

Excel Spreadsheet in Mechanical Engineering Technology Education

Prof. Ti Lin, Liu

Department of Manufacturing & Mechanical Engineering Technology
Rochester Institute of Technology, Rochester, NY 14623

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Abstract:

In the last three decades Excel Spreadsheet has become a very popular and effective computational tool for performing engineering calculations. It is a great challenge on educators to apply this tool towards improving our engineering teaching and to provide high quality, learning-center education. Using spreadsheets provide a unique learning experience on the relationship between the component of an equation—an understanding that is essential in engineering analysis. However, the traditional teaching method and manual computation of equations and modelling do not always prove to be effective. Excel Spreadsheet has been successfully used to promote conceptual change in mechanical system design and analysis. In Excel Spreadsheet Student can perform alternative design and analysis. Student can better understand and interpret the solution using fundamental theoretical and numerical concepts. In this paper, the author is going to introduce his experience how to teach the courses in mechanical engineering technology at RIT using Excel spreadsheets. The case study in engineering mechanics, vibration, machine design, and others will be discussed in this paper.

The case study in this paper is listing below.

Case Study 1. Strength of Materials for Beam (Shaft) Design and Analysis

Case Study 2. Strength of Materials for Combined Stress in unsymmetrical Bending

Case Study 3. Strength of Materials for Combined Stress in column with eccentric load.

Case Study 4. Strength of Materials for Combined Stress in I Beam to find the bending stresses in flange and web of I beam

Case Study 5. Damping Vibration analytical solution

Case Study 6. Gear Box kinematic and shaft design in machine design Case Study

Case Study 7. Numerical Integration for Forced Vibration with Damping in Spring-Mass System.

Case Study 8. Numerical Differentiation for linkage analysis in Dynamics of Machinery.

Case Study 9. Jet Engine Thermodynamic analysis

Case Study 10. Long-hand-calculation of Stiffness Matrix for two dimensional triangular three-node-element in CAE study.

Case Study 1. Strength of Materials for Beam (Shaft) Design and Analysis

First, create an Input in excel. Which will drive all your calculation and create alternative design and solution by change the input data only. For example, if the value of concentrated force P or uniformly distributed load w change, the alternative solution can be found immediately in excel. Also, you can find the solution of stresses and deflection as functions of load P or w.

1	Design a W-shaped I Beam based on $\sigma_{all}=8000$ -psi, and find the deflection (v)A, (v)C,														
2	Slope $\theta_A, \theta_B, \theta_D$ for the beam. Material steel A36.														
3															
4															
5															
6															
8	Solution:														
9															
10															
15	1. Input:														
16	Length-a	a= 2 ft													
17	Length-L	L= 10 ft													
18	Load-P	P= 1275 lb													
19	Load-w	w= 300 lb/ft													
20	Allowable Stress σ_{all}	8000 psi													
21	Steel Material E	30000000 psi													
22	2. Calculation--Reactions and V & M Diagrams														
23	Resultant Force $R_w=(L+a)*w$	3600 lb													
24	$\Sigma M_B=0$	$-R_w*(L-a)/2+RD_y*L+P*a=0$													
25	$RD_y=(R_w*(L-a)/2+P*a)/L$	1185 lb													
26	$\Sigma MD=0$	$-RBy*L+(a+L)+Rw(L+a)/2=0$													
27	$RBy=[P(a+L)+Rw(L+a)]/L$	3690 lb													
28	Double Check by $\Sigma F_y=RD_y+RBy-P-Rw$	0													
29	A1=	-3150													
30	A2=	5490													
31	A3=	-2340													
32	Double Check by ΣA_i	0													
33	3. Design I Beam														
34	Section Modulus $S>M_{max}/\sigma_{all}$	4.725 in ³													
35	Select W6x12 Section Modulus S	7.31 in ³													
36	Moment of Inertia I	22.1 in ⁴													
37	Maximum Bending Stress $\sigma_{max}=M_{max}/S$	5170.999 psi													
38															
39	6. Conclusion of Advanced study--Deflections versus the Concentrated Load P														
40	6A. Add Linear Trendline and Display Equation on Chart,, we have														
41	6B. The Use of Trail-and-Error Method to find														
42	A. When P=1275-lb, (v)A=0, (v)C=0.0505-in														
43	B. When P=2825-lb, (v)C=0, (v)A=0.06464-in														
44	Which will meet the special requirement of industry, if the zero deflection is required.														
4	4. Deflection and Slope of Beam														
1	1. w load on BCD Beam														
2	$v_C1=5wL^4/(384EI)= 0.10181$ in ↓														
3	$\theta_{A1}=\theta_{B1}=\theta_{D1}=wL^3/(24EI)= 0.002715$ Rad (cw)														
4	$v_{A1}=\theta_{A1}*a= 0.065158$ in ↑														
5	$\theta_{A1}=\theta_{B1}=-\theta_{D1}=wL^3/(24EI)$														
2	2. w and P load on AB Beam for BCD Beam														
3	$v_C2=[Mx(L/2-x^2)/(6EI*L)]_{x=L} 0.051312$ in ↑														
4	$\theta_{D2}=ML/(6EI)= 0.00114$ Rad (cw)														
5	$\theta_{A2}=\theta_{B2}=ML^2/(3EI)= 0.002281$ Rad (ccw)														
6	$v_{A2}=\theta_{A2}*a= 0.054733$ in ↓														
3	3. Cantilever Beam AB														
4	$v_{A3}=Pa^3/(3EI)+wa^4/(8EI)= 0.010425$ in ↓														
5	$\theta_{A3}=Pa^2/(2EI)+wa^3/6EI 0.000641$ Rad (ccw)														
4	4. Deflection and Slope														
1	1. BCD Beam														
3	3. Cantilever Beam AB														
4	Superposition Method														
5	$v_C=v_{C1}+v_{C2}= 0.050498$ in ↓														
6	$v_A=v_{A1}+v_{A2}+v_{A3}= 0$ in ↓														
7	$\theta_A=\theta_{A1}+\theta_{A2}+\theta_{A3}= 0.000206$ Rad (ccw)														
8	$\theta_B=\theta_{B1}+\theta_{B2}= -0.00043$ Rad (ccw)														
9	$\theta_D=\theta_{D1}+\theta_{D2}= 0.001575$ Rad (ccw)														
5	5. Advanced study--Deflections versus the Concentrated Load P (0<P<3000-lb)														
29	A1=	-3150													
30	A2=	5490													
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44	Which will meet the special requirement of industry, if the zero deflection is required.														

Deflections versus the Concentrated Load P (0<P<3000-lb)

Concentrated Force P-lb (0<P<3000-lb)

◆ Deflection vC ◆ Deflection vA
— Linear (Deflection vC) — Linear (Deflection vA)

Case Study 2. Strength of Materials for unsymmetrical Bending.

Which is the combination of vertical and horizontal bending. With superposition method, the stresses and deflection could be solved. The stress distribution at the fixed end provide the dangerous stresses for the beam. The combined deflection due to horizontal and vertical bending is solved as well.

1	Find the stresses at the four corner of the fixed end for a cantilever beam with hollow rectangular cross section h=4-in, b=1.5-in, thickness t=0.25-in.																																																																																																																																																							
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Case Study 3. Strength of column with eccentric load.

