Session 2666

EXPERIMENTAL METHOD OF CAUSTICS FOR CIVIL AND MECHANICAL ENGINEERING STUDENTS

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Abstract

The ever-increasing industrial demand for more sophisticated structural and machine components requires a good understanding of the concepts of stress, strain, and behavior of materials. In particular, stress concentrations are a major concern for engineers. Experimental investigations provide the required tool to understand the development of stress concentrations. This paper proposes the use of the optical method of caustics to study the development of stress concentration around circular holes. The goal of these demonstrations is to improve the students' comprehension in mechanics of materials as well as in structural and machine design.

Introduction

At the sophomore level, students in a civil and mechanical engineering programs are introduced to the concepts of stress and strain in a solid body through the Strength of Materials course. In the first Machine Design course, junior mechanical engineering students learn to get a stress concentration factors (SCF) for practical problems from a chart. A senior civil engineering student utilizes similar charts to extract SCF in a steel design course.

The theory usually deals with infinite members. Kirsch developed the theoretical stress distribution in the vicinity of a circular hole in an infinite elastic isotropic plate[1]. This theory predicts a stress-concentration factor (SCF) of 3.0 for the hole with the maximum tensile and compressive stresses being 0 and 90 degrees from the horizontal axis of the hole, respectively. In the field of stress concentrations, the limited established theory does not give an insight for the understanding of the development of stresses in the vicinity of a discontinuity. Thus, experimental work is required to enhance the learning such as stress concentrations[2,3,4]. The solution for the circular hole in a finite-width plate under uniaxial tension was published by Howland[5] in 1930. In more resent years, experimental solutions have been obtained for a wide variety of hole shapes under different loading conditions[6]. Electrical-strain gages are widely used devices to measure the strains in stressed members. However, the averaging effect of a strain gage is problematic in regard to measuring the strain in the vicinity of discontinuties[7,8].

It is important that the students visualize the nature of the quantities being computed. Therefore, the enhancement of the student's overall understanding of the concept of stress concentrations is

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discussed in this paper. This is accomplished by utilizing the experimental method of caustics. The determination of SCF is beyond the scope of this paper.

Optical method of caustics

The method of caustics is relatively new as it was developed in the last 30 years. The methods of transmitted and reflected caustics in various investigations have proven to be a powerful method to measure stress intensity factor at a crack tip in static and dynamic fracture mechanics problems[9,10]. In 1991, the use of the method by undergraduate students was suggested to extract mode I stress intensity factor[11]. The optical method of caustics is a technique based on geometrical optics. The method is accurate, simple and economical because the optical bench has relatively few components. The overwhelming majority of studies that utilize the method of caustics use specimens that are made of Plexiglas; it is assumed that Plexiglas is an optically and mechanically isotropic material at room temperature. The accuracy of the proposed experiments derives from the fact that the physical stress models must obey the practical laws of physics.

The principle of the method is simple in concept. The formation of the caustics image is dependent on the stresses in a structural member or machine component. Therefore, it is an ideal method to be used for when there is a SCF since high stress gradients produce large deflection of the light rays and an image with distinguishing characteristics. The advantage of caustics relative to other optical experimental techniques is that the same equipment can be used in either a reflection or transmission arrangement.

Setting

The proposed experiments introduce the students to a practical experimental technique of caustics for the understanding of stress concentrations in the vicinity of a hole in a plate. The determination of SCF is beyond the scope of this paper. The objectives of the experiments that use specimens in uniaxial tension are to show the students the following:

- The development of stress in a member. (ABET outcomes e, k)
- The region where the theoretical axial stress equation is valid. (ABET outcome a)
- Design of experiments in the stress analysis field. (ABET outcome b)
- The importance of understanding the optics laws and physics in the civil and mechanical engineering fields. (ABET outcomes e, k)
- Availability of experimental stress analysis techniques that are not part of a curriculum. (ABET outcome i)

Equipment setup and calibration

The schematic and suggested arrangements of the optical system for the experimental transmitted and reflected caustics are shown in figures 1 and 2, respectively. Briefly, a monochromatic and coherent light beam emitted from a point source He-Ne laser, which was widened by spatial lens, impinges normally on the specimen. The light beam has to fulfill only one, very important requirement, the light beam has to be parallel. The cost of equipment is less than $2000. To achieve this property, the light source must have the essential features of a point source. Divergent light is used primarily to enlarge the caustic image. The direct recording of the caustics image is possible in transmission arrangements as well as in reflection arrangements. This was accomplished by projecting the reflected light rays on screens. In the reflected case the light beam was slightly tilted with respect to the normal axis to the specimen in order to separate the reflected beam from the impinging beam. The rotation of the model produced a light beam that was not perpendicular to the specimen. This rotation created only a translation of the caustic without effecting the size, shape and relative position of the caustics. However, a rotation of the screen distorts the caustic image. Therefore, the screen should be always parallel to the model. The live caustic image can be captured by a camera.

