AC 2010-1666: EXPANDED USE OF DISCONTINUITY AND SINGULARITY FUNCTIONS IN MECHANICS

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Professor Soutas-Little received his B.S. in Mechanical Engineering 1955 from Duke University and his MS 1959 and PhD 1962 in Mechanics from University of Wisconsin. His name until 1982 when he married Patricia Soutas was Robert William Little. Under this name, he wrote an Elasticity book and published over 20 articles. Since 1982, he has written over 100 papers for journals and conference proceedings and written books in Statics and Dynamics. He has taught courses in Statics, Dynamics, Mechanics of Materials, Elasticity, Continuum Mechanics, Viscoelasticity, Advanced Dynamics, Advanced Elasticity, Tissue Biomechanics and Biodynamics. He has won teaching excellence awards and the Distinguished Faculty Award. During his tenure at Michigan State University, he chaired the Department of Mechanical Engineering for 5 years and the Department of Biomechanics for 13 years. He directed the Biomechanics Evaluation Laboratory from 1990 until he retired in 2002. He served as Major Professor for 22 PhD students and over 100 MS students. He has received numerous research grants and consulted with engineering companies. He now is Professor Emeritus of Mechanical Engineering at Michigan State University.

Expanded Use of Discontinuity and Singularity Functions in Mechanics

Abstract

W. H. Macauley published *Notes on the Deflection of Beams*[1] in 1919 introducing the use of discontinuity functions into the calculation of the deflection of beams. In particular, he introduced the singularity functions, the unit doublet to model a concentrated moment, the Dirac delta function to model a concentrated load and the Heaviside step function to start a uniform load at any point on the beam. Stephen H. Crandall and Norman C. Dahl incorporated these functions into their text on *An Introduction to the Mechanics of Solids*[2] in 1959. The discontinuity functions allow writing a discontinuous function as a single expression instead of writing a series of expressions. The traditional approach requires that the different expressions be written for each region where a discontinuity appears, and when integrated, must be matched by evaluating the constants of integration. This is a mathematically laborious task that becomes more complex as the number of discontinuous functions increases. The Macauley functions are used to start a polynomial loading $\langle x-a\rangle^n = \begin{cases} (x-a)^n & x \geq a \\ 0 & x < a \end{cases}$ at some point on the beam. These discontinuity functions appear in many, if not most, of the current mechanics of materials texts. There are two problems with the Macauley functions; first, they are very limited in the type of load functions that they model and second, for orders above n=1, they are difficult to stop if the region of application is only between $a \leq x \leq b$, where b is less than the length of the beam. The difficulty arises in introducing the negative of higher order polynomials at the point b. A method will be presented to analyze any continuous load function *w(x)* applied on the interval between $a \leq x \leq b$. Therefore, a single expression will be written for any beam loading. This expression will be integrated to determine the shear, moment, slope and deflection. Examples of different beam loadings are presented for a complete use of discontinuity functions. The use of discontinuity functions will be expanded to axial loadings, torsion of circular rods and particle dynamics.

Discontinuity Functions

The discontinuity functions were first introduced by the German mathematician A. Clebsch (1833-1872) in 1862[3]. Walter D. Pilkey gives a complete history of Clebsch's method in his 1964 article [4]. He traced its long popularity in foreign countries and predicted it "will indubitably receive more attention in the future as the current trend is toward the engineering student becoming better equipped mathematically to cope with such analytical techniques." Although the discontinuity functions, as defined by Macauley [1], appear in most mechanics of materials texts, they have not received the attention that Pilkey predicted. These functions were further developed by Oliver Heaviside (1850-1925), an English physicist and electrical engineer, and Paul A. M. Dirac (1902-1984), the 1933 Nobel Laureate in Physics.

There are two types of discontinuity functions; those that are singular at the point of discontinuity (singularity functions) and those that are not singular at the point of discontinuity.