Where the stresses of two direction bending and axial compression are calculated separately, and then combined together. The summation of stresses provide the actual stress distribution for column with eccentric load. With the power of Excel, we find the solution of stresses as the function of column width b, which is changed from 1 to 4-in. Both tabular solution and chart form solution are provided.

1	Find the stress distribution in the column with eccentric load.																																																																																																																																																																																																																																																																	
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Case Study 4. Strength of Materials for Combined Stress in I Beam to find the bending stresses in flange and web of I beam at dangerous cross section.

1 Find the maximum stresses and stress distribution in cantilever beam with
 2 Unsymmetrical bending. The I beam is w-shaped (W24x76)
 3
 4
 5
 6
 7
 8
 9
 10
 11
 12
 13 **1. Input Data:**
 14 Deam W24x76 Section Modulus
 15 Sy= 176 in³
 16 Sz= 18.4 in³
 17 Deam W24x76 Moment of Inertia
 18 Iy= 2100 in⁴
 19 Iz= 82.5 in⁴
 20 Load P= 4000 lb
 21 angle θ= 30 deg
 22 Length L= 48 in
 23 Steel Modulus of Elasticity
 24 E= 30000000 psi
 25 **2. Calculation**

G. Advanced Study
 The Increment of stresses (%) and Deflection (%) Versus the Inclined Angle θ

angle θ (deg)	Stresses	Deflection
0	0	0
5	83	143
10	165	353
15	244	566
20	321	776
25	395	979
30	465	1176

26 A. Bending Moment
 27 Vertical Bending
 28 Horizontal Bending
 29 My=P*cosθ*L = 166281
 30 Mz=P*sinθ*L = 95994
 31 B. Bending Stress σ1=+/-My/Sy σ2=+/-Mz/Sz
 32 C. Combined Stresses
 33 A B C D
 34 σ1-psi 945 945 -945 -945
 35 σ2-psi 5217 -5217 5217 -5217
 36 Σσ=(σ1+σ2)-psi 6162 -4272 4272 -6162
 37 D. Deflection
 38 v=P*cosθ*L^3/(3EIy)= 0.0020 in
 39 h=P*sinθ*L^3/(3EIz)= 0.0298 in
 40 Combined Deflection
 41 f=(v^2+h^2)^0.5= 0.0299 in
 42 E. Set θ=0 for Vertical Bending only, we have
 43 f=v= 0.0023 in
 44 σ=σ1= 1091 psi
 45 where the maximum bending stress increases 465 %
 46 where the deflection at the free end increases 1176 %

Case Study 5. Damping Vibration analytical solution.

By solving differential equation of damping vibration, the general solution of underdamping vibration of a spring-block system could be found. In Excel, the analytical solution could be solved in both tabular and chart form. By changing the value of damping factor C, the deduction of vibration amplitude as a function of time could be solved and to meet the requirement of industry. In this analysis, the time step 0.02-sec is selected.

	A	B	C	D	E	F	G	H	I	J
1	Damping Vibration in Microsoft Excel with Analytical Solution									
2	Input Data:									
3	Stiffness of Spring	k=	30	lb/in				g=	386.4	in/s ²
4	Weight of Block	W=	10	lb				e=	2.7183	
5	Initial Displ.	x0=	0.4	in				Diff Eq. Damping Vibration		
6	Initial Vel.	v0=	5	in/sec				$m\ddot{x} + c\dot{x} + kx = 0$		
7										
8	PreCalculation for Underdamping:									
9	Natural Frequency	wn=	(k/m) ^{0.5} =(k*g/W) ^{0.5}	34.047	rad/sec					
10	Critical Damping Coeff. Ccr=	2m*wn=2W/g*wn=	1.764	lb/in/sec						
11	Damping Factor for Underdamping (Given)	C=	0.120	lb/in/sec						
12	Ratio Zeta=C/Ccr=	Zeta=p	C/Ccr=	0.068				(1-p ²) ^{0.5} =	0.99768	
13	Frequency for Damping Vibration	wd=	wn*(1-p ²) ^{0.5} =	33.968	rad/sec					
14		Zeta*wn=p*wn=	2.316	rad/sec						
15	Factor 1	K=((v0+p*wn*x0)/wd)=	0.174							
16	Period	T=	(2*3.1416)/wd=	0.185	sec					
17	Time Step dt=		0.020	sec						
18	Underdamping C=0.12									
19	$x_1(t) = e^{-\zeta\omega_n t} \left(\frac{v(0) + \zeta\omega_n x(0)}{\omega_n \sqrt{1-\zeta^2}} \sin \sqrt{1-\zeta^2} \omega_n t + x(0) \cos \sqrt{1-\zeta^2} \omega_n t \right)$									
20	$= e^{-\zeta\omega_n t} (K * \sin \omega_d t + x(0) \cos \omega_d t)$									
21										
22										
23	Underdamping									
24	Time	Amplitude	Displ. 1							
25	t (sec)	e ^{-pWn*t}	X1(t)							
26	0	1	0.400							
27	0.020	0.9547361	0.402							
28	0.040	0.91152102	0.232							
29	0.060	0.87026203	-0.021							
30	0.080	0.83087058	-0.243							
31	0.100	0.79326214	-0.342							
32	0.120	0.757356	-0.286							
33	0.140	0.72307512	-0.114							
34	0.160	0.69034592	0.092							
35	0.180	0.65909817	0.241							
36	0.200	0.62926482	0.273							
37	0.220	0.60078184	0.187							
38	0.240	0.57358811	0.028							
39	0.260	0.54762528	-0.128							
40	0.280	0.52283762	-0.216							
41	0.300	0.49917195	-0.204							
42	0.320	0.47657749	-0.106							
43	0.340	0.45500573	0.028							
44	0.360	0.43441104	0.139							
45	0.380	0.41474729	0.181							
46	0.400	0.39597421	0.142							
47	0.420	0.37805087	0.046							
48	0.440	0.36093882	-0.061							
49	0.460	0.34460132	-0.132							
50	0.480	0.32900332	-0.141							
51	0.500	0.31411135	-0.080							