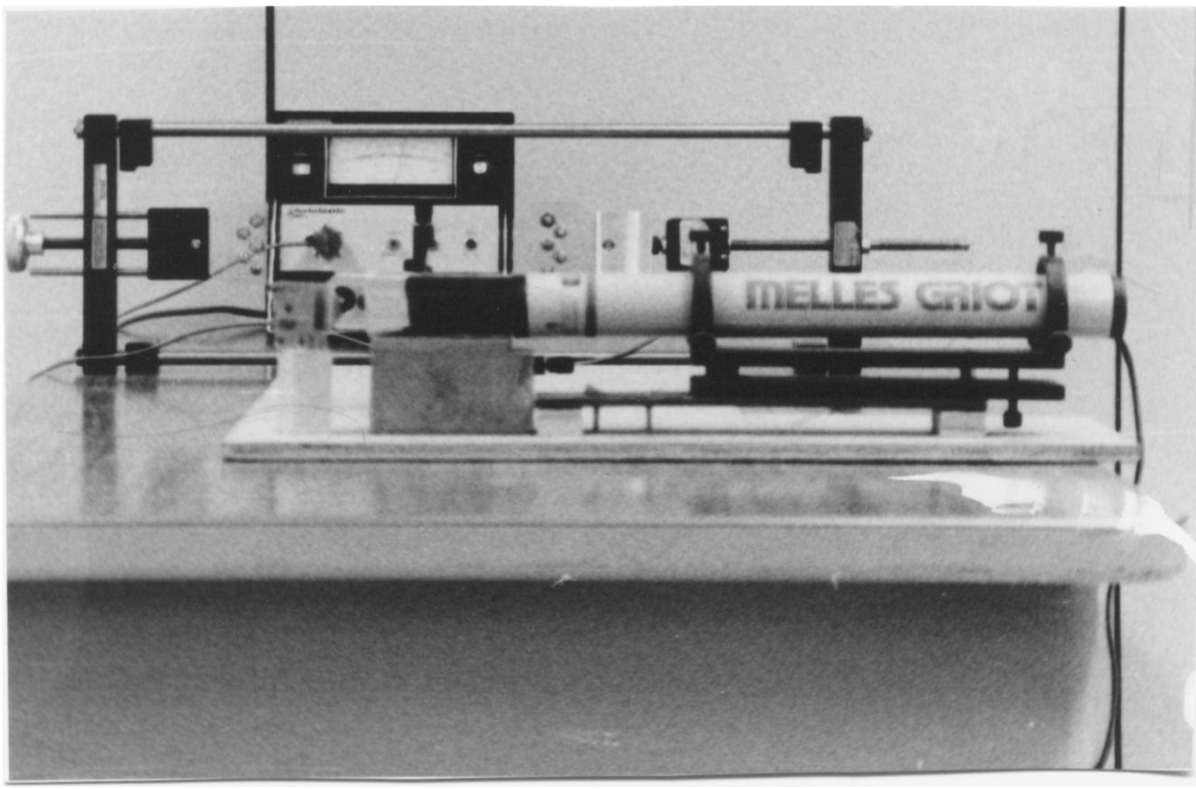

Fig. 1 Experimental transmitted caustic setup

The magnification factor (M) can be determined by using the following formula:

$$M = \frac{\text{any length in the reference plane}}{\text{corresponding length in the image plane}} \quad (1)$$

However, if the screen is not parallel to the model, an error in the evaluation of magnification factor is obtained. This can be eliminated by using the well known divergent light magnification factor law:

$$M = \frac{Z_o + Z_i}{Z_i} \quad (2)$$

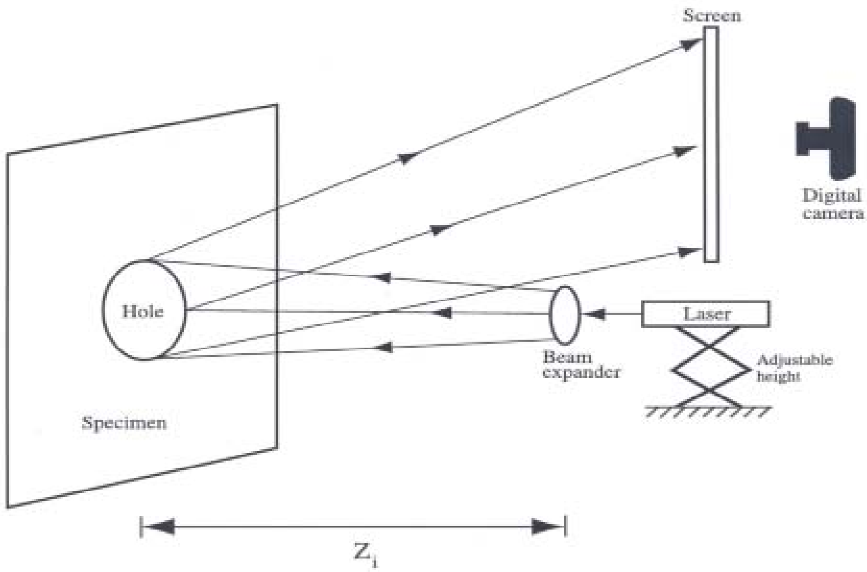

Fig. 2 Schematic reflected caustic experimental setup

Where Z_i is the distance between the divergent light source and the model, and Z_o is the distance between the model and screen. The difference between the calculated magnification factor from equations 1 and 2 indicates the extent of the errors. The main error is that the screen is not parallel to the model and can be easily corrected.

Stresses

The optical method of caustics is particularly convenient for the study of singular stress fields. The stress singularity of the elastic field is transformed into optical one represented by a highly illuminated surface that contains the necessary information for determining the applied stress in this study. The sum of in-plane principal stresses $\mathbf{F}_1$ and $\mathbf{F}_2$, in the vicinity of a hole of radius a, in terms of the polar coordinates, r and **2**, is given by Kirsch's solution as:

$$\sigma_1 + \sigma_2 = \sigma_r + \sigma_\theta = \frac{\sigma}{2}(2 + \frac{4a^2}{r^2}\cos 2\theta) \qquad (3)$$

It is important to remember that the above equation is for infinite plate. If a specimen that contains a central hole is loaded, the state of stress in the vicinity of the hole is much higher than the stresses along the rest of the specimen. If a monochromatic and coherent light beam impinges on the lateral face of transparent specimen, it is partially reflected from the front and eventually the rear face of the specimen. The deviation vector **D** resulting from the light ray transmitted or reflected from the area very close to a hole in an optically isotropic medium is shown in Fig. 3. The direction and magnitude of the deviation vector are correlated to the change in the optical path length **)** sand it is given by Eikonal[12] equation as:

$\mathbf{D} = Z_o \text{ grad } \mathbf{)} s(r, \mathbf{2})$ (4)

and

$\mathbf{)} s = ct (\mathbf{F}_1 + \mathbf{F}_2)$ (5)

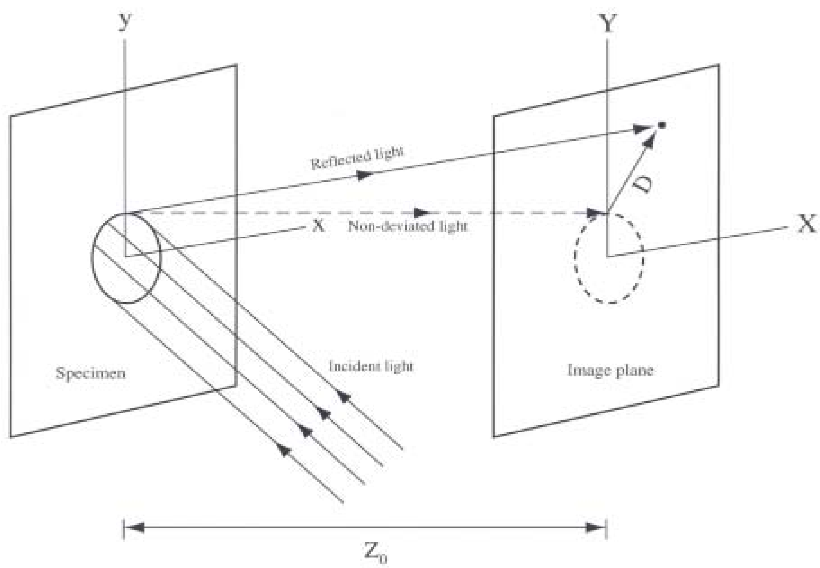

Fig. 3 Vector configuration

Where c is the stress optical constant of the Plexiglas and t is the thickness of the specimen. As an envelope, the caustic is a singular curve of the image equation and the necessary condition for the existence of such a singularity is that the Jacobian determinant (J) is zero. Substituting the above relationships into the image equation and the evaluation of J = 0, yields the initial curve equation[13]:

$r = (12\ \sigma\ Z_o c a^2)^{0.25}$ (6)

Relation (6) shows that the radius, which defines the envelope of the highly stressed zone of the specimen, is constant. Within the framework of linear elasticity, it should be observed that the deformed shape of the specimen surface in the vicinity of the hole is proportional to the applied force. Thus, the transmitted or reflected light patterns obtained from the specimen surface near the hole provide a direct measure of the desired stress. Mapping Eq. (4) becomes:

$x = M\,r\,(\cos \mathbf{2} + 1/3 \cos 3\mathbf{2})$ (7a)
$y = M\,r\,(\sin \mathbf{2} + 1/3 \sin 3\mathbf{2})$ (7b)

The range for the angle θ is from 0 to 2π and the theoretical caustics image is shown in figure 4.

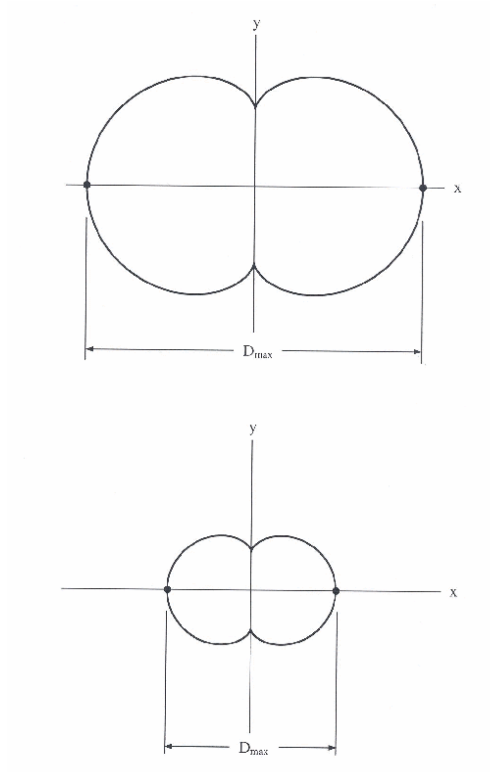

Fig. 4 Theoretical form of the caustic formed around a circular hole

Learning engineering optics

Most mechanical and civil engineering students learn the fundamental of optics in a physics class. Engineering students at Indiana University Purdue University Fort Wayne are required to take physics class: Electricity and Optics. The students are introduced to the geometrical and physical optics in a physics class and most likely will not use the knowledge learned in the rest of the curriculum. Perhaps, most mechanical and civil engineering students are not aware of the use of optics, such as in the area of stress analysis, in their fields of studies. The following two exercises enhance the student's learning of optics.

1. Study the effect of changing the magnification factor. This can be accomplished by varying the distance between the light source and the specimen, the distance between the model and the screen, or both. The effects of changing M using equations 2, 6, and 7 can be compared to the experimental set up on the optical bench. Also, the student can plot the maximum diameter versus the tilt angle, 15-20 degrees, of the screen.

2. Learn the diffraction effect in the engineering applications. Figures 5 and 6 show the experimental transmitted and reflected caustic images, respectively.