The two singularity functions that are of importance in beam analysis are the unit doublet function, also called the dipole function or the unit moment function, and the unit impulse function, also called the Dirac delta function or the concentrated load function. The Dirac delta function can be represented graphically as follows:

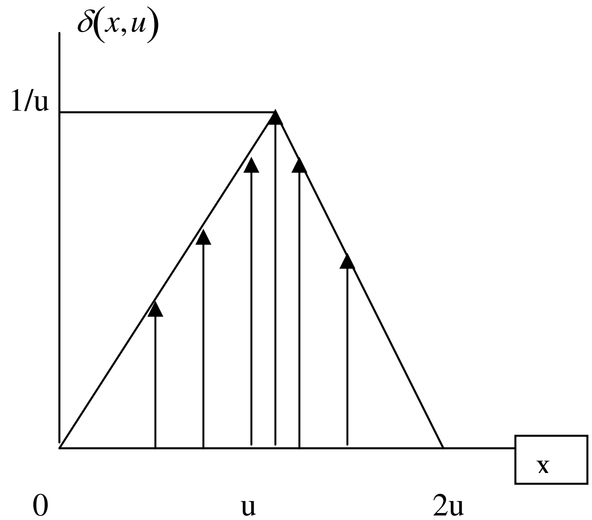

Figure 1: Graphical display[2] of the Dirac delta function at x = 0.

The Dirac delta function $\delta(x-0)$ can be defined as:

$$\delta(x-0)=\lim_{u\to 0}\delta(x,u)\begin{cases}0 & -\infty \le x < 0\\ \dfrac{x}{u^2} & 0 < x < u\\ \dfrac{2}{u}-\dfrac{x}{u^2} & u < x < 2u\\ 0 & 2u < x < \infty\end{cases} \qquad (1)$$

and represents a concentrated load of magnitude one [2] at $x=0$. Macauley represented the Dirac delta function at $x=a$ as a bracket with a subscript of –1.

$$\langle x-a\rangle_{-1}=\delta(x-a)=0 \quad x\neq a \qquad (2)$$

The unit doublet, or concentrated couple or moment can be represented graphically as follows:

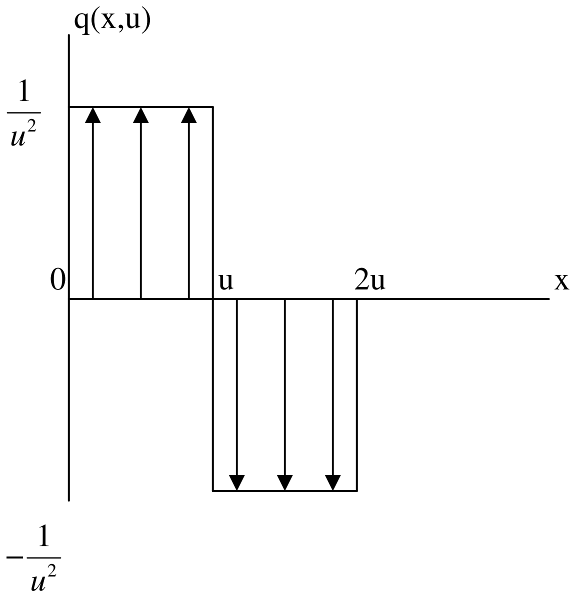

Figure 2: Graphical display[2] of the unit doublet function at x=0.

The unit doublet function at $x = 0$ is defined as:

$$q(x-0) = \lim_{u \to 0} q(x,u) \begin{cases} 0 & -\infty \le x \le 0 \\ \frac{1}{u^2} & 0 < x < u \\ -\frac{1}{u^2} & u < x < 2u \\ 0 & 2u < x < \infty \end{cases} \tag{3}$$

The unit doublet function[2] represents a concentrated moment of magnitude one at $x = 0$. Macauley used the notation $\langle x-a \rangle_{-2} = q(x-a)$ for a unit moment acting at $x = a$.

The basic discontinuity function is the Heaviside step function or unit step function. Many notations are used for this function by different authors. Some of the notations are as follows:

$$\Phi(x-a) = H(x-a) = \langle x-a \rangle^0 = \begin{cases} 0 \text{ for } -\infty < x < a \\ 1 \text{ for } a < x < \infty \end{cases} \tag{4}$$

Macauley used the notation $\langle x-a \rangle^0$.