Case Study 6. Gear Box kinematic and shaft design in machine design

Which involved 1. The kinematic design of gear box, 2. The three dimensional forces and stresses analysis of the shaft in gear box based on Formulas of American Society of mechanical engineers (ASME) and American National Standard Institute (ANSI). Based on reversed bending and steady torsion for both solid and hollow shaft. 3. The ball bearing selection to find the life of ball bearing Ld. 4. The spur gear design and analysis based on bending stress from AGMA Standard 908-B89 formula.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	Gear-Shaft-Bearing Design Project for Mechanical Design in SOM																					
2	Objective: A gear-shaft-Bearing system is shown, where the driver gear1 on shaft I is received P-hp and n1-rpm																					
3	from an electrical motor and delivers to Gear 2 on shaft II. Again, Gear3 on shaft II delivers the power to Gear 4																					
4	on shaft III with required speed n3-rpm for a pump system speed n3-rpm. Applying SOM theory to design and																					
5	analyze mechanical engineering design problems with "real life" situations.																					
6	Input Data:																					
7	Output Speed n3=	150	rpm																			
8	Speed Motor n1=	3000	rpm																			
9	Power =	25	hp																			
10	Dia.Pitch Pd=	6	1/in																			
11	Pressure Angle φ=	20	deg																			
12	Angle θ1=	40	deg																			
13	Angle θ2=	60	deg																			
14	Length a=	10	in																			
15	Length b=	12	in																			
16	Length c=	8	in																			
17	Length L=a+b+c=	30	in																			
18	Part 1. Gear & Shaft Force Calculation(Norton-Ch.11)																					
19	Gear Train Velocity Ratio mV=(-N1/N2)(-N3/N4)=n3/n1=	0.05																				
20	Ratio of Teeth for Gear R=(1/mv)^(1/k)=	4.47																				
21	Select Teeth for pinion N1=N3=	17.00																				
22	Teeth for Gear N2=N4=R*N1=	76																				
23	Actual Output Speed n3=n1*(N1/N2)^2=	150.00	rpm	Eq.#																		
24	Speed for Shaft n2=n1*(N1/N2)=	671	rpm	1																		
25	Torque T=P/ω=P-hp(6600in-lb/s/hp)/(n2-rpm(2π/60rad/s/rpm))=	2350	lb-in	2																		
26	Pitch of Diameter of gears d2=d4=N2/pd=	12.7	in	6a																		
27	Pitch of Diameter of gears d1=d3=N1/pd=	2.8	in	6b																		
28	Tangential Force Component on gear2--Wt2=2T/d2=	371	lb	7a																		
29	Tangential Force Component on gear3--Wt3=2T/d3=	1659	lb	7b																		
30	Radial Force Component on gear2--Wr2=Wt2*tan20°=	135	lb	7c																		
31	Radial Force Component on gear3--Wr3=Wt3*tan20°=	604	lb	7d																		
32	Resultant Force on gear2 W2=Wt2/cos20°=	395	lb	7e																		
33	Resultant Force on gear3 W3=Wt3/cos20°=	1765	lb	7f																		
34	Inclined Angle of shaft Forces α2=	30	deg	8a																		
35	Inclined Angle of shaft Forces α3=	10	deg	8b																		
36	Horizontal Forces on Shaft	W2H=W2*cosα2=	342	lb	8c																	
37		W3H=W3*cosα3=	1738	lb	8d																	
38	Vertical Forces on Shaft	W2V=W2*sinα2=	197	lb	8e																	
39		W3V=W3*sinα3=	307	lb	8d																	
40	Part 2. Draw Shear force and bending moment diagrams (Hibbeter SOM)																					
41	Horizontal Bending																					
42																						
43	RAV=W3H(b+c)/L-W2H*c/L= 1067.8 lb																					
44	RBV=W3Ha/L-W2H(a+b)/L= 328.8 lb																					
45	Vy(x) = 670																					
46	-1067.8																					
47	-328.8																					
48	Vertical Bending																					
49																						
50	RAZ=W3V(b+c)/L+W2V*c/L= 256.97 lb																					
51	RBZ=W3V*a/L+W2V(a+b)/L= 246.89 lb																					
52	Vz(x) = 257																					
53	257																					
54	-247																					
55	MY(x) = 2569.69																					
56	1975.159																					
57	-247																					
58	Part 3. A. Shaft Design based on Fully Reversed Bending and Steady Torsion with ANS/ASME Standard B106.1M-1985 (Norton-Ch.9)																					
59	Definition:																					
60	Nf--Safety Factor in fatigue 3																					
61	kf--Fatigue Stress Concentration Factor 2.5																					
62	Ma--Maximum Bending Moment in Shaft 10983 lb-in																					
63	Tm--Torque in Shaft 2350 lb-in																					
64	Sf--Corrected Endurance Limit, Fatigue Strength 29250 lb-in																					
65	Sut--Ultimate Tensile Strength, 65000 lb-in																					
66	Sy-- Yield Strength 38000 lb-in																					
67	Material of Shaft: Inexpensive, Low-Carbon, Cold-Rolled Steel SAE1020																					
68	Sf=C1(Load)*C2(Size)*C3(Surface for machining--fine-ground)*C4(Temperature)*C5(Reliability)(0.5Sut)=																					
69	We have d= 3.06 in																					
70	B. Neglect the Torque in Torque, we have only consider the bending moment Mmax																					
71	$d = \left[\frac{32 N_f}{\pi} \left(k_f \frac{M_a}{S_f} \right)^2 + \frac{3}{4} \left(\frac{T_m}{S_y} \right)^2 \right]^{\frac{1}{3}}$																					
72	We have d= 3.06 in																					
73	C. SOM Formula																					
74	$d = \left[\frac{32 N_f}{\pi} \left(k_f \frac{M_{max}}{S_y} \right)^2 \right]^{\frac{1}{3}}$																					
75	We have d= 2.80 in where Sf is replaced by Sy																					
76	Upper right, we Set Nf=10 to consider fatigue stress concentration, and Kf=1,																					
77	We have d= 3.08 in																					
78																						

The corrected Formula for Hollow Shaft is Set $\alpha = 0.8$

$$d_o = \left[\frac{32 N_f}{\pi (1 - \alpha^4)} \left[\left(k_f \frac{M_a}{S_f} \right)^2 + \frac{3}{4} \left(\frac{T_m}{S_y} \right)^2 \right]^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

Outer Diameter $d_o = 3.647$ in
 Inner Diameter $d_i = \alpha d_o = 2.917$ in

Select $d = 3.2$ in for Solid Shaft
 and $d_o = 3.75$ in $d_i = 3$ in for Hollow Shaft

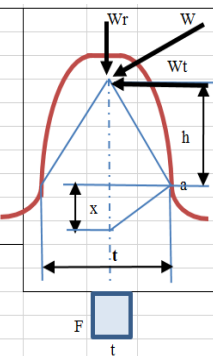
Part 4. Bearing Selection--Select 6300 Series Ball Bearing for A only, which has the max. reaction force (Norton Ch.10)
 $RA = (RAY^2 + RAZ^2)^{0.5} = (1068^2 + 257^2)^{0.5} = 1098$ lb
 Fatigue Life L (in millions of revolutions) is inversely proportional to the third power of the load for ball bearings
 Solid Shaft--Bearing Number #6314
 Bore Diameter $d_b = 2.7559$ -in, Dynamic Load Rating $C = 18000$ -lb
 For Bearing A: $L = (C/P)^3 = (18000/1098)^3 = 44066$ Revs
 Pumps works 24 hours a day, 24-hrs*60mins*671-rpm=0.9666 Revs/day,
 We have the life for Bearing A
 $L_d = 4561$ -days=152-months=12.7-years

Hollow Shaft--Bearing Number #6317
 Bore Diameter $d_b = 3.3465$ -in (70mm), Dynamic Load Rating $C = 21600$ -lb
 For Bearing A: $L = (C/P)^3 = (21600/1098)^3 = 76136$ Revs
 $L_d = 7881$ -days=263-months=22-years