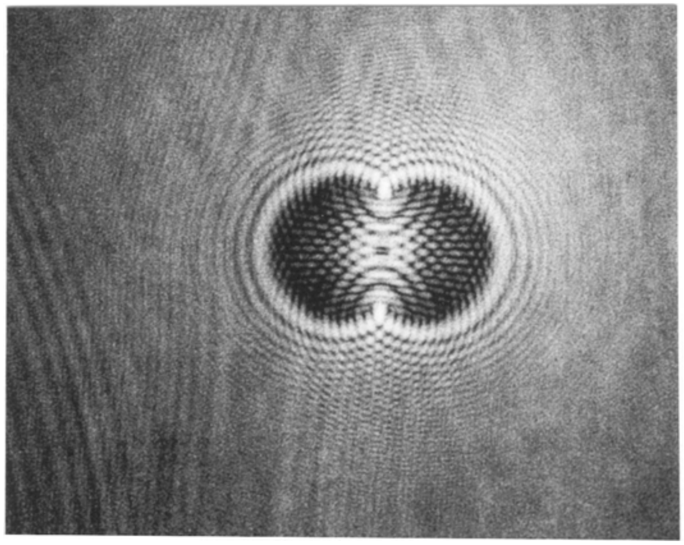

Fig. 5 Experimental transmitted caustic image

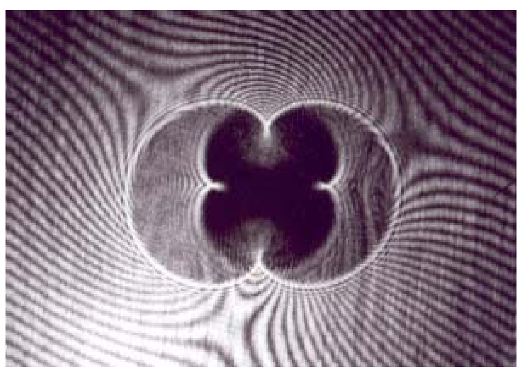

Fig. 6 Experimental reflected caustic image

Theoretically the relevant caustic line should be defined by the transition from the dark inner region to the bright rim of the caustic pattern. However, due to the light diffraction effects, the caustic rim will have a band shape rather than the theoretical fine line. The students should consider the inner diameter, outer diameter, and the average diameter of the caustics image in the calculations to appreciate the effects of diffraction.

Understanding stress concentration

Relation (6) shows that the radius, which defines the envelope of the highly loaded zone of the specimen, is constant. For isotropic and homogeneous specimens containing holes, it can be noticed that the initial curve is always circular when the specimens are subjected to an arbitrary in plane axial loads. Within the framework of linear elasticity, it should be observed that the deformed shape of the specimen surface in the vicinity of the hole is proportional to the axial force. Thus, the transmitted and reflected light patterns obtained from the specimen surface near the hole provide a direct measure of the applied stress.

1. Learning the effect of the load. The applied load affects the size of the optical image. On the top of figure 4, the theoretical applied load, and hence the stress, is twice that of the bottom one. The students can use different values of the applied stress σ in relations (6) and (7) to find the effect on the size of the lens, r, in the model. Also, the students can verify the relationship between the maximum transverse diameter of the caustic image and the initial curve, the lens. It can be shown that:

$$D_{max} = 2.67\ r \qquad (8)$$

2. Calculating the value of applied stress. Engineers are required to determine the actual loads that are transmitted to a structural member or a machine element in many practical design and analysis situations. The interaction between members and elements as well as the effect of assembly stresses necessitate the performance of this task experimentally. The need for the use of the method of caustics to measure the applied load in polymeric members is discussed in reference[13]. Substituting equation (8) into the initial curve equation leads to

$$\sigma = \frac{(D_{max})^4}{610 Z_o c a^2 d M^3} \qquad (9)$$

Equation (9) represents the basic relationship underlying the optical method of measuring the stress. It is clear that the applied axial stress can be determined simply by measuring the transmitted and reflected caustic maximum diameters since d, a, c, A_o, Z_o, and **8** are known parameters in a typical experiment.

A small drill was used to slowly bore the holes in the models. No residual stresses were noticed in the experiments. The residual stress can be detected from the small pseudo caustic it produces. All models were taken from the same plexiglas sheet. This experimentally applied stress should be compared to the known applied stress (σ = P/A). Some of the results obtained are shown in Table 1. The difference between the experimental and actual stresses is due to the stress concentration factor.

Stress concentration

Due to the high stress concentration in the region surrounding the circular hole, both the thickness and refractive index of the material change. As a consequence, the area surrounding the circular hole acts similar to a divergent lens which is also called the initial curve. Due to the presence of lens effect very close to a stress concentration, the transmitted or reflected light rays are deviated outwards. These deviated rays are concentrated along a strongly illuminated surface in space, which forms the caustic surface.

By observing the caustics image, one can determine the region that is affected by the presence of stress concentration. The effect of the hole size upon the determination of the calculated applied stress can be investigated by varying the ratio of the hole size a to the plate width W. Also, each model can be subjected to five different loads. The students learn the effect of the interaction of the finite width of the plate and the hole. At this stage, they can establish the validity of relation (3) in regard to the infinity issue. Thus, students can design caustic experiments with specified magnification factor and valid a/W ratio to measure a physical quantity.

Table 1. Comparison between actual and experimental results

a/W	Applied Stress MPa	Optical Stress MPa	% Difference
0.01	8.81	8.72	0.82
0.018	7.33	7.14	2.70
0.025	8.43	8.38	0.67
0.031	6.28	6.17	1.83
0.05	4.69	5.13	8.84
0.056	6.40	6.78	5.88
0.65	6.03	7.0	14.79

Conclusion

Stress analysis experiments and exercises are proposed for the enhancement of learning some of the engineering optics and stress concentration fundamentals. The experiments described in this paper are a valuable addition to an undergraduate mechanical and civil engineering laboratory content. One of the objectives of the proposed experiments will help the students to recognize the need for life-long learning.

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