For the rest of this paper, the following notations will be used:

Unit moment at $x = a$: $q(x-a)$

Dirac delta function at $x = a$: $\delta(x-a)$ (5)

Heaviside step function at $x = a$: $\Phi(x-a)$

If computational software is used, always check to see functional notations. Some of the common software notations[4,5] are shown in Table 1.

	MATLAB	Mathcad	Maple	Mathmatica
Heaviside step	Heaviside(x-a)	$\Phi(x-a)$	Heaviside(x-a)	Unitstep(x-a)
Dirac delta	Dirac(x-a)	Dirac(x-a)	Dirac(x-a)	Diracdelta(x-a)

Table 1 Computational Software Symbols

Integrals and Differentials

The differential of the Dirac delta function is:

$$\frac{d}{dx}\delta(x-a) = q(x-a) \tag{6}$$

The differential of the Heaviside step function is:

$$\frac{d}{dx}\Phi(x-a) = \delta(x-a) \tag{7}$$

In a similar manner, the integrals of these functions are:

$$\begin{aligned}\int_{-\infty}^{x} q(x-a)dx &= \delta(x-a)\\ \int_{-\infty}^{x} \delta(x-a)dx &= \Phi(x-a)\end{aligned} \tag{8}$$

General Discontinuity Function

The Macauley functions are developed from integrals of the Heaviside step function and are correct but limited in the functions that can be represented. A general discontinuity function is defined as:

$$w(x) = \Phi(x-a)F(x) - \Phi(x-b)F(x) \tag{9}$$

Note that this function starts at $x = a$ and ends at $x = b$.

A more general integral will be developed which greatly increases the use of these functions. Consider the integral of the product of the Heaviside step with a general function F(x):

$$\int_{-\infty}^{x} \Phi(\xi-a)F(\xi)d\xi$$

The product $\Phi(x-a)F(x)$ will start any function $F(x)$ at $x = a$ and continue the function to $+\infty$. This will be used to start or end any function. To be useful the general integral of this product must be evaluated. Consider the general integral:

$$\int_{a}^{b} udv = uv\Big|_{a}^{b} - \int_{a}^{b} vdu \tag{10}$$

Let

$$u(\xi)=\Phi(\xi-a) \quad dv=F(\xi)d\xi$$
$$du=\delta(\xi-a)d\xi \quad v=\int F(\xi)d\xi=G(\xi) \tag{11}$$

Therefore

$$\int_{-\infty}^{x}\Phi(\xi-a)F(\xi)d\xi=\Phi(x-a)G(x)-\Phi(x-a)G(a) \qquad \underline{\text{Basic Integral}}^{5} \tag{12}$$

Macauley treated this integral only when $F(x)=\Phi(x-a)C(x-a)^n \quad n\geq 0$. A special case of the basic integral is when $F(\xi)=C$ a constant. This would lead to the Macauley functions.

$$\int_{-\infty}^{x}\Phi(\xi-a)Cd\xi=\Phi(x-a)C(x-a) \tag{13}$$

The Macauley functions can now be developed.

$$\int_{-\infty}^{x}\Phi(\xi-a)C(\xi-a)^n d\xi=\Phi(x-a)C\frac{(x-a)^{n+1}}{n+1} \tag{14}$$

Using the Macauley notation, this integral can be written as:

$$\int\langle x-a\rangle^n dx=\frac{\langle x-a\rangle^{n+1}}{n+1} \tag{15}$$

The second term in the basic integral, (Eq. 12) in this case is zero.

Application to Beam Problems

The shear, moment, slope and deflection of a beam can be obtained by continued integration of the distributed load function, $w(x)$, which is perpendicular to the long axis of the beam. If y, the deflection of the beam, is positive up, these equations are:

$$V(x)=\int w(x)dx \quad \text{the shear distribution}$$
$$M(x)=\int V(x)dx \quad \text{the moment distribution}$$
$$EI\frac{dy}{dx}(x)=\int M(x)dx \quad \text{the slope of the beam} \tag{16}$$
$$EIy(x)=\int\frac{dy}{dx}dx \quad \text{where y(x) is the deflection}$$

If the load distribution function is continuous across the length of the beam, these integrations are straight forward and the constants of integration will yield the beam reactions and the initial slope and deflection. Frequently, for statically determinant problems, the reactions are solved for initially and the process will start with the moment equation[5]. A difficulty arises when $w(x)$ is not a continuous function across the full length of the beam. In these cases, the beam must be divided into sections over which the load can be expressed as a continuous function. The constants of integration are then used to insure that the beam slope and deflection are continuous.