BEARING NUMBER	BOUNDARY DIMENSIONS				SNAP RING DIMENSIONS			MAX. FILLET RADIUS r_f INCH	APPROX. WEIGHT lb.	S ₁ LIMITING SPEED rpm	C DYNAMIC RATING	C ₀ STATIC RATING lb.		
	BORE mm	BORE INCH	O. DIAM mm	O. DIAM INCH	H	S	t							
6300	10	.3937	35	1.3780	11	.4331	.125	1.562	.044	.025	.13	22000	1400	850
6301	12	.4724	37	1.4567	12	.4724	.125	1.625	.044	.025	.15	20000	1700	1040
6302	15	.5906	42	1.6535	13	.5118	.125	1.811	.044	.040	.20	18000	1930	1200
6303	17	.6693	47	1.8504	14	.5512	.141	2.074	.044	.040	.25	16000	2320	1460
6304	20	.7874	52	2.0472	15	.5906	.141	2.276	.044	.040	.34	14000	3000	1930
6305	25	.9843	62	2.4409	17	.6693	.195	2.665	.067	.040	.58	11000	3800	2500
6306	30	1.1811	72	2.8346	19	.7480	.195	3.091	.067	.040	.83	9500	5000	3400
6307	35	1.3780	80	3.1496	21	.8268	.195	3.406	.067	.060	1.07	8500	5700	4000
6308	40	1.5748	90	3.5433	23	.9055	.226	3.799	.097	.060	1.41	7500	7350	5300
6309	45	1.7717	100	3.9370	25	.9843	.226	4.193	.097	.060	1.95	6700	9150	6700
6310	50	1.9685	110	4.3307	27	1.0630	.226	4.587	.097	.080	2.50	6000	10600	8150
6311	55	2.1654	120	4.7244	29	1.1417	.271	5.104	.111	.080	3.30	5300	12900	10000
6312	60	2.3622	130	5.1181	31	1.2205	.271	5.498	.111	.080	3.81	5000	14000	10800
6313	65	2.5591	140	5.5118	33	1.2992	.304	5.892	.111	.080	4.64	4500	16000	12500
6314	70	2.7559	150	5.9055	35	1.3780	.304	6.286	.111	.080	5.68	4300	18000	14000
6315	75	2.9528	160	6.2992	37	1.4567	.304	6.679	.111	.080	6.60	4000	19300	16300
6316	80	3.1496	170	6.6929	39	1.5354	.346	7.198	.122	.080	9.33	3800	21200	18000
6317	85	3.3465	180	7.0866	41	1.6142	.346	7.616	.122	.100	11.00	3400	23100	18600
6318	90	3.5433	190	7.4803	43	1.6929	.346	7.986	.122	.100	11.60	3400	23200	20000
6319	95	3.7402	200	7.8740	45	1.7717	.346	8.380	.122	.100	13.38	3200	24500	22400
6320	100	3.9370	215	8.4546	47	1.8504	.346	8.786	.122	.100	16.34	3000	26500	24700
6321	105	4.1338	225	8.8582	49	1.9291	---	---	---	.100	17.8	2800	30500	30000
6322	110	4.3307	240	9.4488	50	1.9685	---	---	---	.100	21.0	2600	32500	32500
6324	120	4.7244	260	10.2362	55	2.1654	---	---	---	.100	32.3	2400	38000	38000
6326	130	5.1181	280	11.0236	58	2.2835	---	---	---	40.1	2200	39000	43000	
6328	140	5.5118	300	11.8110	62	2.4409	---	---	---	48.1	2000	44000	50000	
6330	150	5.9055	320	12.5984	65	2.5590	---	---	---	57.8	1900	49000	60000	

Part 5. Spur Gear Bending Stresses
 AGMA Standard 908-B89 replaced Y by a new geometry factor J, includes effects of stress concentration at root of gear tooth. Many K factors are modifiers to account the various conditions, includes dynamics, load distribution application, size, rim thickness, and idler factors. In first approximation, set the Product of k factors equals 1.
 For Gear 3 has the highest value of Wt, we have
 The bending geometry Factor for the 20°, 17-tooth pinion in mesh with the 76-tooth gear found from Chart Y= 0.3 (Juvinall)P.479
 Select the middle of the recommended range $F = 12/pd = 2$
 We have the bending stress at the low speed gear 3
 $\sigma = 16588.29$ psi

Formulation of Bending Stress (Lewis Eq.) in Spur Gear at the root of the gear. Point a, the bending stress $\sigma = Mc/I = Wt^*h/s = 6Wt^*h/(Ft^2)$ (a)
 where F--the width of tooth, t--the height of teeth the range of width of tooth $8/pd < F < 16/pd$
 With similar triangles, $(t/2)/x = h/(t/2)$, or $t^2/2h = 4x$ (b)
 Sub. (b) into (a), we have $\sigma = 6Wt/(4Fx) = Wt^*pd/(F^*Y)$ --Lewis Equation (c)
 where Wt is the tangential force on the tooth
 pd--the diametral pitch, F--the width of tooth



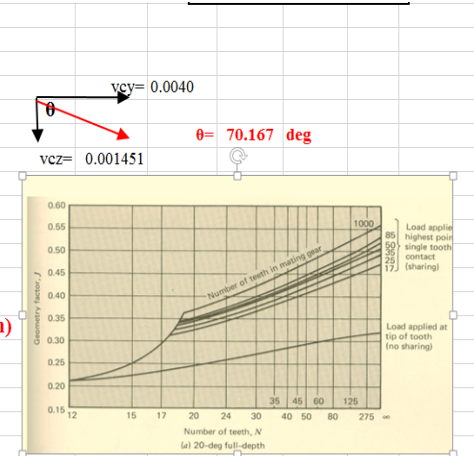
Part 6. Check the Maximum Deflection of Shaft at section C for Shaft (Hibbeler SOM)
 Where Elastic Modulus $E = 3E+07$ psi
 Solid Shaft Moment of Inertia $I = \pi d^4/64 = 5.1446$ in⁴
 Hollow Shaft Moment of Inertia $I = \pi d_o^4(1-\alpha^4)/64 = 5.7282$ in⁴
Hibbeler Deflection Table for Simply Supported Beam with Concentrated Load

Formulas:

$$v_c = \sqrt{(v_{cy})^2 + (v_{cz})^2}$$

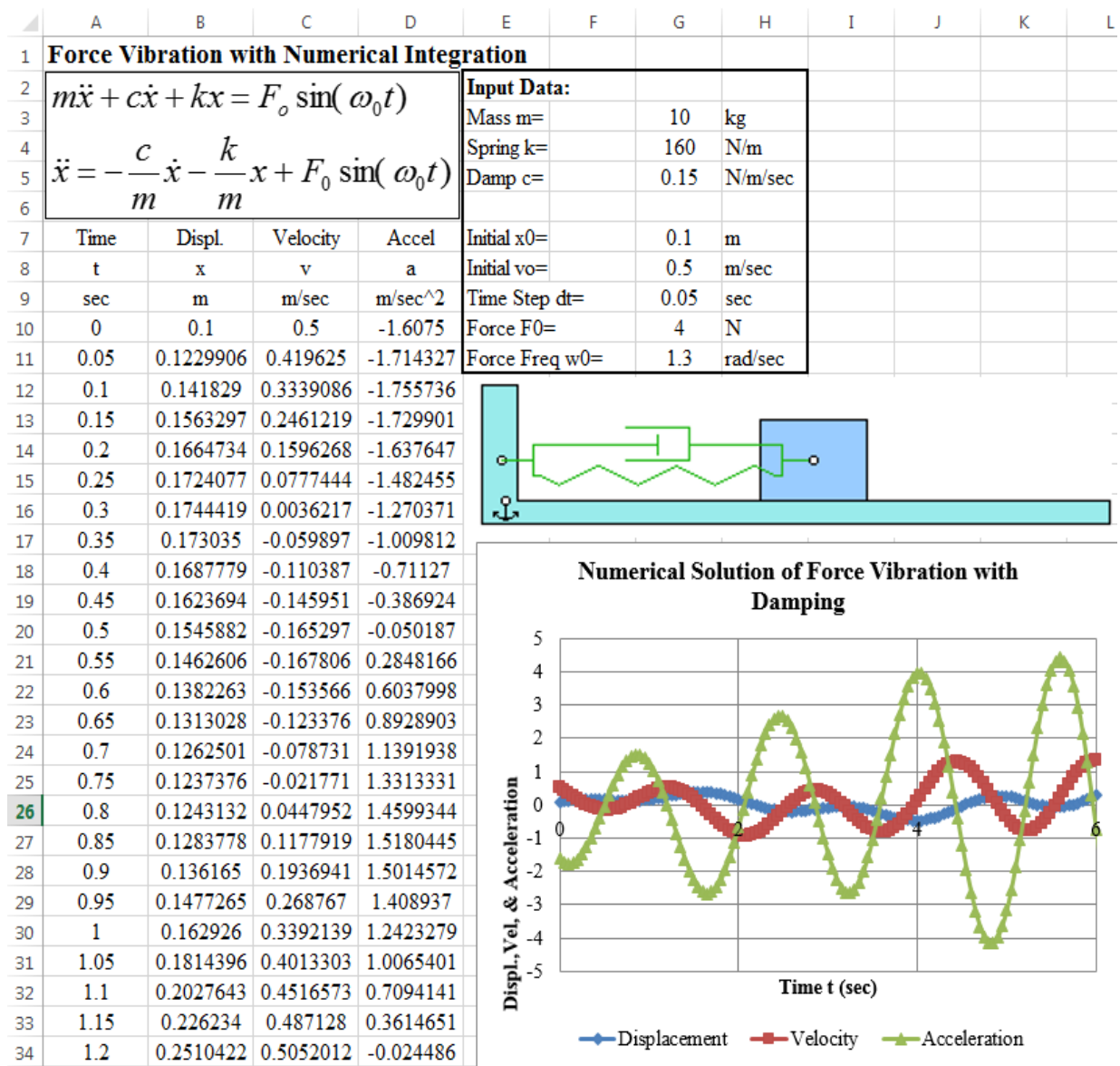
$$\theta = \tan^{-1} \left(\frac{v_{cy}}{v_{cz}} \right) * 57.3$$

Solid Shaft:
 Deflection at C due to Horizontal Bending
 $v_{cy} = (v_{cy})W_3H + (v_{cy})W_2H$
 $= W_3H^3(b+c)a/(6EIL) * (L^2 - (b+c)^2 - a^2) - W_2H(b+c)a/(6EIL)(L^2 - (b+c)^2 - a^2)$
 $= (W_3H - W_2H) * (b+c)a / (L^2 - (b+c)^2 - a^2) / (6EIL) = 0.0040$ in
 Deflection at C due to Vertical Bending
 $v_{cz} = (v_{cz})W_3V + (v_{cz})W_2V$
 $= W_3V^3(b+c)a/(6EIL) * (L^2 - (b+c)^2 - a^2) + W_2V(b+c)a/(6EIL)(L^2 - (b+c)^2 - a^2)$
 $= (W_3V + W_2V) * (b+c)a / (L^2 - (b+c)^2 - a^2) / (6EIL)$
 $= (WV_3 + WV_2) / (WH_3 - WH_2) * C121 = 0.0015$ in
 Using Cartesian Vector Method to find the total Deflection and its direction at C
Hollow Shaft:
 $(v_c)_{Hollow} = (v_c)_{Solid} * (I)_{Solid} / (I)_{Hollow} = 0.0036$ in



Part 7. Discussion: (Student need answer all the questions given in this Discussion)
 1. What is the relation between SOM and Mechanical Design/Analysis?
 2. How to reduce the stress and deflection for shaft, gear?
 3. How to increase the life of ball bearing?
 4. Any suggestion of improvement for this project?
 5. Your design decision

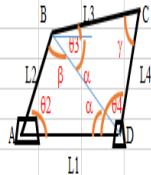
Case Study 7. Numerical Integration for Forced Vibration with Damping in Spring-Mass System. Driven by Input Data sheet with different input mass m, spring stiffness k, damping coefficient c, initial displacement x0 and initial velocity v0, external force magnitude F0, frequency w0, and the time step dt for numerical integration, the solution of forced vibration with damping can be solved in a few second for different system. Where the first equation is the differential equation of forced vibration with damping, the second equation is the solution for acceleration at the given time. Then, the velocity and displacement of system could be solved numerically. The use smaller time step will have better numerical solution of the system. The accuracy and convergence of the solution should be considered. In this example, time step dt=0.05-sec. The chart solution shows the displacement, velocity and acceleration solutions in the first 6 min. But the tabular solution shows only in the first 1.2 min due to the space of this paper.



Case Study 8. Numerical Differentiation for linkage analysis in Dynamics of Machinery

