plot was generated.

Table 4: Experimental gains and position commands used in NERMLAB

Gain (V/rad)	Magnitude of Square Wave (rad)
1	8
25	0.32
100	0.08

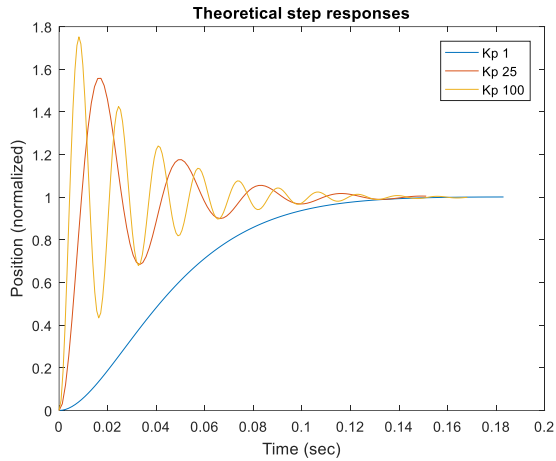


Figure 7: Theoretical step responses of NERMLAB for increasing gains

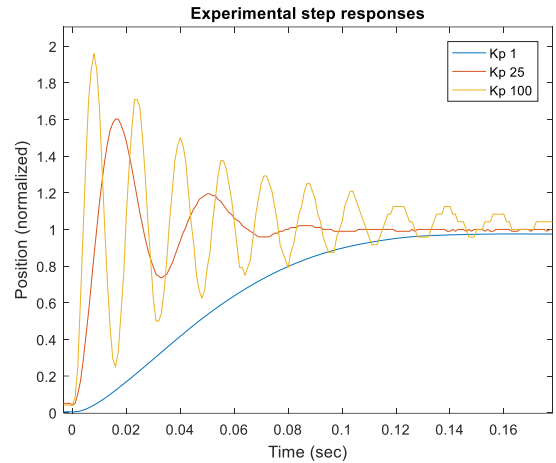


Figure 8: Experimental step responses of NERMLAB for increasing gains

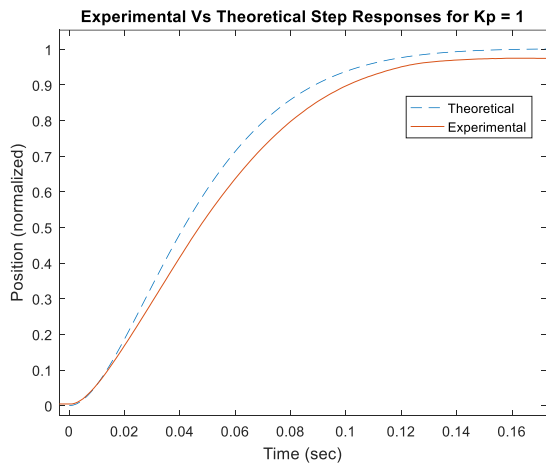


Figure 9: Theoretical vs experimental step responses of NERMLAB for  $K_p=1$

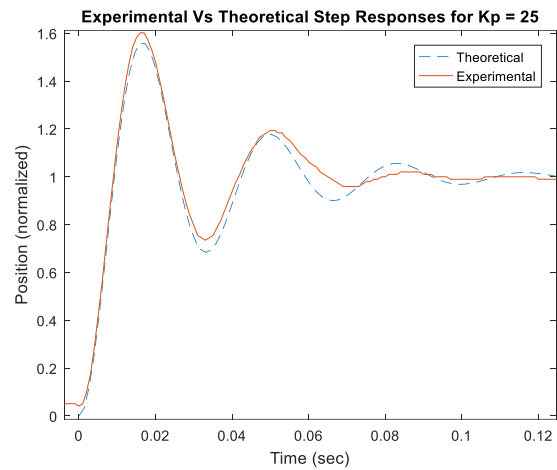


Figure 10: Theoretical vs experimental step responses of NERMLAB for  $K_p=25$

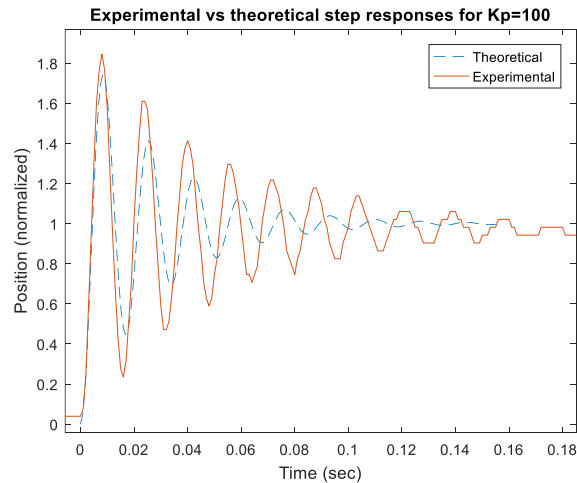


Figure 11: Theoretical vs experimental step responses of NERMLAB for  $K_p=100$



Experimental data was collected using three gains and plotted in Figure 7-11. From these figures it can be seen that there is a good match between the experimental data and the theoretical model. Figure 7 and 8 shows theoretical and experimental responses respectively for increasing gain. Increasing gain in position control results only in increasing oscillations without improving the settling time. This is because with increasing the gain the poles only change parallel to the  $j\omega$  axis in s-plane. In Figures 9-11 experimental data was plotted with theoretical model for  $K_p = 1$ ,  $K_p = 25$  and  $K_p = 100$ . Overall the data taken follows the theoretical model pretty accurately.

## Conclusion

Inexpensive hardware can be beneficial to students as they can carry this to their home environment and perform experiments. This can help them improve at their own pace. But inexpensive hardware can confuse students with poorly produced data which can be caused from cheap encoders, noisy signals from sensors and too much static friction. So, care must be taken while using such hardware. The NERMLAB is an example of inexpensive hardware that avoids these issues. In spite of being cheap hardware, from the experiments performed it can be seen that NERMLAB shows a good relation between experimental data and theoretical model. This is necessary for students at introductory level to connect theoretical lectures with real life scenario. Thus, NERMLAB can be a good fit as introductory control theory laboratory equipment.

## Reference

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