This can be a mathematically tedious task. Macauley [1] suggested the use of discontinuity functions for these cases and introduced definitions for these functions and a notation for the functions. The discontinuity Macauley, non-singular, functions are defined as follows:

$$\langle x-a\rangle^n = \begin{cases} (x-a)^n & x \ge a \\ 0 & x < a \end{cases} \tag{17}$$

Note that the pointed brackets are used for these functions.

Sample Beam Problems

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Example 1

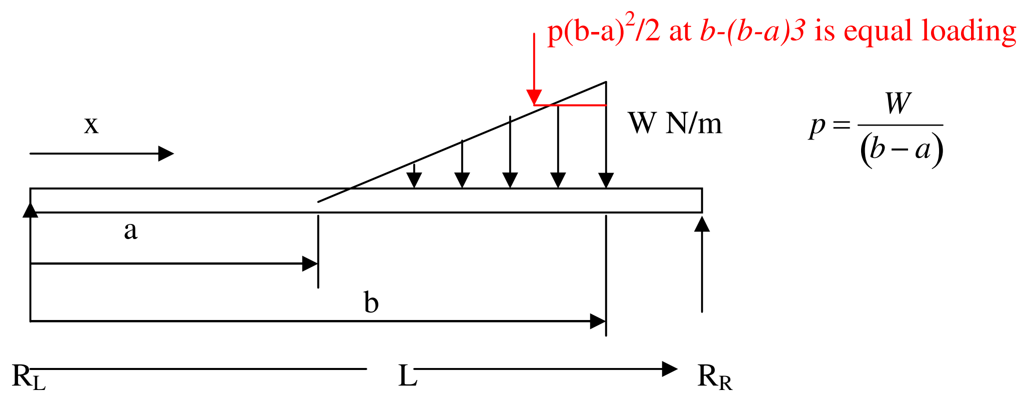

Figure 3. Example 1 beam loading

The beam distributed load function is given by:

$$w(x) = R_L\delta(x-0) - \Phi(x-a)p(x-a) + \Phi(x-b)p(x-a) + R_R\delta(x-L) \tag{18}$$

The shear function is obtained by integration of Eq. (18)

$$V(x) = R_L\Phi(x-0) - \Phi(x-a)p\frac{(x-a)^2}{2} + \Phi(x-b)p\frac{(x-a)^2}{2}$$
$$-\Phi(x-b)p\frac{(b-a)^2}{2} + R_R\Phi(x-L) + C_1 \tag{19}$$

The constant of integration is evaluated using

$$V(-0) = 0 \qquad \therefore C_1 = 0 \tag{20}$$

The equation of static equilibrium is obtained by noting that the shear is zero for all values of x greater than L.

$$V(L+) = 0 \qquad R_L + R_R - \frac{p(b-a)^2}{2} = 0 \tag{21}$$

The moment equation is obtained by integration of Eq. (19).

$$\begin{aligned} M(x) = {} & R_L \Phi(x-0)x - \Phi(x-a)p\frac{(x-a)^3}{6} + \Phi(x-b)p\frac{(x-a)^3}{6} \\ & - \Phi(x-b)p\frac{(b-a)^2}{2}(x-b) - \Phi(x-b)p\frac{(b-a)^3}{6} + \Phi(x-L)R_R(x-L) + C_2 \end{aligned} \tag{22}$$

The second constant of integration is evaluated by noting that the moment is zero at the origin.

$$\mathrm{M}(0) = 0 \qquad \therefore C_2 = 0 \tag{23}$$

The second equation of static equilibrium is obtained by noting that the moment is zero for all values of *x* greater than L.

$$M(L+) = 0 \qquad LR_L - p\frac{(b-a)^2}{2}(L-b) - p\frac{(b-a)^3}{6} = 0 \tag{24}$$

The total load acting down is $p\frac{(b-a)^2}{2}$ and it acts at a point one-third the distance from *b* to *a*.