In general, the displacement of a given linkage could be found analytically, where a four bar linkage is illustrated below for the mathematical formulas in the first six equations. Link 2 is driven by a constant motor, by selecting time steps, with the input angle of link 2, the angular displacement of Link 3, and 4 can be solved. Then, using center difference formulas equations 7 and 8 for numerical differentiation, the velocity and acceleration of output link 3 and 4 can be solved numerically in tabular form and then plot into chart forms as show.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA			
1	Position Analytical Analysis of Four-Bar-Linkage																													
2	1. Formulas for Position Analytical Analysis of Four-Bar-Linkage with Laws of Cosine and Sine														Center Difference Formulas for Numerical Differentiation															
3	Triangle ABD,		$BD^2=L1^2+L2^2-2L1*L2*cos\theta_2$																											
4	Find BD	$BD=(L1^2+L2^2-2L1*L2*cos\theta_2)^{0.5}$																									Eq(1)			
5	Triangle BCD,		$BD^2=L3^2+L4^2-2L3*L4*cos\gamma$																											
6	Find γ	$\gamma=cos^{-1}((L3^2+L4^2-BD^2)/(2L3*L4))$																									Eq(2)			
7	Triangle ABD,		$sin\alpha=L2*sin\theta_2/BD$																											
8	Find α	$\alpha=sin^{-1}(L2/BD)*sin\theta_2$																									Eq(3)			
9	Triangle BCD,		$sin(\theta_4-\alpha)=L3*sin\gamma/BD$																											
10	Find θ_4	$\theta_4=sin^{-1}(L3/BD)*sin\gamma+\alpha$																									Eq(4)			
11	Find θ_3	$\theta_3=sin^{-1}(L4/BD)*sin\gamma-\alpha$																									Eq(5)			
12	Double Check by																													
13	Triangle BCD,		$\theta_3+\alpha+\gamma+\theta_4-\alpha=0$																											
14			$\theta_3+\theta_4+\gamma=180-deg$																										Eq(6)	
15																														
16	2. Input:																													
17	Motor Speed $\omega_2=$	1	rad/sec	(ccw)	L1=	10	in																							
18	Time step dt=	0.1	sec	L2=		2	in																							
19	$d\theta_2=d\omega_2*dt=$	5.73	deg/sec	(ccw)	L3=	8	in																							
20					L4=	10	in																							
21	Table of Calculation with Numerical Differentiation for velocity and acceleration																													
22	Input:														Double Numerical Differentiation															
23	Angle θ_2	Length BD	Angle γ	Angle α	Angle θ_3	Angle θ_4	Check	Ang. Vel. ω_3	Ang. Acc. α_3	Ang. Vel. ω_4	Ang. Acc. α_4																			
24	eq(1)	eq(2)	eq(3)	eq(4)	eq(5)	eq(6)	eq(7)	eq(8)	eq(7)	eq(8)																				
25	deg	in	deg	deg	deg	deg	deg	deg/s	deg/s ²	deg/s	deg/s ²																			
26	0	8	51.322	11.302	62.6239	66.0677	180.013																							
27	5.73	8.0125	51.413	11.299	62.6003	65.9998	180.013	-1.3509	-13.4223	-0.4734	-4.7404																			
28	11.46	8.0497	51.686	11.288	62.5292	65.7976	180.013	-2.6678	-12.9143	-0.9491	-4.7740																			
29	17.19	8.1109	52.137	11.270	62.4105	65.4662	180.013	-3.9189	-12.1095	-1.4285	-4.8144																			
30	22.92	8.1950	52.756	11.245	62.2435	65.0138	180.013	-5.0776	-11.0640	-1.9114	-4.8419																			
31	28.65	8.3004	53.534	11.213	62.0282	64.4507	180.013	-6.1230	-9.8436	-2.3952	-4.8347																			
32	34.38	8.4254	54.460	11.174	61.7645	63.7892	180.013	-7.0410	-8.5156	-2.8756	-4.7731																			
33	40.11	8.5677	55.518	11.128	61.4531	63.0425	180.013	-7.8238	-7.1416	-3.3463	-4.6420																			
34	45.84	8.7253	56.694	11.075	61.0952	62.2244	180.013	-8.4695	-5.7727	-3.8001	-4.4326																			
35	51.57	8.8958	57.972	11.015	60.6931	61.3486	180.013	-8.9806	-4.4478	-4.2288	-4.1424																			
36	57.3	9.0768	59.336	10.950	60.2495	60.4283	180.013	-9.3626	-3.1929	-4.6246	-3.7741																			



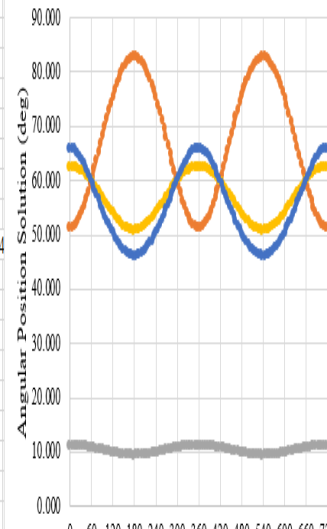
$$\left(\frac{dy}{dx}\right)_k = \frac{y_{k+1} - y_{k-1}}{2\Delta x_i}$$

Eq(7)

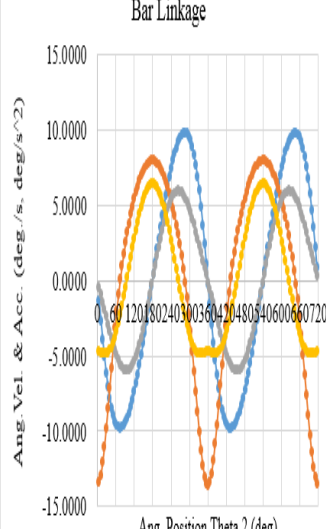
$$\left(\frac{d^2y}{dx^2}\right)_k = \frac{y_{k+1} - 2y_k + y_{k-1}}{(\Delta x_i)^2}$$

Eq(8)

Position Analytical Solution of Four-Bar-Linkage



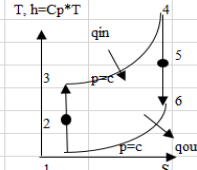
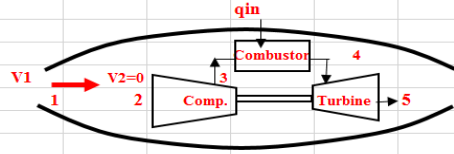
Numerical Differentiation Solution for Angular Velocity and Acceleration of Four Bar Linkage



Case Study 9. Jet Engine Thermodynamic analysis.