Solving Equation (24) yields;

$$R_L = \frac{p(b-a)^2}{2}\left[\frac{L - b + \frac{b-a}{3}}{L}\right] \tag{25}$$

Equation (21) yields:

$$R_R = \frac{p(b-a)^2}{2}\left[\frac{b - \frac{b-a}{3}}{L}\right] \tag{26}$$

This example was chosen, because for the region $b \le x \le L$, the shear and moment could be obtained by substituting the distributed load by a concentrated load on the ramp function. The slope and deflection of the beam is obtained by integrating the moment equation twice.

Example 2

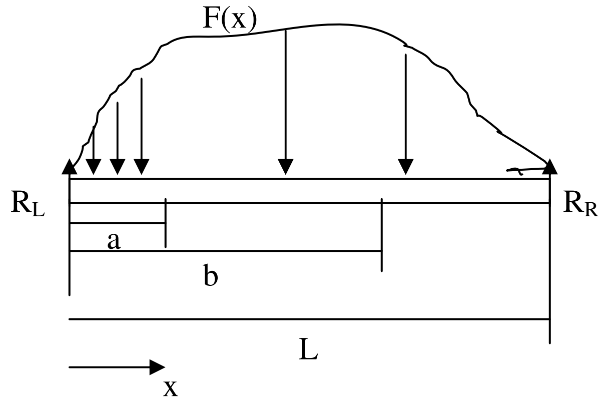

Figure 4. Simply supported beam under general loading

Consider a simply supported beam subjected to a sinusoidal loading $F(x) = -W \sin\frac{\pi x}{L} \quad a \le x \le b$

The distributed load function is

$$w(x) = R_L\delta(x) - \Phi(x-a)W\sin\frac{\pi x}{L} + \Phi(x-b)W\sin\frac{\pi x}{L} + R_R\delta(x-L) \tag{27}$$

Using the basic integral, the shear may be written as

$$\begin{aligned} V(x) &= R_L\Phi(x-0) - \Phi(x-a)\frac{WL}{\pi}\cos\frac{\pi x}{L} + \Phi(x-a)\frac{WL}{\pi}\cos\frac{\pi a}{L} \\ &+ \Phi(x-b)\frac{WL}{\pi}\cos\frac{\pi x}{L} - \Phi(x-b)\frac{WL}{\pi}\cos\frac{\pi b}{L} + R_R\Phi(x-L) + C_1 \end{aligned} \tag{28}$$

Integrating a second time yields the moment equation.

$$\begin{aligned} M(x) &= R_L x + \Phi(x-a)\frac{WL^2}{\pi^2}\sin\frac{\pi x}{L} - \Phi(x-a)\frac{WL^2}{\pi^2}\sin\frac{\pi a}{L} + \Phi(x-a)\frac{WL}{\pi}\cos\frac{\pi a}{L}(x-a) \\ &+ \Phi(x-b)\frac{WL^2}{\pi^2}\sin\frac{\pi x}{L} - \Phi(x-b)\frac{WL^2}{\pi^2}\sin\frac{\pi b}{L} \\ &- \Phi(x-b)\frac{WL}{\pi}\cos\frac{\pi b}{L}(x-b) + R_R\Phi(x-L)(x-L) + C_1 x + C_2 \end{aligned} \tag{29}$$

Setting the shear and moment equal to zero to the left of the origin to evaluate the constants of integration yields

$$\begin{aligned} V(0^-) &= 0 \quad C_1 = 0 \\ M(0^-) &= 0 \quad C_2 = 0 \end{aligned} \tag{30}$$

The reactions at 0 and L can be evaluated by setting the shear and moment equal to zero at x=L.

$$V(L^+) = R_L + \frac{WL}{\pi}\left[\cos\frac{\pi a}{L} - \cos\frac{\pi b}{L}\right] + R_R \tag{31}$$

Note, the total load on the beam is $P = \frac{WL}{\pi}\left(\cos\frac{\pi b}{L} - \cos\frac{\pi a}{L}\right)$. (32)

Setting the moment at L^+ equal to zero yields:

$$M(L^+) = R_L L - \frac{WL^2}{\pi^2}\left[\sin\frac{\pi a}{L} - \sin\frac{\pi b}{L}\right] + \frac{WL}{\pi}\left[\cos\frac{\pi a}{L}(L-a) - \cos\frac{\pi b}{L}(L-b)\right] = 0 \qquad (33)$$

$$R_L = \frac{WL}{\pi^2}\left[\sin\frac{\pi b}{L} - \sin\frac{\pi a}{L}\right] - \frac{WL}{\pi}\left[\cos\frac{\pi a}{L}\left(1-\frac{a}{L}\right) - \cos\frac{\pi b}{L}\left(1-\frac{b}{L}\right)\right] \qquad (34)$$

$$R_R = -\frac{WL}{\pi^2}\left[\sin\frac{\pi b}{L} - \sin\frac{\pi a}{L}\right] - \frac{WL}{\pi}\left[\frac{a}{L}\cos\frac{\pi a}{L} - \frac{b}{L}\cos\frac{\pi b}{L}\right] \qquad (35)$$

The moment equation can be integrated two more times to obtain the slope and deflections and the constants of integration are evaluated by setting the deflection equal to zero at x equals 0 and L.

Other Applications of Discontinuity Functions

Although this paper mainly shows applications of the discontinuity functions to beam problems, they are equally applicable to axial loading of rods, torsion loading of circular bars and dynamic loads on a particle. Pilkey[4] wrote "If the method's current popularity can be accepted as a measure of the trend for the future, then the already established universal equations for most of the remaining problems on small deflection of classical strength of materials, such as simple extensions or torsion of bars." Extension and torsion present simpler applications of discontinuity functions as there is only one integration the load distribution function and the extension and the angle of twist.

Axial Loading

Consider a rod under an axial distributed loading $f(x) \quad 0 \le x \le L$. The force at any point on the rod is:

$$F(x) = \int_0^x f(x)dx \qquad (36)$$

The deformation of the rod is:

$$\Delta(x) = \frac{1}{AE}\int_0^x F(x)dx \qquad (37)$$

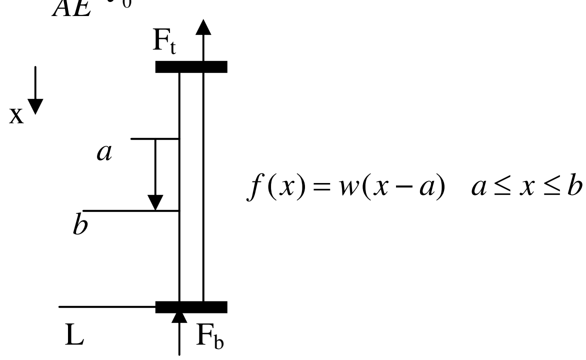

Figure 5. Distributed axial loading

The axial load distribution function is:

$$f(x) = -F_t\delta(x-0) + \Phi(x-a)w(x-a) - \Phi(x-b)w(x-a) - F_b\ \delta(x-L) \qquad (38)$$

The axial loading functions is obtained by integration

$$F(x) = \left[-F_t\Phi(x) + \Phi(x-a)w\frac{(x-a)^2}{2} - \Phi(x-b)w\frac{(x-a)^2}{2} + \Phi(x-b)w\frac{(b-a)^2}{2} - \Phi(x-L)F_b\right] + C_1$$

$$F(0^-) = 0 \qquad C_1 = 0$$

$$F(L^+) = -F_t + w\frac{(b-a)^2}{2} - F_b = 0$$

This is the equilibrium equation (39)

$$\therefore F_t + F_b = w\frac{(b-a)^2}{2}$$

The elongation of the rod is $\Delta(x) = \frac{1}{AE}\int_0^x F(x)dx$

$$\Delta(x) = \frac{1}{AE}[-F_t x + \Phi(x-a)w\frac{(x-a)^3}{6} - \Phi(x-b)w\frac{(x-a)^3}{6} + \Phi(x-b)w\frac{(b-a)^3}{6}$$
$$+ \Phi(x-b)w\frac{(b-a)^2}{2}(x-b) - \Phi(x-L)F_b(x-L)] + C_2 \qquad (40)$$