Jet engine thermodynamic analysis in both SI and English units is processed in Excel--Assume aircraft is stationary and the air is moving towards the aircraft at a velocity of $V_1=280\text{-m/s}$. Ideally, the air will leave the diffuser with a negligible velocity ($V=0$). Air is treated as an ideal gas with constant specific heats $C_p=1.005\text{-kJ/kg-K}$. Student can change any input to find the alternative design in jet engine of aircraft.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Jet Engine--Assume aircraft is stationary and the air is moving towards the aircraft at a velocity of $V_1=280\text{-m/s}$													
2	Ideally the air will leave the diffuser with a negligible velocity ($V=0$)													
3	Air is treated as an ideal gas with constant specific heats $C_p=1.005$													
4	Input (SI Unit):													
5	Air Inlet Temp. $T_1=$				260	K								
6	Air Inlet Pressure $p_1=$				48	kpa								
7	Air Inlet velocity $V_1=$				280	m/sec								
8	Pressure Ratio $PR=p_3/p_2=$				13									
9	Max. Temp $T_4=$				1300	kpa								
10	Specific Heat $C_p=$				1.005	kJ/kg-K								
11	Ratio $C_p/C_v=k=$				1.4									
12	Constant $Z=(k-1/k)=$				0.286									
13														
14	Solution:													
15	Step 1. Diffuser 1-2													
16	$q\dot{d}\text{-}W\dot{d}=h_2\text{-}h_1+(V_2^2\text{-}V_1^2)/2$					1st Law Thermody.								
17	where $q\dot{d}\text{-}W\dot{d}=V_2=0$ in diffuser, we have													
18	$C_p(T_2\text{-}T_1)\text{-}V_1^2/2=0$													
19	$T_2=T_1+V_1^2/(2C_p)=260+(280\text{-m/s})^2/(2C_p)(1\text{-kJ/kg}/1000\text{m}^2/\text{s}^2)=$				299.0	K								
20														
21	$p_2=p_1*(T_2/T_1)^{(1/Z)}$				78.3	kpa								
22	Step 2. Compressor 2-3													
23	$p_3=p_4=p_2*PR=$				1017.8	kpa								
24	$T_3=T_2(p_3/p_2)^Z=$				622.2	K								
25	Step 3. Turbine 4-5													
26	$W_{c,in}=W_{t,out}$	$h_3\text{-}h_2=h_4\text{-}h_5$												
27		$T_3\text{-}T_2=T_4\text{-}T_5$												
28	$T_5=T_4\text{-}T_3+T_2=$				976.8	K								
29	$p_5=p_4*(T_5/T_4)^{(1/Z)}$				374.2	kpa								
30	Step 4. Nozzle 5-6													
31	$T_6=T_5*(p_6/p_5)^Z=$				543.2	K								
32	$q\dot{d}\text{-}W\dot{d}=h_6\text{-}h_5+(V_6^2\text{-}V_5^2)/2$					1st Law Thermody.								
33	where $q\dot{d}\text{-}W\dot{d}=V_5=0$, we have $C_p(T_6\text{-}T_5)\text{-}V_6^2/2=0$													
34	$V_6=((2*C_p(kJ/kg\text{-}K)*(T_5\text{-}T_6)K(1000\text{m}^2/\text{s}^2)*(1\text{-kJ/kg}))^{(1/2)}=$				933.5	m/s								
35														
36	Step 5. Propulsive Work and Propulsive efficiency													
37	Propulsive Work													
38	$W_p=(V_{exit}\text{-}V_{inlet})*V_{aircraft}=(V_6\text{-}V_1)(m/s)*V_1(m/s)*(1\text{-kJ/kg}/(1000\text{-m}^2/\text{s}^2))$													
39		182.99				kJ/kg								
40	Propulsive efficiency $\eta_p=W_p/q_{in}=$				0.269	26.9%								
41	where $q_{in}=h_4\text{-}h_3=C_p(T_4\text{-}T_3)=$				681.2	kJ/kg								
42	The fuel consumption $(m\dot{d})_{fuel}=Q\dot{d}/q_{HV}=$				0.0158	kg								
43	q_{HV} is the heating value of the fuel (kerosene)=				43,000	kJ/kg								
44	where $(m\dot{d})_{fuel}/(m\dot{d})=E_{42}/1\text{-kg mass of Air}$				0.0158	1.58%								
45	Step 6. Required Propulsive Work W^* for Aircraft													
46	$W^*=$	4,000				kw								
47	The Req. mass flow rate of air $m\dot{d}=$				21.859	kg/s								
48	which need to design the of Jet Passage Area based on Aerodynamics													
49	The mass flow rate of fuel $m\dot{d}=$				0.3463	kg/s								
50	Total Mass of fuel in Tank $M_f=$				20000	kg								
51	Nonstop fly Hours of Aircraft $H=$				16.0	hours								
52														
53														
54	Step 5. Propulsive Work and Propulsive efficiency													
55	Propulsive Work													
56	$W_p=(V_{exit}\text{-}V_{inlet})*V_{aircraft}=(V_6\text{-}V_1)(m/s)*V_1(m/s)*(1\text{-Btu/lbm}/(25037\text{-ft}^2/\text{s}^2))$													
57		84.50				Btu/lbm								
58	Propulsive efficiency $\eta_p=W_p/q_{in}=$				0.254	25.40%								
59	where $q_{in}=h_4\text{-}h_3=C_p(T_4\text{-}T_3)=$				332.6	Btu/lbm								
60	The fuel consumption $(m\dot{d})_{fuel}=Q\dot{d}/q_{HV}=$				0.017	lbm/s								
61	q_{HV} is the heating value of the fuel (kerosene)=				19300	Btu/lbm								
62	where $(m\dot{d})_{fuel}/(m\dot{d})=M_{58}/1\text{-lbm of Air}$				0.0172	1.72%								



Case Study 10. Long-hand-calculation of Stiffness Matrix for two dimensional triangular three-node-element in CAE study.

There are three topics are show in this spreadsheet. 1. With matrix analysis tool in Excel, the calculation of stiffness matrix for a two dimensional triangular three-node-element is created. 2. For given nodal displacement of the element, to find the stresses in the element. 3. For given nodal forces of the element to find the stresses in the element. Which provides the basic formulation of finite element analysis in two dimensional problems.

Stiffness Matrix for Given Element Logan Example 6.1. (P.322)

Vector Column Matrices of Stress and Strain

Strain-Displacement Relationship

Constitutive Matrix (6.1.8)

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$$

Stress/Strain Relationship (6.1.7)

$$\{\sigma\} = [D] \{\epsilon\}$$

Constant-Strain Triangular Element Assumed Displacement Function (6.2.2)

$$u(x,y) = a_1 + a_2 x + a_3 y$$

$$v(x,y) = a_4 + a_5 x + a_6 y$$

Strain/Nodal Displacement Relationship (6.2.10)

$$\{\epsilon\} = \frac{1}{2A} \begin{bmatrix} \beta_i & 0 & \beta_j & 0 & \beta_m & 0 \\ 0 & \gamma_i & 0 & \gamma_j & 0 & \gamma_m \\ \gamma_i & \beta_i & \gamma_j & \beta_j & \gamma_m & \beta_m \end{bmatrix} \{d\}$$

Where

$\alpha_i = x_j y_m - y_j x_m$, $\beta_i = y_j - y_m$, $\gamma_i = x_m - x_j$

$\alpha_j = y_1 x_m - x_1 y_m$, $\beta_j = y_m - y_1$, $\gamma_j = x_1 - x_m$

$\alpha_m = x_1 y_j - y_1 x_j$, $\beta_m = y_1 - y_j$, $\gamma_m = x_j - x_1$

Stress/Nodal Displacement Relationship (6.2.36)

$$\{\sigma\} = [D] \{d\}$$

Stiffness Matrix for Given Element Logan Example 6.1. p.322

Coordinates Displacement Thickness

$x_i = 0$	$u_i = 0$	Area $A = 1/2 * \sum \alpha_i - \alpha_j - \alpha_m = 2$	1	in^2
$x_j = -1$	$v_j = 0.0025$	Material (Steel-A36)	2	in^2
$x_m = 2$	$u_m = 0.0012$	Modulus Elasticity $E = 30000000$	30000000	psi
$x_1 = 0$	$v_1 = 0$	Poisson's Ratio $\nu = 0.25$	0.25	
$x_2 = 0$	$u_2 = 0$			
$x_3 = 1$	$v_3 = 0.0025$			

Step 1. Calculate α, β, γ

Strain/Nodal Displacement Matrix (6.2.30)

$$B = 1/4 \begin{bmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & -1 & 0 & 2 & 2 & -1 \end{bmatrix}$$

Constitutive Matrix (6.1.8)

$$D = \frac{32000000}{1-0.25^2} \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 0.375 \end{bmatrix}$$

Stiffness Matrix for Given Element (6.2.52)

$$k = t * A * B^T * D * B = 1-in * 2-in^2 * 32000000/16 * \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 0.375 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & -1 & 0 & 2 & 2 & -1 \end{bmatrix}$$