The elongation at $x = 0$ is zero and therefore $C_2 = 0$.
The elongation of the bar at $x = L^+$ is zero, yielding:

$$\Delta(L^+) = -F_t L + w\frac{(b-a)^3}{6} + w\frac{(b-a)^2}{2}(L-b) = 0 \qquad (41)$$

The reactions can now be obtained.

$$F_t = \frac{1}{L}w\frac{(b-a)^2}{2}\left[\frac{b-a}{3} + L - b\right]$$
$$F_b = \frac{1}{L}w\frac{(b-a)^2}{2}\left[b - \frac{(b-a)}{3}\right] \qquad (42)$$

Torsion of a Circular Rod

The study of a distributed torque along a circular rod is mathematically identical to that of axial load on a rod. Consider a rod built in at both ends. If t(x) is the torque distribution along the rod, then the torque at any point x is

$$T(x) = \int_0^x t(x)dx \qquad (43)$$

The total twist angle of the rod at any point x is:

$$\theta(x) = \frac{1}{GJ}\int_0^x T(x)dx \qquad (44)$$

Consider a rod of length L with a concentrated torque at $x = a$ of magnitude T_a.

$$t(x) = T_L\delta(x) - T_a\delta(x-a) + T_R\delta(x-L) \qquad (45)$$

The torque at any point on the rod is:

$$T(x) = T_L\Phi(x) - T_a\Phi(x-a) + T_R\Phi(x-L) + C \qquad (46)$$

The torque at $x = 0^-$ *is zero,* and the constant $C = 0$. (47)
The torque at $x = L^+$ is zero, therefore:

$$T_L - T_a + T_R = 0 \qquad (48)$$

This is the static equilibrium equation. The angle of twist at any point x is:

$$\theta(x)=\frac{1}{GJ}\left[T_L x-\Phi(x-a)T_a(x-a)+\phi(x-L)T_L(x-L)+C_1\right] \quad (49)$$

The angle of twist at $x = 0$ is zero, therefore $C_1 = 0$.
The right end of the rod is built in and the angle of twist at that point is zero.

$$T_L L - T_a(L-a) = 0 \quad (50)$$

Therefore:

$$T_L = T_a \frac{L-a}{L} \quad (51)$$

Using the equilibrium equation:

$$T_R = T_a - T_L = T_a \frac{a}{L} \quad (52)$$

Applications to Particle Dynamics

The application of discontinuity functions to particle dynamics problems presents an opportunity to investigate a broad range of applications. Although discontinuity functions are a powerful tool in dynamics, they are not presented in most dynamics texts and therefore in most courses. Many of the applications using these functions lead to differential equations that require numerical solutions. If computational software is used in the course, these equations can be solved using the Euler method. This method is easy for students at this level to understand. All the software programs include the higher order Runge-Kutta method.[7]

Dynamics Problem 1

The easiest applications involve applying an impulse force to the particle. The governing differential equation in this case is:

$$m\frac{d^2x}{dt^2} = F(t) \quad (53)$$

This is the impulse distribution function.

$$mv(t) = m\frac{dx}{dt} = \int_0^t F(t)dt \quad (54)$$

$$mx(t) = \int v(t)dt \quad (55)$$

Consider an impulse

$$F(t) = \Phi(t)p - \Phi(t-30)p + \Phi(t-60)p - \Phi(t-90)p \quad (56)$$

that represents a impulse from 0 to 30 seconds and another one from 60 to 90 seconds. The velocity function can be written as:

$$\begin{aligned} mv(t) &= \Phi(t)pt - \Phi(t-30)p(t-30) + \Phi(t-60)p(t-60) \\ &\quad - \Phi(t-90)p(t-90) + C_1 \end{aligned} \quad (56)$$

If the initial velocity is zero, the constant is zero; $C_1 = 0$. The displacement at any time is:

$$\begin{aligned} mx(t) &= \Phi(t)p\frac{t^2}{2} - \Phi(t-30)p\frac{(t-30)^2}{2} + \Phi(t-60)p\frac{(t-60)^2}{2} \\ &\quad - \Phi(t-90)p\frac{(t-90)^2}{2} + C_2 \end{aligned} \quad (57)$$

If the initial position is zero, the second constant is zero.

This is a very simple problem of rectilinear motion where the force acting on the particle is a function only of time. In general, during rectilinear motion, the force will be a function of time, position and velocity. Consider a much more complex problem in particle dynamics.

Dynamics Problem 2

A stuntman of mass *m* jumps from a platform at a height *h* and lands on a mat to break his fall. The mat has a spring constant of *k* and a damping *c* linearly proportional to the velocity. Note that the damping occurs only while the mat is being compressed. Write the equation of motion for the fall of the stuntman, and plot his motion with time. Determine the force exerted during contact with the mat.

The coordinate origin is taken at the platform and the coordinate *x* is positive in the downward direction. The force acting on the stuntman during the fall is:

$$m\frac{d^2x}{dt^2} = F(v,x) \tag{58}$$

$$F(v,x) = mg - \Phi(x-h)\left[k(x-h) + \Phi(v)cv\right] \tag{59}$$

Note, the Heaviside step function $\Phi(v)$ is equal to zero for all velocities equal to or less than zero. The mat damping does not affect the rebound of the stuntman. The differential equation is nonlinear and can be solved numerically using computational software[7].

A Mathcad program is used to generate a solution for a 200 lb man falling 20 ft onto a mat as a numerical example. The Mathcad worksheet is shown in Figure 6 using an Euler method to solve the equation. This problem example is solved with some estimated values of the coefficients *k* and *c*. These can be adjusted to change the mat properties to produce desired results for the stuntman.

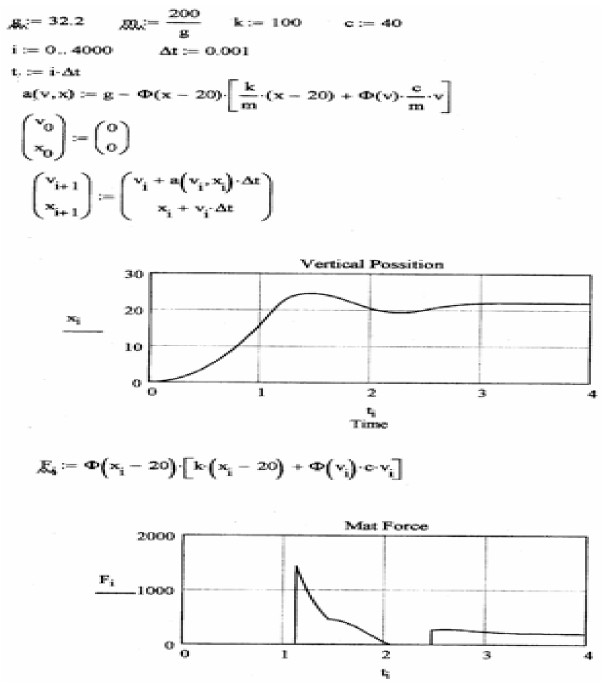

Figure 6. Mathcad solution to Dynamics Problem 2

When the problem is solved in this manner, the student can adjust the parameters and see the results. This gives great insight into the problem and may be used in design. The author has used computational software to solve the linear and non-linear dynamic vector differential equations in dynamic problems for years. The traditional approach in dynamics courses is to consider many of the problems in a "quasistatic manner", that is solve for the acceleration at the instant shown or the position shown. This gives the student little understanding of the resultant motion.[8,9,10]

Conclusions

Using these functions in introductory courses in Statics, Dynamics and Mechanics of Materials may, at first glance, seem too mathematical of an approach. The author has used the Macauley functions since 1958 and discontinuity functions in dynamics since 1996. The students have minimum difficulty in grasping this approach and especially in Dynamics, enjoy completely solving the equations of motion. Without using computational software (MATLAB, Mathcad, Maple and Mathematica), most dynamics problems are presented in a quasi-static manner; that is, "find the acceleration at the instant shown or the position shown." The student gains little information about the actual motion. Discontinuity functions can also be used in Mechanics of

Materials problems to account for change in shape of the cross section of the structure. These functions allow a great increase in the type of functions that can be considered in basic mechanics problems. Although Pilkey[4] predicted increased use of these functions in 1964, their use is still limited in the United States.

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