Resulting Stiffness Matrix:

$$k = 4E+06 \begin{bmatrix} -1 & -0.25 & -0.75 \\ -0.5 & -2 & -0.375 \\ 2 & 0.5 & 0 \\ 0 & 0 & 0.75 \\ -1 & -0.25 & 0.75 \\ 0.5 & 2 & -0.375 \end{bmatrix} = 4000000 \begin{bmatrix} 2.5 & 1.25 & -2 & -1.5 & -0.5 & 0.25 \\ 1.25 & 4.375 & -1 & -0.75 & -0.25 & -3.63 \\ -2 & -1 & 4 & 0 & -2 & 1 \\ -1.5 & -0.75 & 0 & 1.5 & 1.5 & -0.75 \\ -0.5 & -0.25 & -2 & 1.5 & 2.5 & -1.25 \\ 0.25 & -3.63 & 1 & -0.75 & -1.25 & 4.375 \end{bmatrix}$$

Case 1. For given Nodal Displacement, to find the Stresses in Element

In Plane Stresses (6.2.36)

$$\{\sigma\} = [D] \{d\} = \frac{1}{4} \begin{bmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & -1 & 0 & 2 & 2 & -1 \end{bmatrix} \begin{bmatrix} 0.000 \\ 0.003 \\ 0.001 \\ 0.000 \\ 0.000 \\ 0.003 \end{bmatrix}$$

Resulting Stresses:

$$\{\sigma\} = 8000000 \begin{bmatrix} 0.0024 \\ 0.0006 \\ -0.001875 \end{bmatrix} = \begin{bmatrix} 19200 \\ 4800 \\ -15000 \end{bmatrix} \text{ psi}$$

Principal Stresses and Principal Direction (6.1.2)

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{max}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{min}$$

Also, the principal angle θ_p , which defines the normal whose direction is perpendicular to the plane on which the maximum or minimum principal stress acts, is defined by (6.1.3)

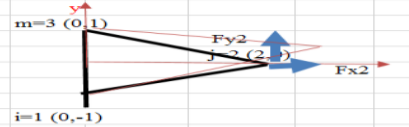
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Resulting Principal Stresses and Angle:

$\sigma_1 = 28639$	psi
$\sigma_2 = -4639$	psi
$\theta_p = -32.18$	deg

81 **Case 2. For given Nodal forces of element to find the stresses**
 82 $u_1=v_1=u_3=v_3=0$ $F_{x2}=500\text{-lb}$, $F_{y2}=200\text{-in}$

84
$$\begin{Bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{16} \\ k_{21} & k_{22} & \dots & k_{26} \\ \vdots & \vdots & \ddots & \vdots \\ k_{61} & k_{62} & \dots & k_{66} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \quad (6.2.55)$$

85 

91
$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ 500 \\ 200 \\ F_{x3} \\ F_{y3} \end{Bmatrix} = 4E+06 \begin{bmatrix} 2.5 & 1.25 & -2 & -1.5 & -0.5 & 0.25 \\ 1.25 & 4.375 & -1 & -0.75 & -0.25 & -3.625 \\ -2 & -1 & 4 & 0 & -2 & 1 \\ -1.5 & -0.75 & 0 & 1.5 & 1.5 & -0.75 \\ -0.5 & -0.25 & -2 & 1.5 & 2.5 & -1.25 \\ 0.25 & -3.625 & 1 & -0.75 & -1.25 & 4.375 \end{bmatrix} \begin{Bmatrix} u_1=0 \\ v_1=0 \\ u_2=? \\ v_2=? \\ u_3=0 \\ v_3=0 \end{Bmatrix}$$

97 **Solving Sub-Matrix with Sub-matrix Method**

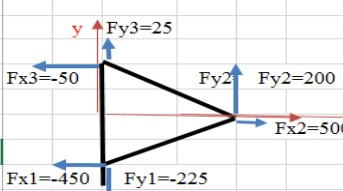
98
$$\begin{Bmatrix} 500 \\ 200 \end{Bmatrix} = 4E+06 \begin{bmatrix} 4 & 0 \\ 0 & 1.5 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \end{Bmatrix}$$

100 We have
$$\begin{Bmatrix} u_2 \\ v_2 \end{Bmatrix} = 1/4E+06 \begin{bmatrix} 0.25 & 0 \\ 0 & 0.66667 \end{bmatrix} \begin{Bmatrix} 500 \\ 200 \end{Bmatrix} = 1/4E+06 \begin{bmatrix} 125.00 \\ 133.33 \end{bmatrix} = \begin{Bmatrix} 3.1E-05 \text{ in} \\ 3.3E-05 \text{ in} \end{Bmatrix}$$

103 **Substituting back to find the unknown nodal forces**

104
$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ 500 \\ 200 \\ F_{x3} \\ F_{y3} \end{Bmatrix} = 4E+06 \begin{bmatrix} 2.5 & 1.25 & -2 & -1.5 & -0.5 & 0.25 \\ 1.25 & 4.375 & -1 & -0.75 & -0.25 & -3.625 \\ -2 & -1 & 4 & 0 & -2 & 1 \\ -1.5 & -0.75 & 0 & 1.5 & 1.5 & -0.75 \\ -0.5 & -0.25 & -2 & 1.5 & 2.5 & -1.25 \\ 0.25 & -3.625 & 1 & -0.75 & -1.25 & 4.375 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 3.1E-05 \\ 3.3E-05 \\ 0 \\ 0 \end{Bmatrix}$$

110
$$= 4E+06 \begin{Bmatrix} -0.00011 \\ -5.6E-05 \\ 0.00013 \\ 0.00005 \\ -1.3E-05 \\ 6.3E-06 \end{Bmatrix} = \begin{Bmatrix} -450 \\ -225 \\ 500 \\ 200 \\ -50 \\ 25 \end{Bmatrix} \text{ lb}$$

116 

117 **Double check by $\Sigma F_x = \Sigma F_y = 0$**

123 **Go back to (6.2.36) to find the stresses**

124 **Stress/Nodal Displacement Relationship**

125
$$\{\sigma\} = [D] \{\epsilon\} = D B \{d\} \quad (6.2.36)$$

Conclusion:

Excel spreadsheet in Microsoft Office allows the integration of computer based projects with traditional mechanical engineering topics. Student is enjoying their spreadsheet learning environment and will bring hundreds spreadsheet calculators to deal with their future engineering design and analysis problems. The easy learning Excel spreadsheets allows our student to get the alternative solution of their interested problems by simply changing the inputs in their Excel calculator. Where the most desirable solution for a given problem could be found by considering the given environment and to improve the design quality. With Excel spreadsheet background, MET student get better understanding and great confidence to deal with their future professional challenge.

Short Bio of author:

Ti-Lin Liu is an associate professor in the Manufacturing and Mechanical Engineering Technology Department at Rochester Institute of Technology. He has been teaching at RIT since 1987. Previously he was the Chair Professor of the Mechanics Department at Shanghai University of Technology. His areas of interest include finite element analysis, computer simulation with working model and Matlab/Simulink, numerical analysis, solid mechanics, heat transfer and system dynamics/vibration. He has been the recipient of teaching awards in China and was the sole recipient of the 2001 Annual Grant Reward from MSC Software Corporation, San Mateo, CA. Research interests include: Finite element analysis, computer simulation with working model and Matlab/Simulink.